Introduction to Hadronic Final State Reconstruction in Collider Experiments

(Part XIV)

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Particle flow inside a jet hints to source

Jet can be a discovery tool by itself

In particular most interesting for boosted (new) heavy particle like Kaluza-Klein excitations
But also interesting for Standard Model particles like boosted top quarks

Usefulness depends on the ability to resolve decay structure

E.g., 2-prong (like W) or 3-prong (top) decays

Resolution scale given by mass of particle (or by particle hypothesis) – to be reflected with detector capabilities

2 – prong decay inside reconstructed jet, e.g. from $W \rightarrow q\bar{q}$ (SM) or heavy new object like $\phi \rightarrow gg$ or $Z' \rightarrow q\bar{q}$ (BSM)

3 – prong decay inside reconstructed jet, e.g. from $t \rightarrow q\bar{q}b$ (SM) or heavy new object like $\phi_{kk} \rightarrow Q\bar{Q}b + X$ or $t' \rightarrow q\bar{q}b$ (BSM)
Jet Mass

Observables and tools

Single jet mass

Mass generated by four-momentum recombination should reflect heavy source

Scales proportional to $p_T$ for light quark or gluon jet

Subject to severe detector effects

Lateral energy spread by individual particle cascades reduces single jet mass resolution

Calorimeter signal definition choices on top of shower spread can enhance or reduce sensitivity to in-jet particle flow and thus improve or worsen single jet mass resolution

\[
\left( \frac{E_{\text{jet}}}{\vec{p}_{\text{jet}}} \right) = \left( \frac{\sum_{\text{constituents}} E_{\text{constituent}}}{\sum_{\text{constituents}} \vec{p}_{\text{constituent}}} \right) \Rightarrow m_{\text{jet}} = \sqrt{E_{\text{jet}}^2 - |\vec{p}_{\text{jet}}|^2}
\]

mass of gluon/light quark jets:

LO 1–parton jets have vanishing mass

NLO 2–parton configurations at given $p_{\text{jet}}$ generate average invariant jet mass:

\[
\langle m_{\text{jet}}^2 \rangle_{\text{NLO}} = \overline{C} \left( p_{\text{jet}} / \sqrt{s} \right) \alpha_s \left( p_{\text{jet}} / 2 \right) p_{\text{jet}}^2 R_{\text{cone}}^2
\]

with:

\[
\overline{C} \left( p_{\text{jet}} / \sqrt{s} \right) \quad \text{pre-function of magnitude } \mathcal{O}(1)
\]

(absorbes color charges and pdf, slowly decreases with rising $p_{\text{jet}}$)

\[
\alpha_s \left( p_{\text{jet}} / 2 \right) \quad \text{strong coupling at scale } \mu = p_{\text{jet}} / 2
\]

⇒ expect linear mass in NLO to scale with $p_{\text{jet}}$:

\[
\sqrt{\langle m_{\text{jet}}^2 \rangle_{\text{NLO}}} \approx \sqrt{\overline{C} \alpha_s} p_{\text{jet}} R_{\text{cone}}
\]

rule of thumb at $\sqrt{s} = 14$ TeV:

\[
\sqrt{\langle m_{\text{jet}}^2 \rangle_{\text{NLO}}} \approx 0.2 \cdot p_{\text{jet}} R_{\text{cone}}
\]
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$\sqrt{M^2_{\text{jet}}} \approx \frac{p_T}{p_T^{\text{jet}}} \eta \leq 2.5$

$\sqrt{s} = 1.96 \text{ TeV, } p\bar{p}$
$\sqrt{s} = 14 \text{ TeV, } pp$

$\rho_{\text{jet}}^{\text{min}} \approx 115 \text{ GeV}$
$\rho_{\text{jet}} = 700 \text{ GeV}$
$\rho_{\text{jet}}^{\text{min}} \approx 685 \text{ GeV}$
$\rho_{\text{jet}} = 4.2 \text{ TeV}$

Jet Mass

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Single jet mass

Mass generated by four-momentum recombination should reflect heavy source
Scales proportional to pT for light quark or gluon jet

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\[
\begin{align*}
\left( \frac{E_{\text{jet}}}{p_{\text{jet}}} \right) &= \left( \sum_{\text{constituents}} \frac{E_{\text{constituent}}}{\sum_{\text{constituents}} p_{\text{constituent}}} \right) \\
\Rightarrow m_{\text{jet}} &= \sqrt{E_{\text{jet}}^2 - |\vec{p}_{\text{jet}}|^2}
\end{align*}
\]

- requires good reconstruction of particle flow in jet by detector signal → depends on chosen calorimeter signal definition, e.g. test

\[
\frac{m_{\text{jet, reco}} - m_{\text{jet, truth}}}{m_{\text{jet, truth}}}
\]

for matching truth and calorimeter jets

- plot on the right shows the spectrum of this relative mass difference for simulated QCD di-jets (kT, R = 0.6) in ATLAS

(cluster jets, tower jets)

\[
\left( m_{\text{jet, reco}} - m_{\text{jet, truth}} \right)/m_{\text{jet, truth}}
\]

(old plot, educational purpose only!)
Jet Mass

Relative mass change

Truncate jet mass

Calculate mass using only constituents above pT thresholds

Compare ratio to unbiased mass

Particle (hadron), cluster, tower jets

**Observables and tools**

**Recombination scales and order in kT-like algorithms**

Jet decomposition tracing back the (recursive) recombination

Can be considered resolving fragmentation to a given scale

Scale of last clustering step relates to mass of source in two-prong decay

Scale of next-to-last clustering step relates to mass of source in three-prong decay

Can be expected to correlate with jet mass in heavy particle decays

But different resolution – likely less sensitive to detector effects!

$y$ – scale in kT algorithms provides a $p_T$ scale at which a given recombination can be undone

**Recall variables:**

$$d_i = p_{T,j}^2 \quad \text{and} \quad d_{ij} = \min(d_i, d_j) \frac{\Delta R_{ij}}{R}$$

**Principal kT clustering rules:**

1. Build list of $d_i$ and $d_{ij}$ from all protojets
2. If common minimum is a $d_i$, call $i$ from list and call it a jet
3. Else combine $i$ and $j$ to a jet and add to list, and remove the previous protojets $i$ and $j$
4. Repeat from (1) until no protojets are left

**Define $y$ – scale**

$$y_{\text{scale}} = y_n \times p_{T,\text{jet}}^2$$, with $n$ being a resolution parameter

**Example:** $n = 2$ refers to the last recombination in the clustering sequence, i.e. $d_{12} < d_1, d_2$:

$$y_{\text{scale}}^{2 \rightarrow 1} = \sqrt{y_2} \times p_{T,\text{jet}} = \sqrt{\min(d_1, d_2)}$$

relates to mass in two-prong decays
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\[ y_{\text{scale}}^{1\rightarrow2} \text{ for jets with } m_{\text{jet}} > 40 \text{ GeV, for QCD and hadronically decaying boosted } W. \]

Note that for QCD \( y_{\text{scale}}^{1\rightarrow2} \) is logarithmically below \( p_{T,jet} \) due to the strong ordering (in \( k_T \)) in QCD evolution, while \( \langle y_{\text{scale}}^{1\rightarrow2} \rangle \approx m_W \) reflects the 2-prong decay of the \( W \) boson.
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**ATLAS simulation:**

\[ Z' \rightarrow t\bar{t}, \quad m_{Z'} = 2(3) \text{ TeV} \]

\[ p_T(t) > 300 \text{ GeV} \]

- \( y_{\text{scale}}^{1\rightarrow2} \) probes top decay,

peaks at \( \approx 100 \text{ GeV} \approx \frac{m_{\text{top}}}{2} \)

- \( y_{\text{scale}}^{2\rightarrow3} \) probes W decay,

peaks at \( \approx 40 \text{ GeV} \approx \frac{m_W}{2} \)
Observables and tools

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\[ \Delta y_{\text{scale}} = y_{\text{scale}}^{1\rightarrow2}_{\text{particle}} - y_{\text{scale}}^{1\rightarrow2}_{\text{calo}} \] for jets with \( m_{\text{jet}} > 40 \text{ GeV} \) from hadronically decaying boosted \( W \).

\( y_{\text{scale}}^{1\rightarrow2}_{\text{calo}} \) is calculated for parameterized, response smearing simulation (fast, no lateral shower spread) and from detailed full simulation \( \rightarrow \) indications that \( y_{\text{scale}}^{1\rightarrow2} \) is little sensitive to details of showering.
**Sub-Jet Analysis**

**Observables and tools**

*Direct attempt to reconstruct sub-jets within jet*

Narrow jet reconstruction in bigger jet motivated by mass drop

Includes signal enhancement strategy

*Requires additional (3rd) jet from gluon radiation in the decay system*

Look for $H \rightarrow b\bar{b}g$ with $p_{T,H}>200$ GeV in $WH / ZH$ production - about 5% of total cross-section:

\[
R_{bb} = \frac{1}{\sqrt{z(1-z)}} \frac{m_H}{p_T}, \quad p_T \gg m_H
\]

use Cambridge/Aachen kT flavour jet finder to find large jet ($R=1.2$), $p_T>200$ GeV for sub-jet analysis

1. break jet $j$ into two subjects $j_1, j_2$, with $m_{j_1} > m_{j_2}$, by undoing last recombination

2. if there is a significant mass drop such that $m_{j_1} < \mu m_j$, and the splitting $j \rightarrow (j_1, j_2)$ is not too asymmetric, i.e.

\[
\min(p_{j_1}^2, p_{j_2}^2)/m_{j_1}^2 \Delta R_{j_1,j_2}^2 > \gamma_{cut},
\]

then the jet $j$ is assumed to be the heavy particle neighbourhood and the analysis stops

3. else, set $j = j_1$ and go back to step (1)

apply filter to all heavy particle neighbourhoods, with a finer angular scale $R_{\text{filter}} < R_{bb}$, e.g., $R_{\text{filter}} = \min(0.3, R_{bb}/2)$ seems to be good for LHC, and take the 3 hardest objects that appear $\rightarrow H \rightarrow b\bar{b}g$, including the hardest ($O(\alpha_s)$) radiation. Tag the $b$ jets and calculate the invariant mass.

Observables and tools

**Direct attempt to reconstruct sub-jets within jet**

Narrow jet reconstruction in bigger jet motivated by mass drop

Includes signal enhancement strategy

Requires additional (3rd) jet from gluon radiation in the decay system

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J.M. Butterworth, A.R. Davison, M. Rubin, G.P. Salam,
Jet Pruning

Jet pruning

Enhancement of jet components to increase substructure resolution

Applied in kT-style jet clustering procedure

Jet trimming

Applies a filter by removing soft sub-jets in a jet

Soft pT cut-off evaluated dynamically jet by jet

Jet Pruning

- attempt to suppress underlying event and pile-up contributions to jets
- cleans jets by vetoing spurious recombinations during clustering \( \rightarrow \) kT and C/A jets only!
- sensitive variables are angular distance \( \phi = \Delta R_{12} \)
  and relative pT hierarchy \( z\equiv \min(p_{T,1},p_{T,2})/p_{T,p} \)
  in recombination \( 1,2 \rightarrow p \)
- suppress large distances and large hierarchies at each clustering iteration

\[ \phi > R_{\text{cut}} \]
\[ z < z_{\text{cut}} \]

works better for heavy particle decays than for QCD:

- not clear what \( R_{\text{cut}} \) is for QCD \( - \) \( R_{\text{cut}} \approx m/p_T \) for heavy particle decays
- also not clear what \( z_{\text{cut}} \) should be \( - \) contamination looks hard early in clustering, especially for kT; for heavy particles, \( z_{\text{cut}} = 0.1(0.15) \) works well for kT(C/A) jets from boosted top
Jet Pruning

- attempt to suppress underlying event and pile-up contributions to jets
- cleans jets by vetoing spurious recombinations during clustering \( \rightarrow \) kT and C/A jets only!
- sensitive variables are angular distance \( \phi = \Delta R_{12} \) and relative \( p_t \) hierarchy \( z = \min(p_{t,1}, p_{t,2})/p_{t,\rho} \), in recombination \( 1,2 \rightarrow p \)
- suppress large distances and large hierarchies at each clustering iteration

\[ \phi > R_{\text{cut}} \]

\[ z < z_{\text{cut}} \]

works better for heavy particle decays than for QCD:

Jet Pruning

- attempt to suppress underlying event and pile-up contributions to jets
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- suppress large distances and large hierarchies at each clustering iteration
  \[ \phi > R_{\text{cut}} \]
  \[ z < z_{\text{cut}} \]

works better for heavy particle decays than for QCD:

Jet Pruning

- enhancement of jet components to increase substructure resolution
- applied in kT-style jet clustering procedure

Jet trimming

- applies a filter by removing soft sub-jets in a jet
- soft pT cut-off evaluated dynamically jet by jet

Observables and tools

Jet Pruning

- refinement of jet components to increase substructure resolution
- applied in kT-style jet clustering procedure

Jet trimming

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- soft pT cut-off evaluated dynamically jet by jet
Jet Pruning

- improves jet mass measurement for boosted top etc.

### Observables and tools

#### Jet pruning
Enhancement of jet components to increase substructure resolution
*Applied in kT-style jet clustering procedure*

#### Jet trimming
Applies a filter by removing soft sub-jets in a jet
*Soft pT cut-off evaluated dynamically jet by jet*

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$m_{\text{jet}}$ (GeV) for boosted top decays ($p_{T,\text{jet}} > 200$ GeV)

**Jet Trimming**

- **main motivation** is removing contaminations from e.g. pile-up and underlying event, from a fully reconstructed jet
- measures softness/hardness of contamination relative to whole jet — no judgements at the clustering stage
- **approach:**
  1. fully reconstruct jet from calorimeter signals
  2. cluster narrow sub-jets, typically with $R_{sub} = 0.2$
  3. discard sub-jets $i$ with $p_{T,i} < f_{cut} \Lambda_{hard}$
  4. rebuild jet from surviving sub-jets
- typical choice for $\Lambda_{hard}$ is $\Lambda_{hard} = p_{T,\text{jet}}$

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**Observables and tools**

**Jet pruning**

- Enhancement of jet components to increase substructure resolution
- Applied in $k_T$-style jet clustering procedure

**Jet trimming**

- Applies a filter by removing soft sub-jets in a jet
- Soft $p_T$ cut-off evaluated dynamically jet by jet
Jet Trimming

Trimmed and variable radius (VR) jets from \[ \phi \rightarrow q\bar{q}, gg \]
(for VR, see D. Krohn, J. Thaler, and L.-T. Wang, 
Jets with Variable R, JHEP 06 (2009) 059)