Lecture 12
Weak Decays of Hadrons

• $\pi^+$ and $K^+$ decays
• Semileptonic decays
• Hyperon decays
• Heavy quark decays
• Rare decays
• The Cabibbo-Kobayashi-Maskawa Matrix
Charged Pion Decay

$\pi^+$ decay by annihilation of the $u\bar{d}$ into a $W^+$ boson
$\pi^-$ decay by annihilation of the $d\bar{u}$ into a $W^-$ boson:

The dominant decay mode is to a muon and a neutrino

$\pi^+ \rightarrow \mu^+ \nu_\mu$ \hspace{1cm} $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$

Why not to an electron and a neutrino?
Charged Pion Lifetime

The matrix element for the weak decay is:

\[ \mathcal{M} = \frac{G_F}{\sqrt{2}} f_\pi q^\mu \left( \bar{u}_\mu \gamma^\mu \frac{1}{2} (1 - \gamma^5) u_\nu \right) \]

where \( f_\pi \) is the charged pion decay constant (probability that quark-antiquark annihilate inside pion)

The matrix element squared in the rest frame of the pion is:

\[ |\mathcal{M}|^2 = 4 G_F^2 f_\pi^2 m_\mu^2 [p_3 \cdot p_4] \]

\[ \Gamma_\pi = \frac{1}{\tau_\pi} = \frac{G_E^2}{8 \pi} f_\pi^2 m_\pi m_\mu \left( 1 - \frac{m_\mu^2}{m_\pi^2} \right)^2 \]

Charged pion mass, lifetime and decay constant:

\[ m_{\pi^+} = 139.6 \text{MeV} \quad \tau_{\pi^+} = 26 \text{ns} \quad f_\pi = 131 \text{MeV} \]
Helicity Suppression in $\pi^+$ Decays

The pion has $J=0$, so the $\mu^+$ and $\nu$ have the same helicities:

The decay to an electron and a neutrino is \textbf{helicity suppressed}

$$R = \frac{\Gamma(\pi \rightarrow e\nu)}{\Gamma(\pi \rightarrow \mu\nu)} = \frac{m_e^2}{m_\mu^2} \frac{1}{(1 - m_\mu^2/m_\pi^2)^2} = 1.275 \times 10^{-4}$$

Experimental result proves $V - A$ theory of weak interactions:

$$R = (1.267 \pm 0.023) \times 10^{-4}$$
Charged Kaon Decay

The main decay modes of the charged Kaon are:

- A leptonic decay (similar to $\pi^+ \to \mu^+ \nu_\mu$)

  \[ \mathcal{B}(K^+ \to \mu^+ \nu_\mu) = 63.4\% \]

- A semileptonic decay (with equal amounts of $\ell = e, \mu$)

  \[ \mathcal{B}(K^+ \to \pi^0 \ell^+ \nu_\ell) = 8.1\% \]

- Weak hadronic decays to pions

  \[ \mathcal{B}(K^+ \to \pi^+ \pi^0) = 21.1\% \quad \mathcal{B}(K^+ \to \pi^+ \pi^+ \pi^-) = 5.6\% \]

Charged Kaon mass, lifetime and decay constant:

\[ m_{K^+} = 494\text{MeV} \quad \tau_{K^+} = 12\text{ns} \quad f_K = 160\text{MeV} \]

$f_K \neq f_\pi$ is example of breaking of $SU(3)$ flavour symmetry
Neutral Kaon Decays

For neutral Kaons the decay eigenstates $K_S$ and $K_L$ are not equal to the flavour eigenstates $K^0$ and $\bar{K}^0$ (more in Lecture 13)

The main decay modes are:

$$\mathcal{B}(K_L \rightarrow \pi^- \ell^+ \nu_\ell) = 67.5\% \quad (\ell = e, \mu)$$

$$\mathcal{B}(K_L \rightarrow \pi^+ \pi^- \pi^0) = 12.6\% \quad \mathcal{B}(K_L \rightarrow \pi^0 \pi^0 \pi^0) = 19.6\%$$

$$\mathcal{B}(K_S \rightarrow \pi^+ \pi^-) = 69.2\% \quad \mathcal{B}(K_S \rightarrow \pi^0 \pi^0) = 30.7\%$$

Neutral Kaon mass and lifetimes:

$$m_{K^0} = 498\text{MeV} \quad \tau_S = 0.09\text{ns} \quad \tau_L = 51\text{ns}$$

There are two very different lifetimes (short and long)!
The Cabibbo Angle

The couplings of the $W$ boson to $u\bar{d}$ and $u\bar{s}$ quarks are described by the **Cabibbo angle** $\theta_C$

Weak coupling becomes $G_F \Rightarrow G_F V_{ud}$ or $G_F V_{us}$

\[
\begin{pmatrix}
  d' \\
  s'
\end{pmatrix}
= 
\begin{pmatrix}
  V_{ud} & V_{us} \\
  V_{cd} & V_{cs}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s
\end{pmatrix}
= 
\begin{pmatrix}
  \cos \theta_C & \sin \theta_C \\
  -\sin \theta_C & \cos \theta_C
\end{pmatrix}
\begin{pmatrix}
  d \\
  s
\end{pmatrix}
\]

Interpret this as a rotation matrix between the flavour eigenstates $d, s$ and the weak eigenstates $d', s'$

The Cabibbo angle is measured to be:

$\theta_C = 12.7^\circ$ \hspace{1cm} $\sin \theta_C = 0.220$ \hspace{1cm} $\cos \theta_C = 0.976$
Semileptonic Decays & Selection Rules

The matrix element for semileptonic Kaon decays is:

\[ \mathcal{M} \propto f_+ \left[ (p_K + p_\pi) \mu \bar{\ell} \gamma^\mu (1 + \gamma^5) \nu \right] + f_- \left[ m_\ell \bar{\ell} (1 + \gamma^5) \nu \right] \]

where \( f_+ \) and \( f_- \) are semileptonic decay form factors which describe the hadronic transitions \( K \rightarrow \pi \)

The \( f_- \) term multiplying \( m_\ell \bar{\ell} \) is negligible for electrons

Semileptonic kaon decays obey the selection rules:

\[ \Delta I = \Delta I_3 = \frac{1}{2} \quad \Delta Q = \Delta S = 1 \]

Hadronic kaon decays obey the selection rules:

\[ \Delta I = 1/2, 3/2 \quad (1/2 \text{ preferred}) \quad \Delta I_3 = \frac{1}{2} \quad \Delta S = 1 \]
Hyperon Decays

Baryons containing strange quarks are known as hyperons

With one exception they all have weak decays:

<table>
<thead>
<tr>
<th>Hyperon</th>
<th>Quark Content</th>
<th>Decay Modes</th>
<th>Lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Λ</td>
<td>uds</td>
<td>pπ⁻, nπ⁰</td>
<td>0.26 ns</td>
</tr>
<tr>
<td>Σ⁺</td>
<td>uus</td>
<td>pπ⁰, nπ⁺</td>
<td>0.80 ns</td>
</tr>
<tr>
<td>Σ⁰</td>
<td>uds</td>
<td>Λγ</td>
<td>7 × 10⁻²⁰ s</td>
</tr>
<tr>
<td>Σ⁻</td>
<td>dds</td>
<td>nπ⁻</td>
<td>0.15 ns</td>
</tr>
<tr>
<td>Ξ⁰</td>
<td>uss</td>
<td>Λπ⁰</td>
<td>0.29 ns</td>
</tr>
<tr>
<td>Ξ⁻</td>
<td>dds</td>
<td>Λπ⁻</td>
<td>0.16 ns</td>
</tr>
<tr>
<td>Ω⁻</td>
<td>sss</td>
<td>ΛK⁻, Ξ⁰π⁻</td>
<td>0.08 ns</td>
</tr>
</tbody>
</table>

Lifetimes ≈ 10⁻¹⁰ s     Decay Lengths ≈ 1 cm are observable
Heavy Quark Decays

Charm quark decays are mainly $c \rightarrow s$ (a few percent $c \rightarrow d$)

Examples $D \rightarrow K \ell \nu$, $D \rightarrow K \pi$, $D \rightarrow K \pi \pi$
Lifetimes $\tau_{D^+} = 1.04\, \text{ps}$, $\tau_{D^0} = 0.41\, \text{ps}$

Bottom quark decays are mainly $b \rightarrow c$ (a few percent $b \rightarrow u$)

Examples $B \rightarrow D \ell \nu$, $B \rightarrow D \pi$, $B \rightarrow J/\psi K^0_S$
Lifetimes $\tau_{B^+} = 1.64\, \text{ps}$, $\tau_{B^0} = 1.53\, \text{ps}$

The proper decay lengths of $b$ and $c$ hadrons are $100 - 500\, \mu\text{m}$

The top quark decays almost completely $t \rightarrow b$

*Its lifetime is too short to form hadrons!*
Semileptonic & Rare $b$ Decays

Inclusive and exclusive semileptonic decays (BaBar/Belle)

\[ \mathcal{B}(b \rightarrow c\ell\nu_\ell) = (10.75 \pm 0.15)\% \]

\[ \mathcal{B}(B \rightarrow D\ell\nu_\ell) = (2.2 \pm 0.1)\% \quad \mathcal{B}(B \rightarrow D^*\ell\nu_\ell) = (5.6 \pm 0.5)\% \]

\[ \mathcal{B}(b \rightarrow u\ell\nu_\ell) = (1.3 \pm 0.1) \times 10^{-3} \quad \mathcal{B}(B \rightarrow \pi\ell\nu_\ell) = (1.4 \pm 0.1) \times 10^{-4} \]

Determine CKM couplings $V_{cb}$ and $V_{ub}$

Flavour-changing neutral currents (CLEO/BaBar/Belle)

\[ \mathcal{B}(b \rightarrow s\gamma) = (3.5 \pm 0.3) \times 10^{-4} \quad \mathcal{B}(B \rightarrow K^*\gamma) = (4.5 \pm 0.2) \times 10^{-5} \]

These are second-order weak decays (“penguin” loops)

Set limits on many New Physics models
Leptonic Decays of Heavy Quarks

Leptonic $D$ decays measured by CLEO-c experiment (2008)

\[
\mathcal{B}(D^+ \rightarrow \mu^+ \nu_{\mu}) = (4.4 \pm 0.6) \times 10^{-4}
\]

\[
\mathcal{B}(D_s^+ \rightarrow \mu^+ \nu_{\mu}) = (6.2 \pm 0.6) \times 10^{-3}
\]

\[
\mathcal{B}(D_s^+ \rightarrow \tau^+ \nu_{\tau}) = (6.6 \pm 0.6) \times 10^{-2}
\]

Leptonic $B$ decay measured by BaBar/Belle experiments (2006)

\[
\mathcal{B}(B^+ \rightarrow \tau^+ \nu_{\tau}) = (1.4 \pm 0.4) \times 10^{-4}
\]

Determine decay constants $f_D, f_{Ds}, f_B$

Set limits on possible charged Higgs couplings
Cabibbo-Kobayashi-Maskawa Matrix

Kobayashi & Maskawa awarded Nobel prize in October 2008!

By extension from the Cabibbo angle, the full description of weak decays of quarks needs the $3 \times 3$ CKM matrix:

$$
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}
= 
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
$$

The CKM matrix is unitary, and its elements satisfy:

$$
\sum_i V_{ij}^2 = 1 \quad \sum_j V_{ij}^2 = 1 \\
\sum_i V_{ij} V_{ik} = 0 \quad \sum_j V_{ij} V_{kj} = 0
$$

The CKM matrix can be written in terms of just four parameters
CKM Parametrizations

With three angles $s_i = \sin \theta_i$, $c_i = \cos \theta_i$, and a complex phase $\delta$:

\[
\begin{pmatrix}
    c_1 & s_1 c_3 & s_1 s_3 \\
- s_1 c_3 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\
 s_1 s_2 & - c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & - c_1 s_2 s_3 + c_2 c_3 e^{i\delta}
\end{pmatrix}
\]

Wolfenstein parametrisation is expansion in powers of $\lambda = \sin \theta_C$:

\[
\begin{pmatrix}
    1 - \lambda^2 / 2 & \lambda & A \lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2 / 2 & A \lambda^2 \\
A \lambda^3(1 - \rho - i\eta) & A \lambda^2 & 1
\end{pmatrix}
\]
Measurements of CKM Elements

- $V_{ud} = 0.976$ from pion and nuclear $\beta$ decays
- $V_{us} = 0.220$ from Kaon and Hyperon decays
- $V_{cs} = 0.97 \pm 0.12$ from $D \to K\ell\nu$ semileptonic decays
- $V_{cd} = 0.224 \pm 0.012$ from neutrino production of charm
- $V_{cb} = 0.0420 \pm 0.0007$ from $b \to c\ell\nu$ semileptonic decays
- $V_{ub} = 0.0044 \pm 0.0004$ from $b \to u\ell\nu$ semileptonic decays
- $V_{td}$ and $V_{ts}$ are measured in $B$ meson mixing (Lecture 13)
- $V_{tb} \approx 1$ is measured in top decays at the Tevatron

Wolfenstein parameters:

$\lambda = 0.2265 \pm 0.0025 \quad A = 0.80 \pm 0.03 \quad \rho = 0.19 \pm 0.08 \quad \eta = 0.36 \pm 0.04$