Lecture 10

Weak interactions, parity, helicity

Weak decay of particles

❑ **The weak interaction is also responsible for the β^+-decay of atomic nuclei, which involves the transformation of a proton to a neutron (or vice versa).**

$$^{10}C \to {}^{10}B^* + e^+ + \nu_e,$$

$$^{14}O \to {}^{14}N^* + e^+ + \nu_e.$$

Here, one of the protons in the nucleus transforms into a neutron via

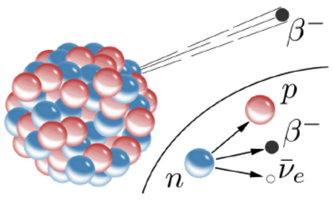
$$p \to n + e^+ + \nu_e \quad (1)$$

For free protons, this is energetically impossible (cf. the particle masses), but the crossed reaction, the β^--decay process

$$n \to p + e^- + \overline{\nu_e} \quad (2)$$

is allowed and is the reason for the neutron's instability (with a mean life-time of 920 sec).

Without the weak interaction, the neutron would be as stable as the proton, which has a lifetime of $\tau_p > 10^{30}$ years.



Weak decay of particles

- The weak decay of π^- and μ^- :

$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu \qquad \text{with } \tau = 2.6 \times 10^{-8} \text{ sec,}$$
$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \qquad \text{with } \tau = 2.2 \times 10^{-6} \text{ sec} \qquad (3)$$

The observed lifetimes of the pion and muon are considerably longer than those of particles which decay either through color (i.e. strong) or electromagnetic interactions:

i.e. particles decay by strong interactions in about 10^{-23} sec and through electromagnetic interactions in about 10^{-16} sec (for example, $\pi^0 \rightarrow \gamma\gamma$).

Note: The lifetimes are inversely related to the coupling strength of these interactions

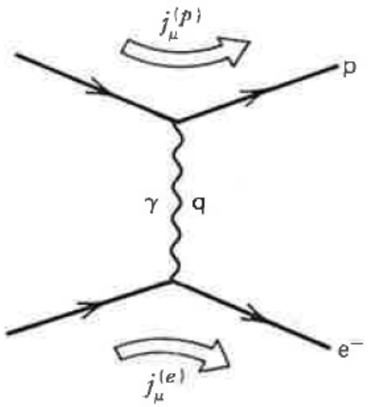
Moreover, pions are the lightest hadrons ➔ cannot decay by the strong interaction, additionally π^- can not decay electromagnetically as π^0 due to the charge.

➔ The pion and muon decays provide evidence for an interaction with an even weaker coupling than electromagnetism ➔ weak interaction

Fermi's theory for the weak interaction

Fermi's explanation of β-decay (1932) was inspired by the structure of the electromagnetic interaction:

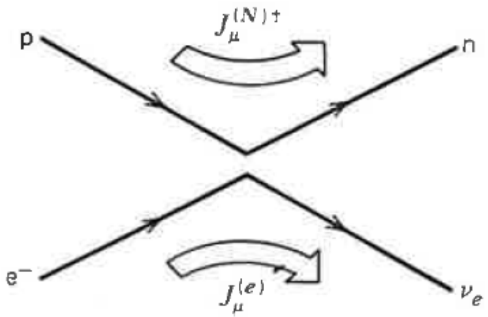
- **The invariant amplitude for electromagnetic electron-proton scattering is**



$$\mathcal{M} = -\frac{e^2}{q^2}\left(j_\mu^{em}\right)_p\left(j^{em\mu}\right)_e. \qquad (4)$$

$$\mathcal{M} = \left(e\bar{u}_p\gamma^\mu u_p\right)\left(\frac{-1}{q^2}\right)\left(-e\bar{u}_e\gamma_\mu u_e\right) \qquad (5)$$

- **β^+-decay process** $p \rightarrow n + e^+ + \nu_e$ **or its crossed form** $p + e^- \rightarrow n + \nu_e$



$$\mathcal{M} = G\left(\bar{u}_n\gamma^\mu u_p\right)\left(\bar{u}_{\nu_e}\gamma_\mu u_e\right) \qquad (6)$$

V-V: vector-vector coupling
(or A-A axialvector-axialvector coupling)

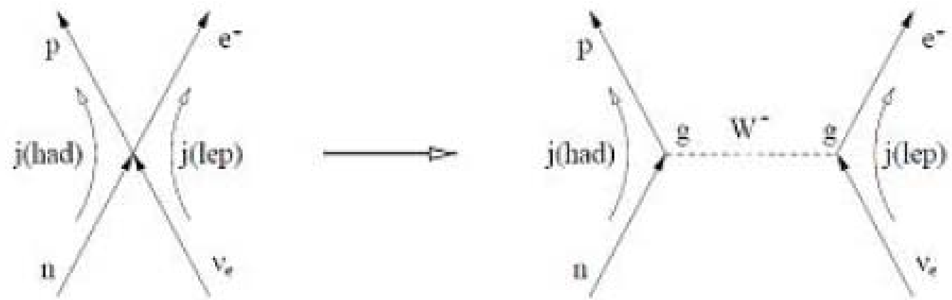
where G is the weak coupling constant which remains to be determined by experiment; G is called the Fermi constant [~1/GeV^2].

GSW theory for the weak interaction

- **Problems with Fermi‘s picture:** V-A-coupling doesn't describe the weak interaction very well, especially at high energies

- **1960s - GSW-theory:** Sheldon Glashow, Abdus Salam and Steven Weinberg propose the theory of the electroweak interaction by the **exchange of vector bosons** with huge masses (~100 GeV):

W^-,W^+ and Z^0 bosons



- **W-bosons – discovered experimentally in 1973 at CERN**

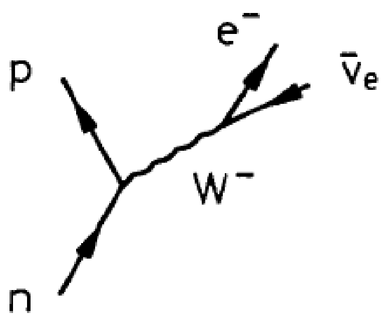
➔ **β-decay processes**

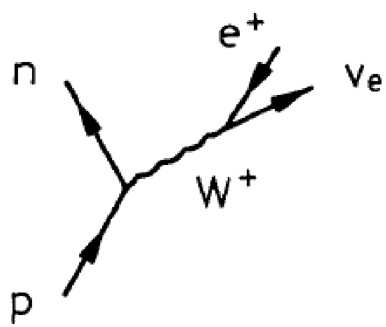
➔ **μ-decay**

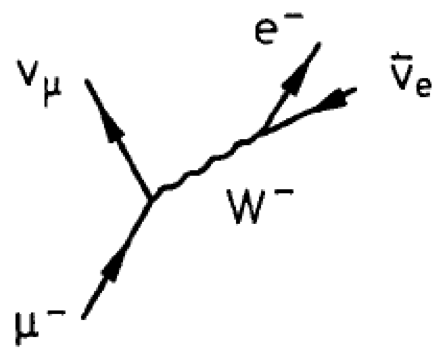
$$p \rightarrow n + e^{+} + \nu_e$$

$$n \rightarrow p + e^{-} + \overline{\nu_e}$$

$$\mu^{-} \rightarrow \nu_{\mu} + e^{-} + \overline{\nu_e}$$





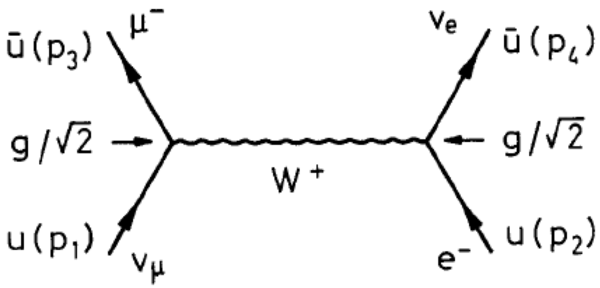


GSW theory for the weak interaction

Photon propagator: $\frac{-g_{\mu\nu}}{q^2}$ ⇒ **Vector boson propagator:** $\frac{-g_{\mu\nu} + q_\mu q_\nu / M_W^2}{q^2 - M_W^2 \quad (+i\epsilon)}$ (7)

- **Feynman diagram for**

$$\nu_\mu + e^- \to \mu^- + \nu_e$$
$$p_1 + p_2 = p_3 + p_4$$



Matrix element: $M_{fi} \sim \frac{g^2}{2} \overline{u}_3 \gamma^\mu u_1 \frac{-g_{\mu\nu} + q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \overline{u}_4 \gamma^\nu u_2$ (8)

- **For small q:** $|q^2| << M_W^2$ $\quad M_{fi} \sim \frac{g^2}{2M_W^2} \overline{u}_3 \gamma_\mu u_1 \, \overline{u}_4 \gamma^\mu u_2$ (9)

➢ **From (6) and (9) ➔** $\quad = \frac{g^2}{2M_W^2} j_\mu(\text{Myon}) \cdot J^\mu(\text{Elektron})$

$$G \sim g^2 / M_W^2$$

➔ **g is a new coupling constant – without dimention**

$$g \approx e \;\Rightarrow\; M_W \approx 100\ GeV$$

Parity violation by the weak interaction

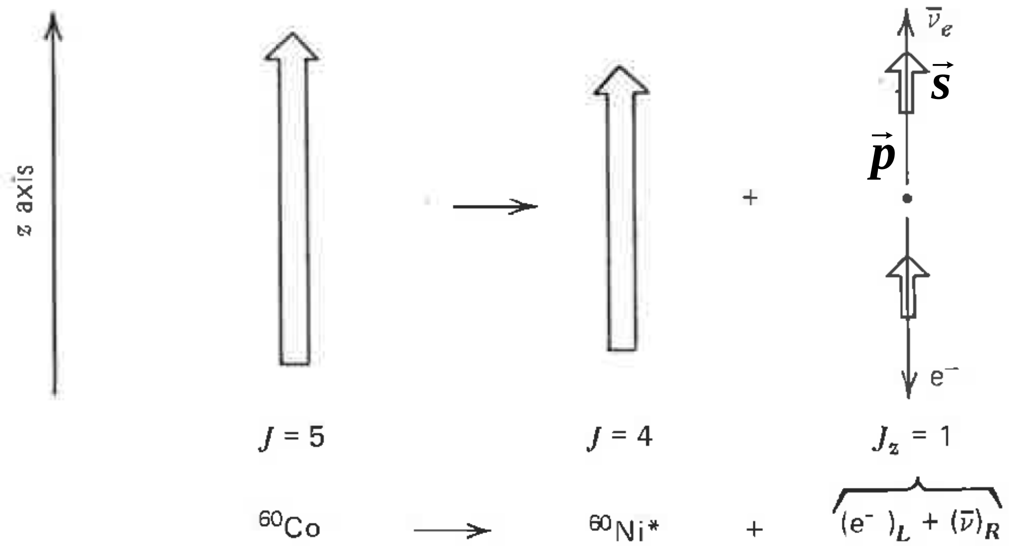
1956 – Lee and Yang prove experimentally that the weak interaction violates parity !

The experiment studied β-transitions of polarized cobalt nuclei:

$$^{60}Co \rightarrow {}^{60}Ni^* + e^- + \bar{\nu}_e.$$

The nuclear spins in a sample of ^{60}Co were aligned by an external magnetic field, and an asymmetry in the direction of the emitted electrons was observed. The asymmetry was found to change sign upon reversal of the magnetic field such that electrons prefer to be emitted in a direction opposite to that of the nuclear spin.

The observed correlation between the nuclear spin and the electron momentum is explained if the required $J_Z = 1$ is formed by a right-handed antineutrino, $\bar{\nu}_R$, and a left-handed electron e_L



right-handed:

$\vec{s} \uparrow\uparrow \vec{p}$

left-handed:

$\vec{s} \downarrow\uparrow \vec{p}$

Parity violation by the weak interaction

The cumulative evidence of many experiments is that indeed only $\bar{\nu}_R$ (and ν_L) are involved in the weak interactions. The absence of the ‚mirror image' states, $\bar{\nu}_L$ and ν_R is a clear violation of parity invariance.

Also, charge conjugation, C- invariance is violated, since C transforms a ν_L state into a $\bar{\nu}_L$ state.

However, the $\gamma^\mu(1-\gamma^5)$ form leaves the weak interaction invariant under the combined CP-operation.

For instance,

$$\Gamma(\pi^+ \to \mu^+ \nu_L) \neq \Gamma(\pi^+ \to \mu^+ \nu_R) = 0 \qquad \text{P violation,}$$

$$\Gamma(\pi^+ \to \mu^+ \nu_L) \neq \Gamma(\pi^- \to \mu^- \bar{\nu}_L) = 0 \qquad \text{C violation,}$$

but

$$\Gamma(\pi^+ \to \mu^+ \nu_L) = \Gamma(\pi^- \to \mu^- \bar{\nu}_R) \qquad \text{CP invariance.}$$

The operator $(1-\gamma^5)/2$ automatically selects a left-handed neutrino (or a right-handed antineutrino).

➔ V-A (vector-axial vector) structure of the weak current

Parity

Consider the parity transformation:

$$x = (t, \vec{x}) \xrightarrow{P} x' = (t, -\vec{x}) \qquad (10)$$

Transformed Dirac spinors

$$\psi'(x') = S_P \psi(x) \qquad (11)$$

(S_P is the parity operator)

should follow the Dirac equation:

$$(i\gamma^\mu \partial'_\mu - m)\psi'(x') = 0 \quad ,$$

$$(i\gamma^0 \frac{\partial}{\partial t} - i\vec{\gamma} \cdot \vec{\nabla} - m) S_P \psi(x) = 0 \qquad (12)$$

By multiplying (12) from the left side by S_p^{-1} we obtain:

$$S_p^{-1}\gamma^0 S_p = \gamma^0 \quad , \quad S_P^{-1}\vec{\gamma} S_P = -\vec{\gamma} \qquad (13)$$

Solution of (13): **Parity operator $S_P = \gamma^0$**

$$\psi'(x') = \gamma^0 \psi(x) \qquad (14)$$

Parity

Parity transformation of Dirac spionors:

$$\psi(x) = \frac{1}{\sqrt{V}} u(p) e^{-ipx} = \frac{1}{\sqrt{V}} \sqrt{E+m} \begin{pmatrix} \varphi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\varphi \end{pmatrix} e^{-iEt} e^{i\vec{p}\cdot\vec{x}}$$

$$\psi'(x') = S_P \psi(x) = \frac{1}{\sqrt{V}} \sqrt{E+m} \begin{pmatrix} \varphi \\ -\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\varphi \end{pmatrix} e^{-iEt} \underbrace{e^{i\vec{p}\cdot\vec{x}}}_{e^{i(-\vec{p})\cdot(-\vec{x}')}} \qquad (15)$$

$\vec{p}$ is replaced by $-\vec{p}$

- **Consider a particle at rest:**

$$u(0) = \frac{1}{\sqrt{V}} \sqrt{2m} \begin{pmatrix} \varphi \\ 0 \end{pmatrix} \qquad \varphi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (16)$$

parity transformation $(S_P = \gamma^0)$ ⇨

$$\gamma^0 u(0) = u(0) \qquad (17)$$

➔ **Particles have a positive parity !**

Parity

- Consider an anti-particle at rest:

$$v(0) = \frac{1}{\sqrt{V}}\sqrt{2m}\begin{pmatrix}0\\ \chi\end{pmatrix} \qquad \chi = \begin{pmatrix}1\\ 0\end{pmatrix} \quad \text{or} \quad \begin{pmatrix}0\\ 1\end{pmatrix} \tag{18}$$

parity transformation $(S_P = \gamma^0)$ ⟹ $\underline{S_P v(0) = -v(0)}$ (19)

➔ **Anti-particles have a negative parity !**

Particles and anti-particles have an opposite parity!

Example:

In the quark model : quarks ***u,d,s,c,b*** have a Parity ***P = +1***

antiquarks $\bar{u},\bar{d},\bar{s},\bar{c},\bar{b}$ have a Parity ***P = -1***

Helicity

Note: property of the γ_5 matrix :

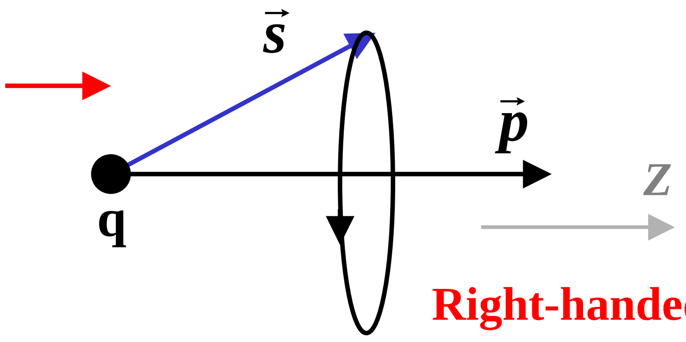
$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad (20)$$

$$\gamma^\mu\gamma_5 = -\gamma_5\gamma^\mu$$

Introduce the helicity λ:

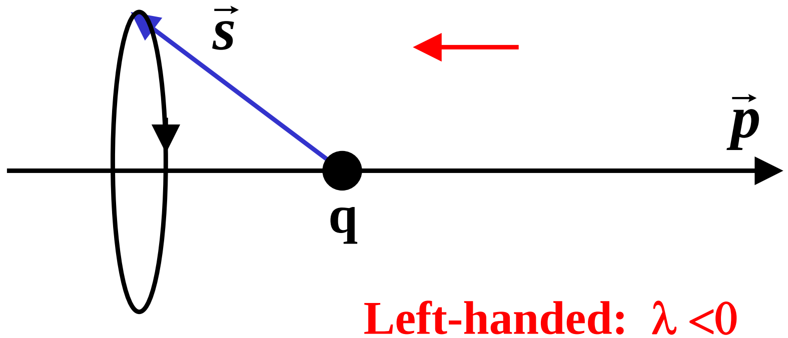
$$\lambda = \vec{s}\cdot\hat{p} \quad , \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|} \quad (21)$$

Helicity = projection of spin on the direction of motion



Right-handed: λ >0

$\vec{s} \uparrow\uparrow \vec{p}$



Left-handed: λ <0

$\vec{s} \downarrow\uparrow \vec{p}$

Helicity

Dirac spinors for positive and negative helicity:

$$u_1 = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ p/(E+m) \\ 0 \end{pmatrix} \quad \text{has} \quad \lambda = +\frac{1}{2} \quad \text{positive helicity} \quad \vec{s} \uparrow\uparrow \vec{p} \qquad (22)$$

$$u_2 = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -p/(E+m) \end{pmatrix} \quad \text{has} \quad \lambda = -\frac{1}{2} \quad \text{negative helicity} \quad \vec{s} \downarrow\uparrow \vec{p}$$

For ***E >> m* :**

$$u_1 = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad , \quad u_2 = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \qquad (23)$$

$$\frac{1}{2}(1-\gamma_5)u_1 = 0 \quad , \quad \frac{1}{2}(1-\gamma_5)u_2 = u_2 \qquad (24)$$

Helicity

Operator $P_L = \frac{1}{2}(1-\gamma_5)$ is the **projection operator for negative helicity**

Operator $P_R = \frac{1}{2}(1+\gamma_5)$ is the **projection operator for positive helicity**

$$P_R u_1 = u_1 \quad , \quad P_R u_2 = 0 \qquad (25)$$

General properties of projection operators:

$$P_L^2 = P_L \quad , \quad P_R^2 = P_R \quad , \quad P_R P_L = P_L P_R = 0 \qquad (26)$$

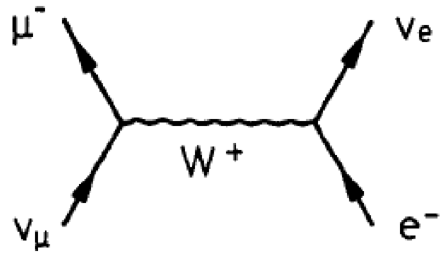
With the help of the projection operators P_L and P_R the spinor can be decomposed into R and L parts:

$$u = \frac{1}{2}(1+\gamma_5)u + \frac{1}{2}(1-\gamma_5)u = u_R + u_L \qquad (27)$$

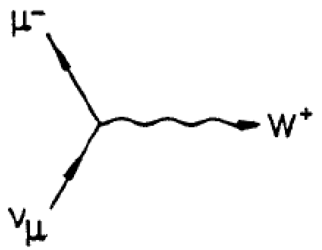
$$\begin{array}{ll} P_R u_R = u_R & P_R u_L = 0 \\ P_L u_R = 0 & P_L u_L = u_L \end{array} \qquad (28)$$

V-A form of the weak current

Consider the process $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$



From experiment ➔ ν_e must be left-handed, i.e.



$$\overline{u}(\mu^-)\gamma_\mu u_L(\nu)$$

$$u_L(\nu) = \frac{1}{2}(1-\gamma_5)u(\nu) \qquad (29)$$

Current

$$\overline{u}(\mu^-)\gamma_\mu u_L(\nu) \Rightarrow \boxed{\frac{1}{2}\overline{u}(\mu)\gamma_\mu(1-\gamma_5)u(\nu)} \qquad (30)$$

- Consider the current (30) under Lorentz transformations:

$$x'^\nu = a^\nu_\mu x^\mu \quad , \quad j'^\nu(x') = a^\nu_\mu j^\mu(x) \qquad (31)$$

$$j^\mu = \overline{\psi}\gamma^\mu\psi$$

V-A form of weak current

From (30) ➔ current in the neutrino-muon vertex has two terms:

1) vector current $V^\mu = \bar{\psi}(\mu)\gamma^\mu\psi(\nu)$ (32)

which under Lorentz transformation (31) transforms as $V'^\mu(x') = a^\nu_\mu V^\mu(x)$

2) axialvector current $A^\mu = \bar{\psi}(\mu)\gamma^\mu\gamma_5\psi(\nu)$ (33)

which under Lorentz transformation (31) transforms as $A'^\mu(x') = a^\mu_\nu A^\nu(x)$

- Thus, the parity transformation (or space reflection) leads to

$$\psi'(x') = \gamma^0\psi(x)$$

$$V'^\mu = \bar{\psi}'\gamma^\mu\psi' = \bar{\psi}\gamma^0\gamma^\mu\gamma^0\psi$$

$$V'^0 = V^0 \quad , \quad \vec{V}' = -\vec{V}$$

$$A'^\mu = \bar{\psi}'\gamma^\mu\gamma_5\psi' = \bar{\psi}\gamma^0\gamma^\mu\gamma_5\gamma^0\psi$$

$$A'^0 = -A^0 \quad , \quad \vec{A}' = +\vec{A}$$

(34)

➔ The neutrino-muon vertex has two terms: (V-A) coupling

$$\frac{1}{2}(\bar{\psi}_{(\mu)}\gamma^\mu\psi_{(\nu)} - \bar{\psi}_{(\mu)}\gamma^\mu\gamma_5\psi_{(\nu)}) = \frac{1}{2}(V^\mu - A^\mu)$$

(35)

Helicity of antineutrinos

❑ Helicity of antineutrinos

We know from experiment that antineutrinos are right-handed ➔ positive helicity

Dirac spinors for antiparticles with positive and negative helicity read:

$$v_1 = \sqrt{|E|}\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \Uparrow \qquad v_2 = \sqrt{|E|}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \Downarrow \tag{36}$$

$$\lambda = +\tfrac{1}{2} \qquad\qquad \lambda = -\tfrac{1}{2}$$

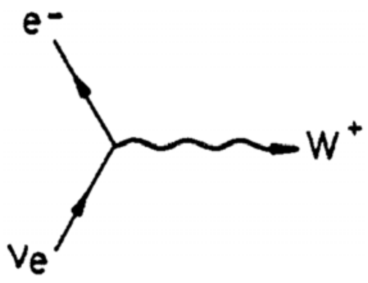
$$P_L v_1 = \tfrac{1}{2}(1-\gamma_5)v_1 = v_1 \;, P_L v_2 = 0 \tag{37}$$

Projection operator $P_L = 1/2(1-\gamma_5)$ **sorts antineutrinos with positive helicity**

Helicity of electrons

❑ Helicity of the electron

The currect for the neutrino-electron vertex:



$(P_L^2 = P_L)$

$$\bar{u}_e \gamma_\mu \frac{(1-\gamma_5)}{2} u_\nu = \bar{u}_e \gamma_\mu \left(\frac{1-\gamma_5}{2}\right)^2 u_\nu \qquad (38)$$

$$= u_e^+ \left(\frac{1-\gamma_5}{2}\right) \gamma^0 \gamma_\mu \left(\frac{1-\gamma_5}{2}\right) u_\nu$$

$$= \overline{(u_e)_L} \gamma_\mu (u_\nu)_L$$

$\bar{u} \equiv u^+ \gamma^0$

$(u_e)_L = \frac{1-\gamma_5}{2} u_e$

Electron with positive helicity reads as:

$$\lambda = +\frac{1}{2} \qquad u_1(p) = \sqrt{E+m} \begin{pmatrix} 1 \\ 0 \\ p/(E+m) \\ 0 \end{pmatrix} \qquad (39)$$

$$\frac{1}{2}(1-\gamma_5)u_1 = \sqrt{E+m}\,\frac{1}{2}\left(1 - \frac{p}{E+m}\right) \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \to 0 \quad \textbf{for} \quad E >> m$$

Left-right-handed assimetry

Electron with negative helicity:

$$\lambda = -\frac{1}{2} \qquad u_2(p) = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -p/(E+m) \end{pmatrix} \tag{40}$$

$$\frac{1}{2}(1-\gamma_5)u_2 = \underbrace{\sqrt{E+m}\frac{1}{2}\left(1+\frac{p}{E+m}\right)}_{\approx\sqrt{E+m} \quad \text{für} \quad E>>m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \rightarrow u_2 \qquad \textbf{for} \quad E>>m$$

Asymmetry in left-right-handed electron production:

$$\frac{W(\lambda=+1/2)-W(\lambda=-1/2)}{W(\lambda=+1/2)+W(\lambda=-1/2)} = \frac{\left(1-\frac{p}{E+m}\right)^2-\left(1+\frac{p}{E+m}\right)^2}{\left(1-\frac{p}{E+m}\right)^2+\left(1+\frac{p}{E+m}\right)^2} = -\frac{p}{E} = -\beta = -\frac{v}{c} \tag{41}$$

➔ **electron has preferentially a negative helicity (i.e. left-handed)**

Pion weak decay

Pion weak decay in the electron channel $\pi^- \to e^- + \bar{\nu}_e$

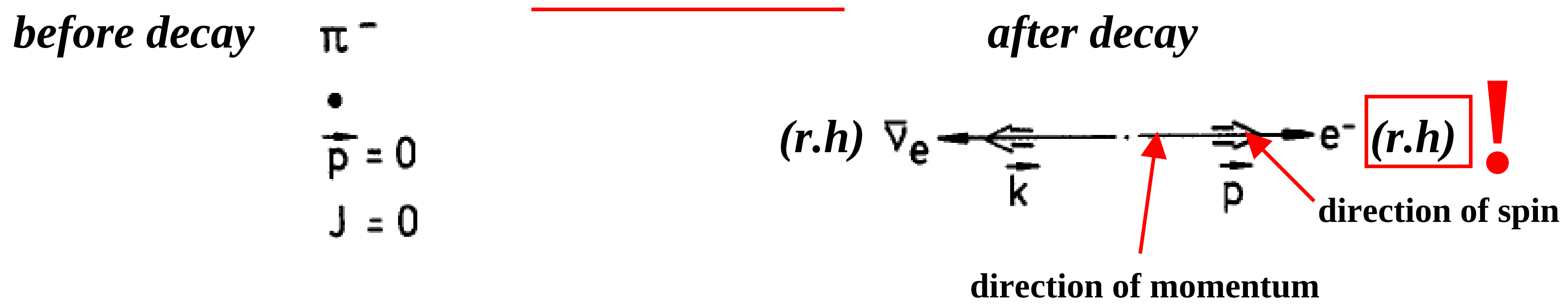
is suppressed by a factor of $1.3 \cdot 10^{-4}$ relative to its decay in the muon channel $\pi^- \to \mu^- + \bar{\nu}_\mu$

whereas from phase-space arguments it should be the opposite since the muon is much heavier than the electron !

The reason: in the decay $\pi^- \to e^- + \bar{\nu}_e$ **an electron must have a positive helicity, i.e. to be right-handed (due to angular momentum conservation), whereas the V-A theory (41) shows that it is suppressed by a factor of (1-υ/c),**

i.e. from V-A theory: particles have a negative helicity – left-handed *(l.h.)* $\vec{s} \downarrow\uparrow \vec{p}$
antiparticles have a positive helicity – right-handed *(r.h)* $\vec{s} \uparrow\uparrow \vec{p}$

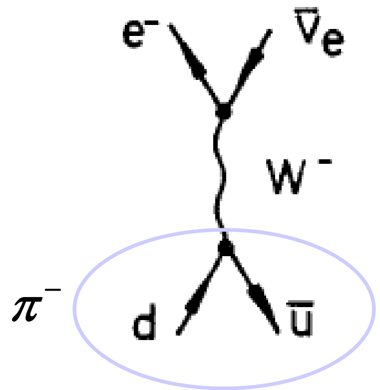
$\pi^- \to e^- + \bar{\nu}$



➔ **Pion weak decay is an experimental check of the V-A theory!**

Pion weak decay

Feynman diagram for $\pi^- \rightarrow e^- + \bar{\nu}_e$



Matrix element:

$$M_{fi} = f_\pi \cdot \frac{G_F}{\sqrt{2}} p_\mu \bar{u}(e)\gamma^\mu(1-\gamma_5)v(\bar{\nu}) \quad (42)$$

- **Pion currect – from Klein-Gordan equation for spinless particles:**

$$j_\pi \sim \frac{p_\mu}{m_\pi} \quad (43)$$

- G_F **is the coupling constant for the 4-point like fermion vertex** $(d\ \bar{u}\ e^-\ \bar{\nu}_e)$

- f_π **is the pion decay constant:** $f_\pi = 93\ MeV$

In the rest frame of pion: $p_\mu = (m_\pi, \vec{0})$

$$M_{fi} = f_\pi \cdot \frac{G_F}{\sqrt{2}} m_\pi \bar{u}(e)\gamma^0(1-\gamma_5)v(\bar{\nu}) \quad (44)$$

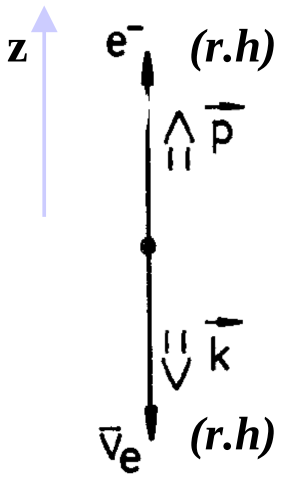
For the decay in the rest frame of the pion $(s = m_\pi^2)$

$$d\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e) = \frac{1}{2m_\pi}|M_{fi}|^2\ dLips(m_\pi^2; p, k) \quad (45)$$

Γ - decay width

***Lips*= Lorentz invariant phase space**

Pion weak decay



❑ **Antineutrino has a positive helicity, i.e. right-handed** $\vec{s} \uparrow\uparrow \vec{k}$

z − direction $\downarrow\uparrow \vec{k}$

$$v_1 = \sqrt{k} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad , \quad k = |\vec{k}| \qquad (46)$$

❑ **Electron for this process must have a positive helicity, too, i.e. to be right-handed** $\vec{s} \uparrow\uparrow \vec{p}$ ➔ u_1 **Dirac spinor:**

$$u_1 = \sqrt{E+m} \begin{pmatrix} 1 \\ 0 \\ p/(E+m) \\ 0 \end{pmatrix} \qquad \Uparrow \qquad \lambda = +\frac{1}{2} \qquad (47)$$

$$u_2 = \sqrt{E+m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -p/(E+m) \end{pmatrix} \qquad \Downarrow \qquad \lambda = -\frac{1}{2}$$

Pion weak decay

Indeed, by considering $\bar{u}_{1,2}\gamma^0(1-\gamma_5)v_1 = u^+_{1,2}\cdot 2v_1$ **(48)**

one finds that $\underline{u^+_2 v_1 = 0}$ ➔ the process $\pi^- \to e^- + \bar{\nu}_e$ is only possible if the electron has a positive helisity

$$2u^+_1 v_1 = 2\sqrt{E+m}\sqrt{k}\left(\frac{p}{E+m}-1\right) \quad (49)$$

Matrix element:

$$|M_{fi}|^2 \sim k(E+m)\left(\frac{p}{E+m}-1\right)^2 \quad ; \quad k^2 = p^2 = E^2 - m^2 \quad (50)$$

$$= 2p(E-p) = 2pE\left(1-\frac{v}{c}\right) \quad .$$

- In the rest frame of pion or CMS $e^- + \bar{\nu}_e$

Lorentz invariant phase space:

$$dLips(m_\pi^2; k, p) = \frac{1}{(4\pi)^2}\cdot\frac{p}{m_\pi}\,d\Omega \quad (51)$$

Pion weak decay

By substituting (50) and (51) into (45), we find

$$\Gamma \sim 2p(E-p)\cdot p \qquad (52)$$

where p is the momentum of the electron in the rest frame of the pion:

$$p = \frac{1}{2m_\pi}(m_\pi^2 - m_e^2) \quad , \quad E = \frac{1}{2m_\pi}(m_\pi^2 + m_e^2) \qquad (53)$$

⇨ decay width for $\pi^- \rightarrow e^- + \bar{\nu}_e$

$$\boxed{\Gamma \sim (m_\pi^2 - m_e^2)^2 m_e^2} \qquad (54)$$

In a similar way (by replacing m_e by m_μ), one can calculate the decay width for $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$

➔ the ratio

$$\frac{\Gamma(\pi^- \to e^- \overline{\nu}_e)}{\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu)} = \frac{m_e^2 \left(1 - \left(\frac{m_e}{m_\pi}\right)^2\right)^2}{m_\mu^2 \left(1 - \left(\frac{m_\mu}{m_\pi}\right)^2\right)^2} = \underline{1{,}28 \cdot 10^{-4}} \qquad (55)$$

(exp.: $(1{,}267 \pm 0{,}023) \cdot 10^{-4}$)

⇨ Pion decay $\pi^- \rightarrow e^- + \bar{\nu}_e$ strongly suppressed relative to $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$