Lecture 7 - The Parton Model

- Parton Distribution Functions
- Bjorken Scaling
- Valence and Sea Quarks
- Neutrino Deep Inelastic Scattering
- Gluons & Scaling Violation
Parton-level Scattering

At high $Q^2$ the underlying process is “elastic” scattering off a “pointlike” parton of mass $m$ and charge $Q_i$:

$$2m\nu + q^2 = 0$$

$$x = \frac{Q^2}{2m_p\nu} = \frac{m}{m_p}$$

$$\frac{d\sigma}{d\Omega} = \frac{Q_i^2 \alpha^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \frac{E_3}{E_1} \left( \cos^2 \frac{\theta}{2} - \frac{q^2}{2m^2} \sin^2 \frac{\theta}{2} \right)$$
Parton Distribution Functions

The probability of a parton of type $i$ having a fraction $x$ of the proton energy is the parton distribution function (pdf) $f_i(x)$.

Pointlike partons have structure functions that are $\delta$ functions:

$$W_1^i = \frac{Q_i^2}{4m^2} \delta(\nu - \frac{Q_i^2}{2m})$$
$$W_2^i = \delta(\nu - \frac{Q_i^2}{2m})$$

The proton structure functions are sums over the parton pdfs:

$$W_1^p = \frac{F_1(x)}{m_p}$$
$$F_1^p(x) = \frac{1}{2} \sum_i Q_i^2 f_i(x)$$

$$W_2^p = \frac{F_2(x)}{\nu}$$
$$F_2^p(x) = \sum_i xQ_i^2 f_i(x)$$

For $S = 1/2$ quarks must have: $2xF_1^p(x) = F_2^p(x)$
Valence Quark Fractions

Proton structure function from valence quarks $uud$:

$$F_2^p(x) = \sum_i xQ_i^2 f_i(x) = \frac{4}{9} xu(x) + \frac{1}{9} xd(x)$$

Neutron has valence quarks $udd$ ⇒ interchange $u(x)$ and $d(x)$

$$F_2^n(x) = \sum_i xQ_i^2 f_i(x) = \frac{4}{9} xd(x) + \frac{1}{9} xu(x)$$

Integrate measured form factors $F_2$ in electron DIS:

$$\int_0^1 F_2^p(x)dx = \frac{4}{9} f_u + \frac{1}{9} f_d = 0.18$$

$$\int_0^1 F_2^n(x)dx = \frac{4}{9} f_d + \frac{1}{9} f_u = 0.12$$

where $f_u = \int_0^1 xu(x)dx = 0.36$ and $f_d = \int_0^1 xd(x)dx = 0.18$

Valence quarks are only 54% of the proton!
Valence and Sea Quarks in Protons

Valence quarks are \( u \) and \( d \) at high \( x \)

Sea contains quark-antiquark pairs \( u\bar{u}, d\bar{d}, s\bar{s} \) at low \( x \)
Neutrino (Anti)quark Scattering

Cross-sections for charged current (CC) scattering of $\nu_\mu$ at the parton level (via virtual $W^+$ or $W^-$ exchange):

$$\frac{d\sigma}{dy}(\nu_\mu d \rightarrow \mu^- u) = \frac{d\sigma}{dy} (\bar{\nu}_\mu \bar{d} \rightarrow \mu^+ \bar{u}) = \frac{G_F^2 x s}{\pi}$$

$$\frac{d\sigma}{dy}(\bar{\nu}_\mu u \rightarrow \mu^+ d) = \frac{d\sigma}{dy} (\nu_\mu \bar{u} \rightarrow \mu^- \bar{d}) = \frac{G_F^2 x s}{\pi} (1 - y)^2$$

Virtual $W^\pm$ boson distinguishes between up and down quarks
Neutrinos and antineutrinos distinguish quarks and antiquarks

Neutrino cross-sections are very small $\approx 10^{-42} m^2$ at $s = 1 GeV^2$
They increase linearly with the CM energy-squared $s$
Quark and Antiquark Distributions

Neutrino-nucleon scattering can be written in terms of parton density functions for quarks, \( Q(x) \), and antiquarks, \( \bar{Q}(x) \)

\[
\frac{d\sigma^{CC}}{dxdy}(\nu N) = \frac{G_F^2 sx}{2\pi} \left[ Q(x) + \bar{Q}(x)(1 - y)^2 \right]
\]

\[
\frac{d\sigma^{CC}}{dxdy}(\bar{\nu} N) = \frac{G_F^2 sx}{2\pi} \left[ \bar{Q}(x) + Q(x)(1 - y)^2 \right]
\]

Ratios of total cross-sections for fermion (antifermion) \( f(\bar{f}) \):

\[
R = \frac{\sigma(\bar{\nu} f)}{\sigma(\nu f)} = \frac{1}{3} \quad \bar{R} = \frac{\sigma(\bar{\nu} \bar{f})}{\sigma(\nu \bar{f})} = 3
\]

Fraction of sea antiquarks is found to be:

\[
\frac{\bar{Q}}{Q} = \frac{3R - 1}{3 - R} \approx 0.1
\]
\[ \frac{d\sigma^{CC}}{dxdy}(\nu N) = \frac{G_F^2}{2\pi} \left[ (1 - y)F_2^\nu(x) + y^2 x F_1^\nu(x) \pm y \left( 1 - \frac{y}{2} \right) x F_3^\nu \right] \]

The \( \pm \) sign refers to \( \nu \) and \( \bar{\nu} \) scattering respectively.

The form factor \( F_2^\nu \) in neutrino DIS is given by:

\[ F_2^{\nu p}(x) = 2x [d(x) + \bar{u}(x)] \quad F_2^{\nu n}(x) = 2x [u(x) + \bar{d}(x)] \]

For a heavy nucleus \( N \) the ratio of \( \nu \) to \( e \) structure functions:

\[ F_2^{\nu N} = \frac{18}{5} F_2^{e N} \quad \text{if} \quad Q_u = +2/3e \quad Q_d = -1/3e \]

Total number of valence quarks obtained by integrating \( F_3 \):

\[ \int_0^1 F_3^{\nu N}(x)dx = \int_0^1 [u(x) - \bar{u}(x) + d(x) - \bar{d}(x)] \, dx = 3 \]
Finding the Gluons

A virtual photon can **hard scatter** off a gluon inside a proton:

and there is **gluon emission** from a scattered quark:

Gluon emission and hard scattering lead to **scaling violation**
Evidence for Gluon Emission

The hadronic jet in DIS has a larger transverse momentum, $p_T^2$, than would be expected from lepton-quark scattering.
Bjorken Scaling:
Structure function $F_2(x)$ is independent of $Q^2$

HERA measured collisions between 30 GeV electrons and 830 GeV protons

Scaling breaks down at low $x$
Structure Functions with Gluons

Gluons change kinematics of lepton-quark scattering

Change in the structure function $F_2$ due to gluon emission:

$$F_2(x, Q^2) = xQ_i^2[q(x) + \Delta q(x, Q^2)]$$

$$\Delta q = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} q(y) P(z) \log \left( \frac{Q^2}{\mu^2} \right)$$

$P$ is the probability of a gluon emission that changes the parton energy from $y$ to $x$, i.e. reduces it by a fraction $z = x/y$

Change in $F_2$ due to hard scattering:

$$\Delta F_2(\gamma^* g \rightarrow q\bar{q}) = Q_i^2 \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} g(y) P(z) \log \left( \frac{Q^2}{\mu^2} \right)$$

where $g(y)$ is the gluon parton density function
H1 1994

NLO QCD fit

\[ Q^2 = 20 \text{GeV}^2 \]

\[ Q^2 = 5 \text{GeV}^2 \]

full error