A COMPOSITE MODEL OF LEPTONS AND QUARKS

Michael A. SHUPE

Physics Department, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

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A heuristic model is presented, treating leptons and quarks as composites of spin 1/2 fields with charges 0, ±e/3. A distinguishability assumption leads to the emergence of three quark colors. The model is extended to consideration of the force-mediating bosons.

1. Introduction. In the past few years, experiment and theory have converged on a description of the submicroscopic world which is astounding in its simplicity and regularity. All known matter can be built from fundamental fermions — leptons and quarks — arranged and structured by four interactions which themselves are surprisingly similar. Gauge field theories may well describe all of these interactions, and their attendant mediating bosons.

The leptons have something akin to a “level” structure. Each charged lepton joins with a corresponding neutrino in obeying a number conservation principle. This effectively disconnects the six leptons into three groups of two, with considerable mass splitting between the charged leptons. The quarks also show level structure. Each level holds one quark of charge +2/3 and one of charge −1/3. The levels are split in mass by the same order of magnitude as the lepton splittings, and the levels would be disconnected — obeying a “quark number” conservation principle — if the Cabibbo angle were zero, instead of 15°.

To date, no direct connection has been observed between quarks and leptons. If the proton were observed to decay, the connection would be established. Nevertheless, all the observed similarities suggest that the leptons and quarks should be grouped together in a pattern of “generations”, split according to mass and number conservation. The first generation consists of e, νe, u, and d; the second, of μ, νμ, c, and s; and the third, of τ, ντ, t, and b. This grouping is the basis of the Georgi–Glashow scheme of grand unification based on SU(5) [1], and has been discussed in detail by Harari [2], among others. As with the periodic table or the hadron multiplets, there is a regular, repeating pattern with no indication that the states of large mass have been exhausted. There is strong suggestion, therefore, that a composite model should be attempted.

In the following sections we consider objections to the composite view, possible properties of constituents, states describing quarks and leptons, states describing the bosons which mediate the various interactions, problems occurring in the present model, and paths for ongoing investigation.

2. Barriers to the composite view. The main objection to describing the leptons and quarks as composite is the success of formulations which treat them as Dirac point particles. The Lamb shift, electron and muon magnetic moments, high-energy electromagnetic scattering, and Bjorken scaling (with QCD corrections) are all accounted for without introducing lepton or quark form factors.

There are two possible escapes from the uncomfortable prospect of doing physics at a point. The first is to assume that all present experiments are at pre-asymptotic values of momentum transfer; that, at smaller distances, quarks and leptons will start to show structure. The second escape is to envision a new kind
of composite structure consisting of pointlike constituents, tied together in an unfamiliar way such that the composite is also pointlike. Without being committed to either view, we can proceed to construct a composite model of leptons and quarks.

3. **Properties of the constituents.** We adopt a philosophy at this point of seeking constituents which are few in number and which depart little from the properties which characterize the fundamental fermions: spin, charge, mass, and the existence of antiparticles, among others. We describe this process initially by a list of assumptions.

**Assumption 1: Spin.** All leptons and quarks are spin 1/2. If the composite states are to be built using the superposition principle, this implies that at least one constituent field has to be fermionic. For simplicity, we will consider only fermions; in particular, only spin 1/2, the most fundamental representation of the rotation group. Forming the leptons and quarks by superposition will involve states such as \( t, tt, tt_-, tt_+ \ldots \) + permutations. We will ignore the first case since it is not composite.

**Assumption 2: Fixed configuration.** We will assume that the number of constituents in a quark or lepton is some fixed, small integer; equal for all quarks and leptons (at least within the first generation). This assumption is made by analogy with the situation at the quark level, where hadronic structure can be investigated by dealing with valence quark composites containing fixed numbers of quarks: \( qqq \) and \( \bar{q}q \).

**Assumption 3: Charge.** We will allow for charged and neutral constituents. A tantalizing peculiarity of the Gell–Mann quark model is the presence of fractional charge. Rather than think of “fractional couplings” to the photon, we choose to redefine the fundamental unit of charge to be \( \pm e/3 \). We therefore assume that lepton and quark constituents may be neutral or carry charge \( \pm e/3 \), the fundamental charge.

**Assumption 4: Antiparticles.** As in the case of the more familiar Dirac spinors, we assume that the positive and negative constituents are antiparticles of each other. Having allowed neutral constituents, we anticipate the problem of accounting for neutrinos and antineutrinos by hypothesizing that a neutral constituent \( \bar{0} \) and its antiparticle \( 0 \) exist.

**Assumption 5: Mass.** We assume that the constituents have no intrinsic mass. In brief, we assume that the masses of the first generation represent self-energy of the photon and gluon fields coupling to leptons and quarks, and that generation to generation splitting is due to some kind of internal “reconnection” of the constituents.

**Assumption 6: Excitation.** We will not allow for radial or angular excitation of the composite states – especially in considering the first generation. All spin configurations will be treated as \( S \) state.

This completes the list of assumptions required to start building composite structures. We refer to these fields hereafter as **quips** (quark inner parts, also in the sense of Webster’s alternate definition: “something curious or odd”). Quips will be denoted by the lower case \( a \), since \( q \) is in common use to denote quark fields. In summary, all quips are spin 1/2 and massless. The four quip fields \( a^+, a^-, a^0, \bar{a} \) have charges \( +e/3, -e/3, 0, \) and \( 0 \), respectively. \( (a^+, \bar{a}^-) \) and \( (a^0, \bar{a}^0) \) are particle–antiparticle pairs.

4. **Constructing the observed states.** We now attempt to reproduce the patterns of quark and lepton states, restricting ourselves to the first generation – \( e, \nu_e, u, \) and \( d \) – and choosing the simplest case allowed by spin, the three-quip states such as \( \bar{a}a\bar{a} \). Dropping the spin arrows for the time being, in table 1 we enumerate all possible charge states which can be formed from three quips. The charge and spin assumptions have led inevitably to the proper spectrum of charges, but the pattern looks too cluttered to associate with the first generation of leptons and quarks. An arbitrary rule is needed to suppress unwanted states:

**No mixing rule.** There is no mixing of quip and antiquip fields in forming lepton and quark composites. States may contain only mixtures of \( a^+ \) and \( a^0 \), or \( a^- \) and \( a^0 \).

Applying this rule reduces the number of charge states to eight, and they may be associated with the first-generation fermions, as shown in table 2.
Three-quip states look promising as a representation of the first-generation quarks and leptons. We note that there is an interesting difference between the charge states of leptons and quarks. The leptons contain three identical quips, whereas quarks contain one quip differing from the other two. This suggests a simple explanation for quark colors. If the ordering of the quips in a quark is important, there are three distinguishable ways of building a quark: baa, aba, and aab (b not to be confused with the bottom quark). There remains only one way of building a lepton: aaa. Assuming that we have stumbled across the color degree of freedom, we assign colors to each quip state: R = baa, B = aba, G = aab.

The hypothetical structure which makes this ordering of quips important will henceforth be referred to as “three-cell structure.” For reference, we display the charges and colors of the first generation of quarks and leptons in fig. 1, introducing a notation where each state is signified by a vertical box containing the three quips (labelled only by their charge). The quip positions in the box are their “cell” locations.

We conclude this section by begging a very large question: What leads to the formation of higher generations? We contend that the question cannot be answered until quip structure is much better understood. Rather than being excitations, these upper families may contain more quips, or be “reconnections” of the three-quip first-generation states. In the next section, we shift our focus to the gauge boson fields mediating the various forces.

5. The mediating bosons. In the same spirit as the rest of the investigation, we will explore the possibility that the mediating bosons can themselves be built from quips. Our prejudice is that the noted similarities of photons, gluons, and weak bosons — for instance, that they are all vector bosons and gauge fields — are reflections of a common property of their quip structures.

We begin by considering the photon. Briefly, we are seeking to construct a vector state from quips which couples with equal strength to a + and a -, but not to a 0 and a 0. The simplest vector state is the two-quip state with spins aligned: ↑↑ or ↓↓. The state must be charge neutral: a+a- or a0a0. The a+a- appears to be a likely state, if we reassess what is meant by “coupling”.

When a photon (containing a+a-) encounters a charged lepton or quark, one of its own charged quips will have the possibility of annihilating a corresponding
antiquip in the lepton or quark. When this occurs, the other quip brought in by the photon will remain to provide the outgoing quip needed by the lepton or quark. The photon has been absorbed! To consider a concrete example, we diagram the process $e^+ + \gamma \rightarrow e^+$ in fig. 2a. This view of the photon has ascribed the nebulous concept of coupling to the more primitive concept of pair annihilation (and for emission processes, pair creation).

Unfortunately, in the present infant state of the model two problems emerge, which may or may not turn out to be serious. The two-quip state which we have discarded, $a^0 a^0$, would make as good a “photon” as $a^+ a^-$, being neutral and having the capability to annihilate $a^0$ and $a^0$ quips in leptons and quarks. Under the influence of this field, neutrinos would have charge of magnitude $e$; $d$ quarks, $2e/3$; $u$ quarks, $e/3$; and electrons, 0. Not having a better understanding of quips, we can do nothing but invoke another rule:

**One-photon rule.** Two-quip vector states $a^+ a^-$ can form and propagate, but $a^0 a^0$ cannot.

The second serious problem with this two-quip view is the possibility of having photons couple to themselves. Clearly, more than simple consideration of charge states will be needed to investigate these problems.

We next consider possible quip structures of the weak vector bosons, beginning with the charged current case. Since $W^+$ and $W^-$ have charge $\pm e$, at least three quips must be used. Adding the spin constraint makes this number 4, 6, 8, ... even numbers of quips. In the process $u + W^- \rightarrow d$, all three quips are transformed as the flavor is changed. The same thing happens in the electron–neutrino transition. By analogy with the photon, the most direct way of doing this is by having the $W$ contain the appropriate combinations to annihilate incoming quips and provide outgoing ones. This can be accomplished by a six-quip state, the “second simplest” allowed by charge and spin. These are “amorphous” states with no three-cell structure, and hence no regard for quip order. Our $u-d$ transition then reduces to a process which balances like a chemical reaction: $u(a^0 a^+ a^-) + W^-(a^- a^- a^- a^0 a^0) \rightarrow d(a^- a^- a^- a^0 a^0)$, as shown in fig. 2b. The same quip assignment for the $W^-$ properly balances in the transition $e^+ + W^- \rightarrow \bar{\nu}_e$. The $W^+$ is found by charge conjugating the $W^-$ state: $W^+ = a^+ a^+ a^0 a^0 a^0$. As a reminder, these six-quip states must be vector states such as $\uparrow \uparrow \uparrow \downarrow \downarrow \downarrow$.

The neutral current case, $Z^0$, is not as simple as the $W$’s. We assume for regularity that the $Z$ is a six-quip structure like the $W$’s, and that coupling is through the same basic mechanism. We find that four $Z$ states are needed to account for neutral current processes such as $Z$ decay to $\nu_e \bar{\nu}_e (000000)$, $e^+ e^- (+-+-+-+-+-)$, $u\bar{u}$ (+-0000), and $d\bar{d}$ (+-0000). Thus the $Z^0$ must exist in any one of four quip states. Like the photon, it consists of equal mixtures of quips and antiquips. Unlike the photon, the states containing $a^0$ and $a^0$ are as common as those containing $a^+$ and $a^-$. There is no equivalent of the “one-photon rule” here. Also, complete processes such as $\nu_e + \bar{\nu}_e \rightarrow Z^0 \rightarrow e^+ + e^-$ indicate that the transition $a^0 a^0 \leftrightarrow a^+ a^-$ is allowed in this case!

We proceed, finally, to consideration of gluon structure. Color change looks, at the quip level, like simple cell exchange; yet we know to expect the non-abelian structure of QCD and the eight gluons corresponding to the Gell–Mann matrices, $\lambda_i, i = 1–8$. By analogy with quarks and leptons, gluons can be represented in

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**Fig. 2.** (a) A positron absorbs a photon, as viewed at the quip level. (b) A $u$ quark absorbs a $W^-$ boson, as viewed at the quip level.
a vertical box notation, but with two quips per cell: one to annihilate the incoming quip and one to provide the outgoing quip (absorption). Like weak vector bosons, gluons in this model are six-quip composites. Unlike the weak bosons, gluons are not amorphous, but have column three-cell structure like quarks and leptons, and row structure like the other vector bosons.

Two sets of gluons are presented in fig. 3. The first presents gluons associated with $u-u$ or $u-\bar{u}$ processes, the second, with $d-d$ and $d-\bar{d}$ processes. The quip on the right in each cell annihilates the incoming quip, while the one on the left provides the outgoing quip (usual ordering for such operators). It is understood that linear combinations of $RR$, $BB$, and $GG$ are to be taken, and $RR + BB + GG$ cast out as color singlet.

Looking more closely at the quip content, we note that in the off-diagonal gluons, quip combinations which raise (+0) or lower (−0) charge are found in the same cell of a given gluon in both the $u$ and $d$ quark tables, but in reverse order. As in the photon case, the quip which annihilates is determined by the antiquip which is encountered. Neutral combinations in the off-diagonal gluons (+− and 00) differ between the $u$ and $d$ table. For a $u$ and $d$ quark to exchange a gluon, the neutral combination must undergo the transition $a+ a^- \leftrightarrow a^0 a^0$, as also occurred in the $Z^0$ case. For the diagonal members of the tables, the situation is messier. When a $u$ and $\bar{d}$ exchange a gluon, all three cells must undergo the neutral transition. This problem of dealing with the neutral states has been encountered in one form or another with all the vector bosons. Clearly more work is needed in this area.

6. Summary of the quip structures. We have attempted to build the observed spectrum of leptons and quarks from two spin 1/2 fields, $a^+$ and $a^0$, and their antiparticles, $a^-$ and $a^0$. We have extended the model to include the bosons mediating the electromagnetic, weak and strong interactions. The following structures, based primarily on spin and charge constraints, have emerged:

- One quip: No free quips are postulated or needed.
- Two quips: The photon: $a^+ a^- \rightarrow$ massless vector. No $a^0 a^0$ states. No scalars.
- Three quips: Leptons and quarks: spin 1/2, three-cell structures, no mixing of quips and antiquips. No spin 3/2 states.
- Four quips: First candidate for the graviton (not discussed).
- Five quips: ?
- Six quips: Vector bosons only: (A) Gluons: three-cell structures with each cell containing one quip and one antiquip. Neutral transitions needed. (B) Weak bosons: amorphous structures. Neutral transitions needed.
- Seven or more: ?

7. Problems, and paths of investigation. A number of serious problems have been swept under the rug in this simple investigation based on spin and charge states, such as the structural differences between quark lepton generations and the abelian nature of the photon. Progress may be made by a more careful consideration of spin and symmetry for the various states, and by seeking a more concrete representation for the quips. Solitons, for example, have some of the properties desired as a representation of fermionic quip—antiquip pairs. Eventually, any model worth its salt should account for the wealth of data on the weak processes, with their apparent SU(2) X U(1) symmetry. Much remains to be done.

Addendum 1. Discussions with L.M. Jones have shown the author that a possible solution to the problems surrounding the photon and the $Z^0$ is to consider the photon as a six-quip state, and to construct the photon and $Z$ states as linear combinations, more in
line with the philosophy of the Weinberg–Salam model. This point will be pursued.

_Addendum 2._ It has come to the author’s attention that a similar model has been proposed by Harari [3].

**References**