Particle Interactions – Examples

Ionization:
- Charged Particle → Electron

Pair production:
- Photon → Positron + Electron

Compton scattering:
- Electron + Photon → Electron + Photon

Energy Loss by Ionization – $dE/dx$

For now assume: $Mc^2 \gg mec^2$

i.e. energy loss for heavy charged particles
[$dE/dx$ for electrons more difficult ...]

Interaction dominated by elastic collisions with electrons ...

\[
- \langle \frac{dE}{dx} \rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\text{max}}}{I^2} - \beta^2 - \frac{\delta(\beta \gamma)}{2} \right]
\]

Bethe-Bloch Formula

$\propto \frac{1}{\beta^2} \cdot \ln(\text{const} \cdot \beta^2 \gamma^2)$
Bethe-Bloch – Classical Derivation

Particle with charge \( ze \) and velocity \( v \) moves through a medium with electron density \( n \).

Electrons considered free and initially at rest.

Momentum transfer:

\[
\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dt}{dx} dx = \int F_\perp \frac{dx}{v}
\]

\[
= \int_{-\infty}^{\infty} \frac{z e^2}{(x^2 + b^2)} \cdot \frac{b}{\sqrt{x^2 + b^2}} \cdot \frac{1}{v} \, dx = \frac{z e^2 b}{v} \left[ \frac{x}{b^2 \sqrt{x^2 + b^2}} \right]_{-\infty}^{\infty} = \frac{2 z e^2}{bv}
\]

Symmetry!

More elegant with Gauss law:
[infinite cylinder; electron in center]

\[
\int E_\perp (2\pi b) \, dx = 4\pi (ze) \rightarrow \int E_\perp \, dx = \frac{2ze}{b}
\]

and then ...

\[
\left\{ \begin{align*}
F_\perp &= e E_\perp \\
\Delta p_\perp &= e \int E_\perp \frac{dx}{v} = \frac{2 z e^2}{bv}
\end{align*} \right.
\]
Energy transfer onto single electron for impact parameter $b$:

$$\Delta E(b) = \frac{\Delta p^2}{2m_e}$$

Consider cylindric barrel $\Rightarrow N_e = n \cdot (2\pi b) \cdot db \cdot dx$

Energy loss per path length $dx$ for distance between $b$ and $b+db$ in medium with electron density $n$:

Energy loss!

$$-dE(b) = \frac{\Delta p^2}{2m_e} \cdot 2\pi nb \cdot db \cdot dx = \frac{4z^2e^4}{2b^2v^2m_e} \cdot 2\pi nb \cdot db \cdot dx = \frac{4\pi n z^2e^4}{m_ev^2} \cdot \frac{db}{b} \cdot dx$$

Diverges for $b \rightarrow 0$; integration only for relevant range $[b_{\text{min}}, b_{\text{max}}]$:

$$-\frac{dE}{dx} = \frac{4\pi n z^2e^4}{m_ev^2} \cdot \int_{b_{\text{min}}}^{b_{\text{max}}} \frac{db}{b} = \frac{4\pi n z^2e^4}{m_ev^2} \ln \frac{b_{\text{max}}}{b_{\text{min}}} \ln \frac{b_{\text{max}}}{b_{\text{min}}}$$

Bethe-Bloch – Classical Derivation

Bohr 1913
Bethe-Bloch – Classical Derivation

Bohr 1913

Determination of relevant range \([b_{\text{min}}, b_{\text{max}}]\):  
[Arguments: \(b_{\text{min}} > \lambda_e\), i.e. de Broglie wavelength; \(b_{\text{max}} < \infty\) due to screening ...]

\[
b_{\text{min}} = \lambda_e = \frac{h}{p} = \frac{2\pi \hbar}{\gamma m_e v}
\]

Use Heisenberg uncertainty principle or that electron is located within de Broglie wavelength ...

\[
b_{\text{max}} = \frac{\gamma v}{\langle \nu_e \rangle}; \quad \left[ \gamma = \frac{1}{\sqrt{1 - \beta^2}} \right]
\]

Interaction time \((b/v)\) must be much shorter than period of the electron \((\gamma/\nu_e)\) to guarantee relevant energy transfer ...  
[adiabatic invariance]

\[- \frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e c^2 \beta^2} n \cdot \ln \frac{m_e c^2 \beta^2 \gamma^2}{2\pi \hbar \langle \nu_e \rangle}\]

Deviates by factor 2 from QM derivation

Electron density: \(n = N_A \cdot \rho \cdot \frac{Z}{A}!!\)

Effective Ionization potential: \(I \sim \hbar \langle \nu_e \rangle\)
Bethe-Bloch Formula

\[ -\langle \frac{dE}{dx} \rangle = K \frac{z^2}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \left( \frac{2m_e c^2 \beta^2 \gamma^2 T_{\text{max}}}{I^2} \right) - \beta^2 - \frac{\delta(\beta \gamma)}{2} \right] \]

\[ K = 4\pi N_A r_e^2 m_e c^2 = 0.307 \text{ MeV g}^{-1} \text{ cm}^2 \]
\[ T_{\text{max}} = \frac{2m_e c^2 \beta^2 \gamma^2}{(1 + 2 \gamma m_e/M + (m_e/M)^2)} \]

\[ z : \text{ Charge of incident particle} \]
\[ M : \text{ Mass of incident particle} \]
\[ Z : \text{ Charge number of medium} \]
\[ A : \text{ Atomic mass of medium} \]
\[ I : \text{ Mean excitation energy of medium} \]
\[ \delta : \text{ Density correction [transv. extension of electric field]} \]

Validity:
\[ .05 < \beta \gamma < 500 \]
\[ M > m_\mu \]

\[ N_A = 6.022 \cdot 10^{23} \]
[Avogadro's number]
\[ r_e = \frac{e^2}{4\pi \varepsilon_0 m_e c^2} = 2.8 \text{ fm} \]
[Classical electron radius]
\[ m_e = 511 \text{ keV} \]
[Electron mass]
\[ \beta = \frac{v}{c} \]
[Velocity]
\[ \gamma = (1-\beta^2)^{-2} \]
[Lorentz factor]
Minimum ionizing particles (MIP): $\beta \gamma = 3-4$

$dE/dx$ falls $\sim \beta^{-2}$; kinematic factor
[precise dependence: $\sim \beta^{-5/3}$]

$dE/dx$ rises $\sim \ln(\beta \gamma)^2$; relativistic rise
[rel. extension of transversal E-field]

Saturation at large $(\beta \gamma)$ due to density effect (correction $\delta$)
[polarization of medium]

Units: MeV g$^{-1}$ cm$^2$

MIP looses $\sim 13$ MeV/cm
[density of copper: 8.94 g/cm$^3$]
Understanding Bethe-Bloch

1/$\beta^2$-dependence:

Remember:

$$\Delta p_\perp = \int F_\perp \, dt = \int F_\perp \frac{dx}{v}$$

i.e. slower particles feel electric force of atomic electron for longer time ...

Relativistic rise for $\beta\gamma > 4$:

High energy particle: transversal electric field increases due to Lorentz transform; $E_y \rightarrow \gamma E_y$. Thus interaction cross section increases ...

 Corrections:

- low energy : shell corrections
- high energy : density corrections
Understanding Bethe-Bloch

Density correction:

Polarization effect ...
[density dependent]

→ Shielding of electrical field far from particle path; effectively cuts of the long range contribution ...

More relevant at high $\gamma$ ...
[Increased range of electric field; larger $b_{\text{max}}$; ...]

For high energies:

$$\frac{\delta}{2} \rightarrow \ln(\hbar \omega / I) + \ln \beta \gamma - 1/2$$

Shell correction:

Arises if particle velocity is close to orbital velocity of electrons, i.e. $\beta c \sim v_e$.

Assumption that electron is at rest breaks down ...
Capture process is possible ...

Density effect leads to saturation at high energy ...

Shell correction are in general small ...
Energy Loss of Charged Particles

Dependence on

Mass A
Charge Z
of target nucleus

Minimum ionization:
ca. 1 - 2 MeV/g cm\(^{-2}\)
[H\(_2\): 4 MeV/g cm\(^{-2}\)]
Stopping Power at Minimum Ionization

Stopping power at minimum ionization for the chemical elements. The straight line is fitted for $Z > 6$.

A simple functional dependence on $Z$ is not to be expected, since $\langle -dE/dx \rangle$ also depends on other variables.
dE/dx and Particle Identification

Measured energy loss

[ALICE TPC, 2009]

Remember: dE/dx depends on $\beta$!