

Introduction to Hadronic Final State Reconstruction in Collider Experiments (Part XII)

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Plots for this session

Most if not all plots shown in this session are meant as examples and for illustration purposes

Educational showcases to highlight certain features of energy scales and calorimeter response

They do not represent the up-to-date estimates for ATLAS jet reconstruction performance

In general much better than the (old) results shown here!

Not many new plots can be shown in public yet!

The performance plots shown are published

Reflection of state-of-art at a given moment in time

No experimental collision data available at that time!



Experiment and simulation

Calorimeter towers

2-dim signal objects from all cells or only cells surviving noise suppression (topological towers in ATLAS)

Calorimeter clusters

3-dim signal objects with implied noise suppression (topological clusters in ATLAS)

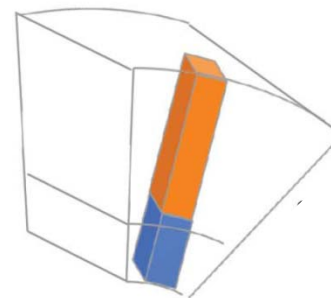
Tracks

Reconstructed inner detector tracks – only charged particles with $p_T > p_{T_{\text{threshold}}} = 500$ MeV – 1 GeV (typically)

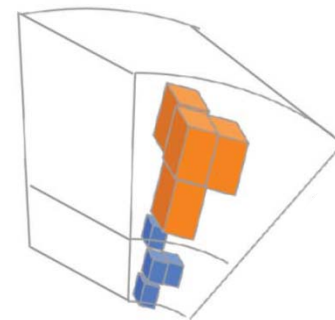
Simulation only

Generated stable particles

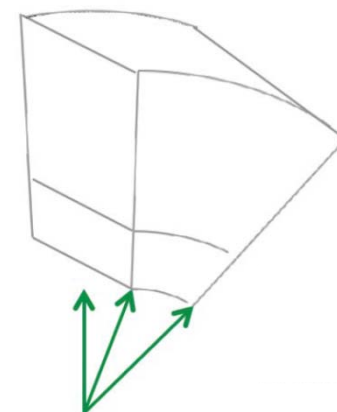
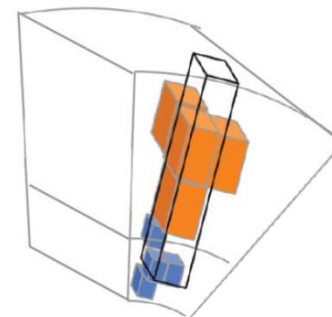
Typically $\tau_{\text{lab}} > 10$ ps to be a signal source



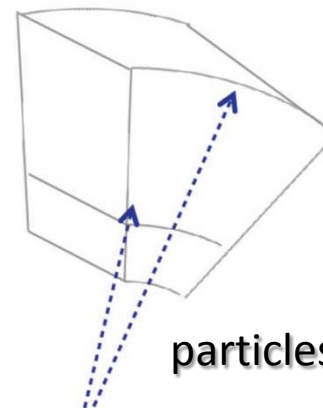
towers



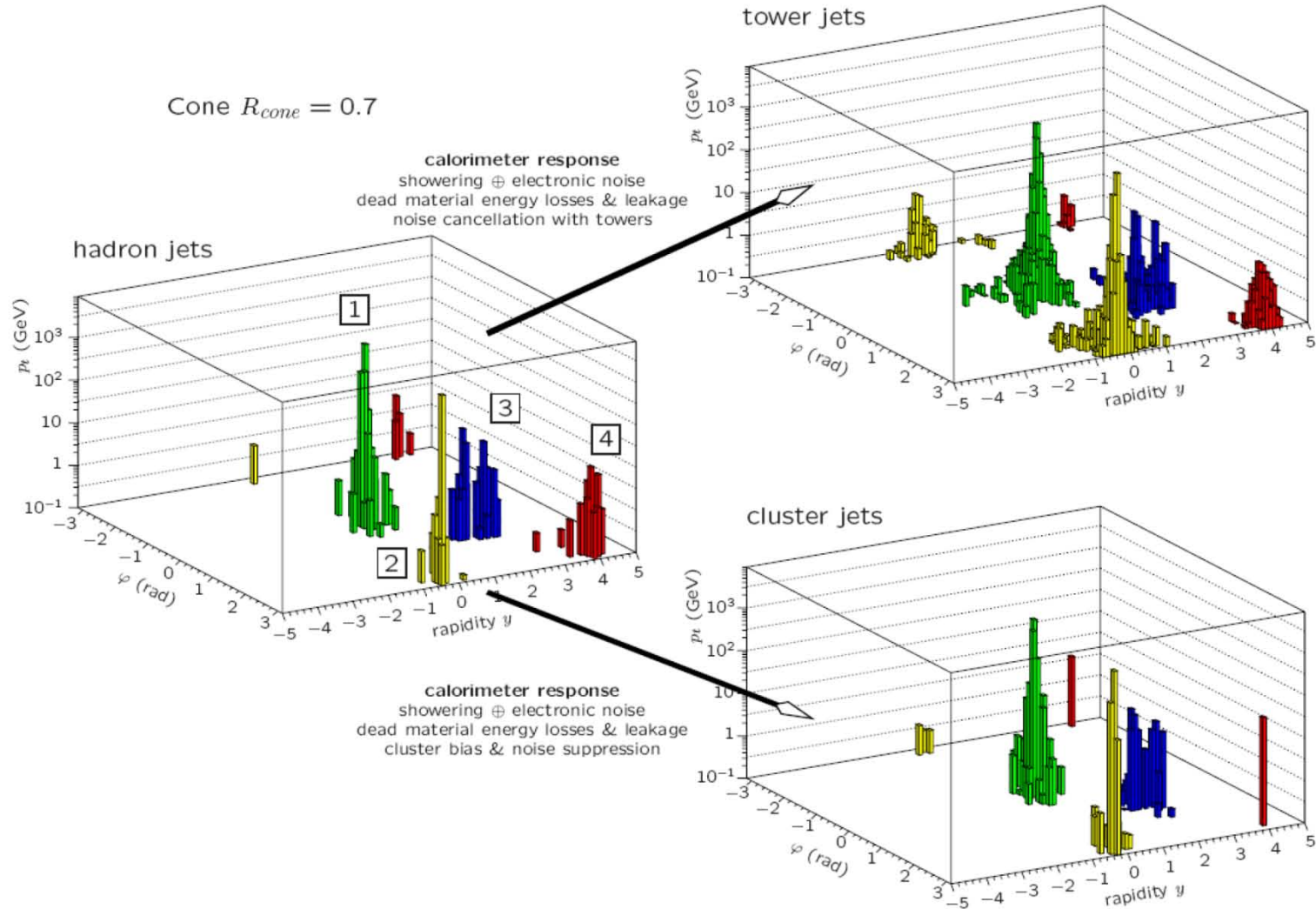
clusters



tracks



particles



Calorimeter jet response

Electromagnetic energy scale

Available for all signal definitions

No attempt to compensate or correct signal for limited calorimeter acceptance

Global hadronic energy scale

All signal definitions, but specific calibrations for each definition

Calibrations normalized to reconstruct full true jet energy in "golden regions" of calorimeter

Local hadronic energy scale

Topological clusters only
No jet context – calibration insufficient to recover calorimeter acceptance limitations – no corrections for total loss in dead material and magnetic field charged particles losses)

Unbiased and noise-suppressed towers:

$$\underbrace{\begin{pmatrix} E_{0,\text{jet}} \\ \vec{p}_{0,\text{jet}} \end{pmatrix}}_{\text{reconstructed calorimeter jet}} = \underbrace{\begin{pmatrix} \sum_{\text{towers in jet}} E_{0,\text{tower}} \\ \sum_{\text{towers in jet}} \vec{p}_{0,\text{tower}} \end{pmatrix}}_{\text{reconstructed calorimeter jet}} < \underbrace{\begin{pmatrix} \sum_{\text{particles in jet}} E_{\text{particle}} \\ \sum_{\text{particles in jet}} \vec{p}_{\text{particle}} \end{pmatrix}}_{\text{matched particle jet (truth reference)}}$$

Topological cell clusters:

$$\underbrace{\begin{pmatrix} E_{0,\text{jet}} \\ \vec{p}_{0,\text{jet}} \end{pmatrix}}_{\text{reconstructed calorimeter jet}} = \underbrace{\begin{pmatrix} \sum_{\text{clusters in jet}} E_{0,\text{cluster}} \\ \sum_{\text{clusters in jet}} \vec{p}_{0,\text{cluster}} \end{pmatrix}}_{\text{reconstructed calorimeter jet}} < \underbrace{\begin{pmatrix} \sum_{\text{particles in jet}} E_{\text{particle}} \\ \sum_{\text{particles in jet}} \vec{p}_{\text{particle}} \end{pmatrix}}_{\text{matched particle jet (truth reference)}}$$

Note at any scale:

$$m_{\text{jet}} = \sqrt{E_{\text{jet}}^2 - \vec{p}_{\text{jet}}^2} > 0 \text{ for } N_{\text{towers}}, N_{\text{clusters}} > 1$$



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Cell based calibration for all calorimeter signals and jets in "golden spot":

$$\begin{aligned}
 & \text{reconstructed calorimeter jet} \\
 & \left(\begin{array}{c} E_{\text{rec,jet}} \\ \vec{p}_{\text{rec,jet}} \end{array} \right) = \left(\begin{array}{c} \sum_{\text{cells in jet}} w(\rho_{\text{cell}}, \mathcal{R}_{\text{cell}}) \cdot E_{0,\text{cell}} + E_{\text{DM}} \\ \sum_{\text{cells in jet}} w(\rho_{\text{cell}}, \mathcal{R}_{\text{cell}}) \cdot \vec{p}_{0,\text{cell}} + E_{\text{DM}} \frac{\vec{p}_{0,\text{jet}}}{|\vec{p}_{0,\text{jet}}|} \end{array} \right) \\
 & = \left(\begin{array}{c} \sum_{\text{particles in jet}} E_{\text{particle}} \\ \sum_{\text{particles in jet}} \vec{p}_{\text{particle}} \end{array} \right) \\
 & \text{matched particle jet (truth reference)}
 \end{aligned}$$

(cells are extracted from unbiased or noise suppressed towers or topological clusters forming the jet)



Calorimeter jet response

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Available for all signal definitions

No attempt to compensate or correct signal for limited calorimeter acceptance

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Calibrations normalized to reconstruct full true jet energy in “golden regions” of calorimeter

Local hadronic energy scale

Topological clusters only

No jet context – calibration insufficient to recover calorimeter acceptance limitations – no corrections for total loss in dead material and magnetic field charged particles losses)

Locally calibrated clusters only:

$$\underbrace{\begin{pmatrix} E_{\text{rec,jet}} \\ \vec{p}_{\text{rec,jet}} \end{pmatrix}}_{\text{reconstructed calorimeter jet}} = \begin{pmatrix} \sum_{\text{clusters in jet}} E_{\text{rec,cluster}} \\ \sum_{\text{towers in jet}} \vec{p}_{\text{rec,cluster}} \end{pmatrix} \leq \underbrace{\begin{pmatrix} \sum_{\text{particles in jet}} E_{\text{particle}} \\ \sum_{\text{particles in jet}} \vec{p}_{\text{particle}} \end{pmatrix}}_{\text{matched particle jet (truth reference)}}$$



Final Jet Energy Scale (JES)

Final jet calibration

All corrections applied

Best estimate of true (particle) jet energy

Flat response as function of p_T

Uniform response across whole calorimeter

Relative energy resolution

Depends on the calorimeter jet response – calibration applies compensation corrections

Resolution improvements by including jet signal features

Requires corrections sensitive to measurable jet variables

Can use signals from other detectors

Determination with simulations

Measure residual deviations of the calorimeter jet response from truth jet energy

Derive corrections from the calorimeter response at a given scale as function of p_T (linearity) and pseudorapidity (uniformity) for all particle jets

Use numerical inversion to parameterize corrections

Conversion from truth variable dependence of response to reconstructed variable response



From simulations

Compare calorimeter response with particle jet energy as function of the particle jet energy

All jets, all regions, full kinematic coverage

Residual deviation from linearity

Depend on calorimeter energy scale – large for electromagnetic energy scale and local calibration due to missing jet level corrections

Small for global calibration due to jet energy normalization

Corrections can be extracted from residuals

A bit tricky – need to use numerical inversion (see later)

From experiment

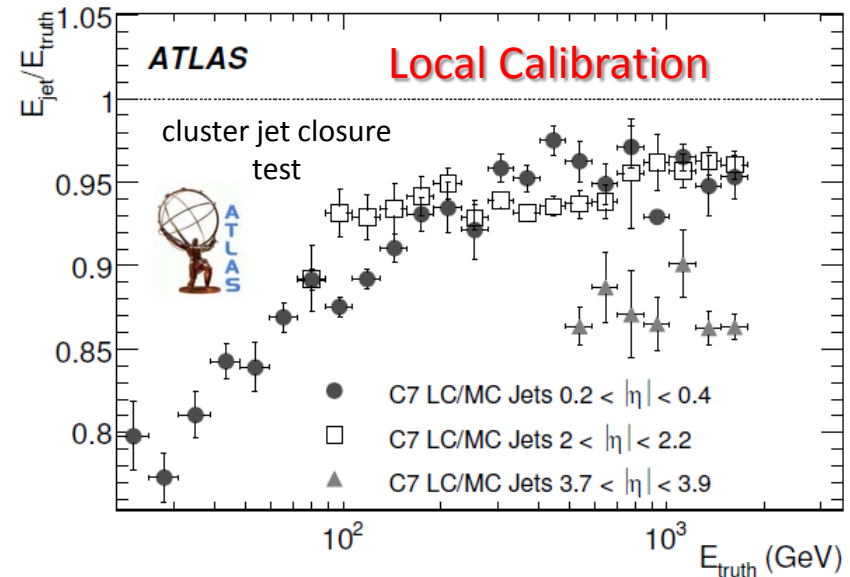
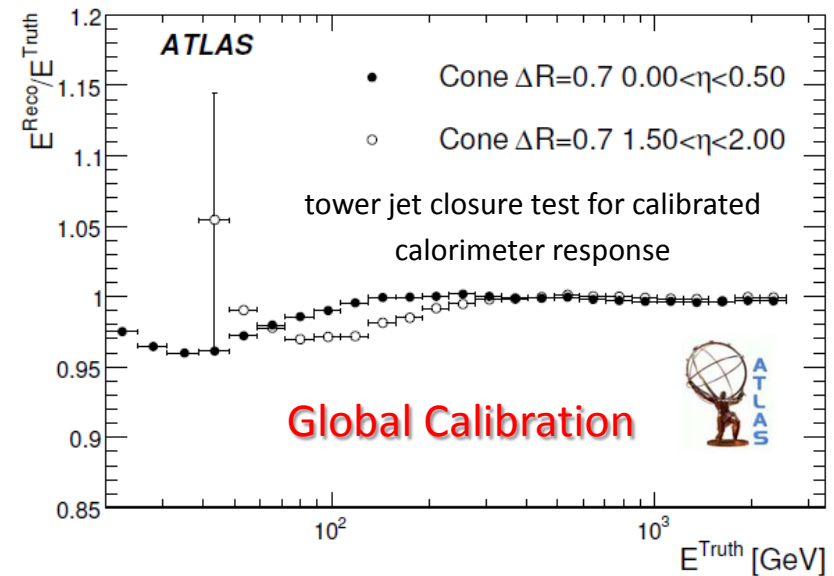
Validate and extract calibrations from collision data

W boson mass in hadronic decay is jet energy scale reference

pT balance of electromagnetic signal (Z boson, photon) and jet

Note change of reference scale

In-situ channels provide interaction (parton) level truth reference!



Simulations

Compare calorimeter response with particle jet energy as function of the jet direction

All jets in full kinematic range

Residual non-uniformities expected in cracks

Only jets in “golden regions” used for calibration

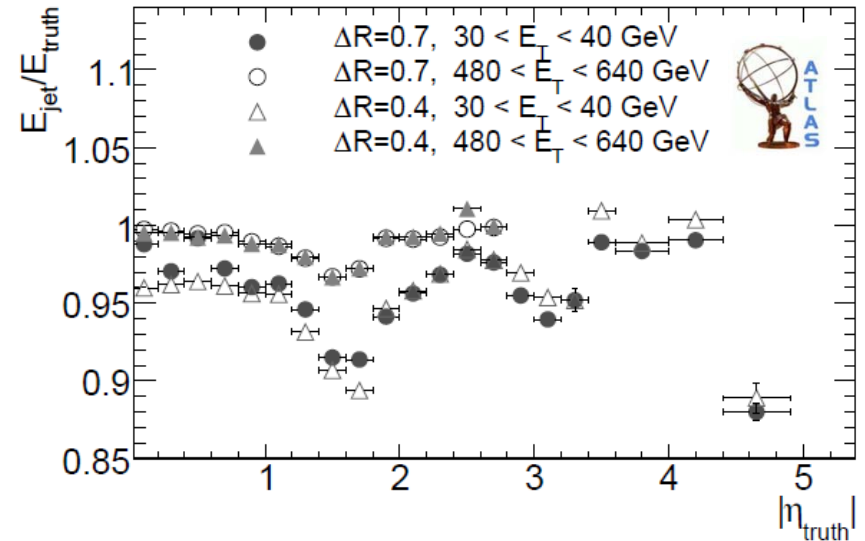
From experiment

Di-jet pT balance

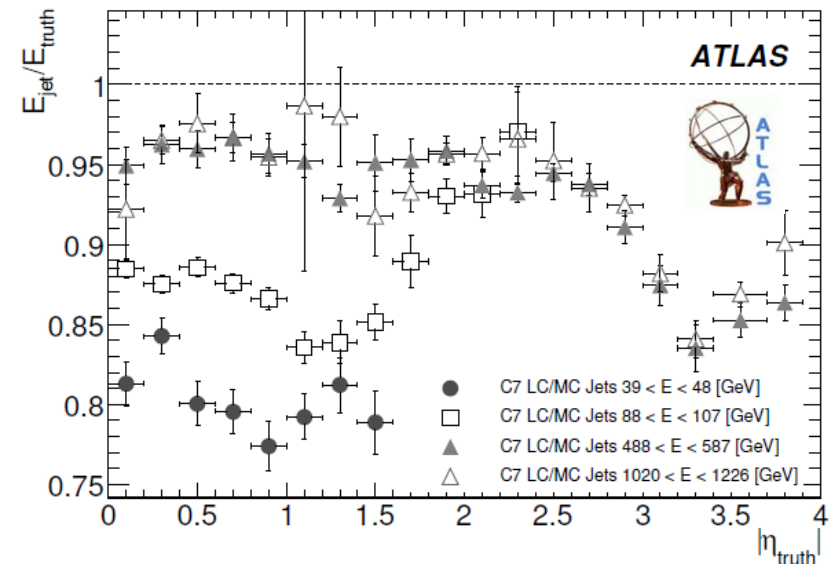
Balance pT of well calibrated jet in “golden region” with jet in other calorimeter regions

Can also use photon pT balance with jets outside of “golden region”

Global Calibration



Local Calibration



Simulations

Measure fluctuations of calorimeter jet energy as function of truth jet energy

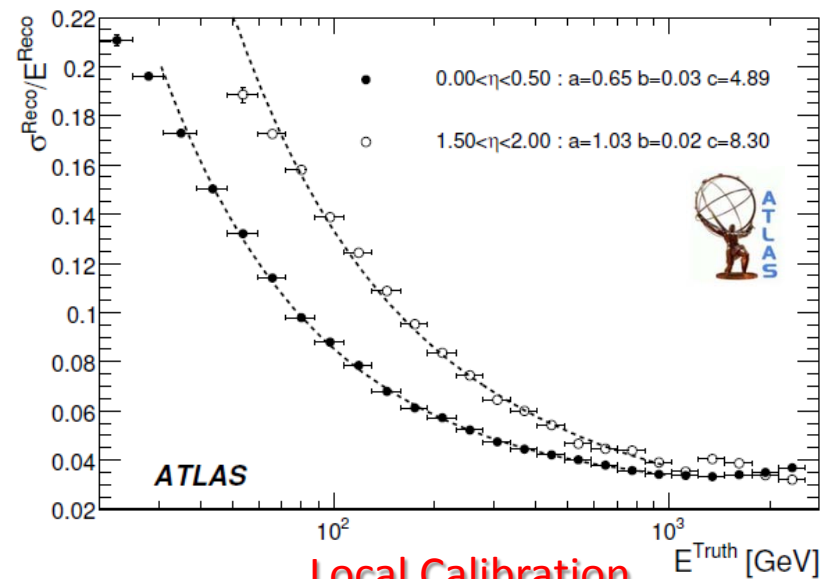
All jets in full kinematic range and in various regions of pseudo-rapidity

From experiment

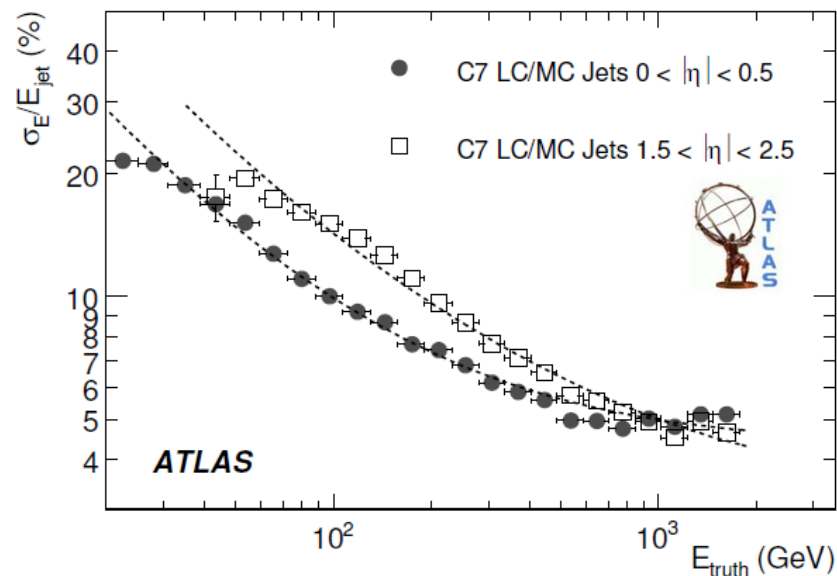
Di-jet final states

Measure relative fluctuations of jet energies in back-to-back (pT) balanced di-jets

Global Calibration



Local Calibration



Golden rule of calorimetric energy measurement

The fully calibrated calorimeter signal is most probably the true jet (or particle) energy

Interpretation holds only for symmetrically distributed fluctuations – mean value is identical to average value

The resolution of the measurement is given by the characteristics of the signal fluctuations

Can only be strictly and correctly understood in case of Gaussian response distributions

We need a normally distributed response!

Problem for all calibration techniques

Residual deviations from expected jet reconstruction performance must be measured as function of true quantities

Only then is the fluctuation of the response $R = E_{\text{reco}}/E_{\text{true}}$ really Gaussian after calibration

But need to apply corrections to measured jets

Need parameterization as function of reconstructed quantities

Simple re-binning does not maintain the Gaussian characteristics of the fluctuations – hard to control error!

Use numerical inversion to transfer the calibrations from true to measured parameters

Maintains Gaussian character



Toy model

Generate flat jet energy spectrum

Uniform energy distribution for E_{jet} in $[E_{\text{min}}, E_{\text{max}}]$

Smear true jet energy with Gaussian

Assume perfect average calibration

Width of distribution follows calorimetric energy resolution function

Calculate the response

In bins of E_{true} and in bins of $E_{\text{smear}} = E_{\text{reco}}$

Repeat exercise with steeply falling energy spectrum

Calibrated response:

$$\langle E_{\text{smear}} \rangle = \langle E_{\text{reco}} \rangle = \langle E_{\text{true}} \rangle$$

Calorimeter resolution function (no noise):

$$\frac{\sigma_E}{E} = \sqrt{\frac{a^2}{E_{\text{true}}} + c^2}$$

Smeared energy:

$$E_{\text{smear}} = E_{\text{true}} + r \cdot \sigma_E$$

r is a random number following the Gaussian PDF:

$$g(r) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}r^2\right]$$

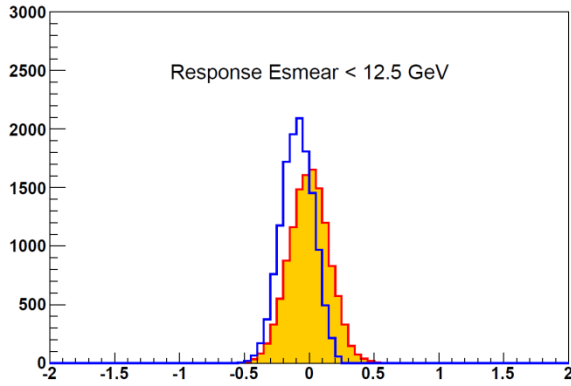
i.e. distributed around 0 with a width of 1

Response fluctuations:

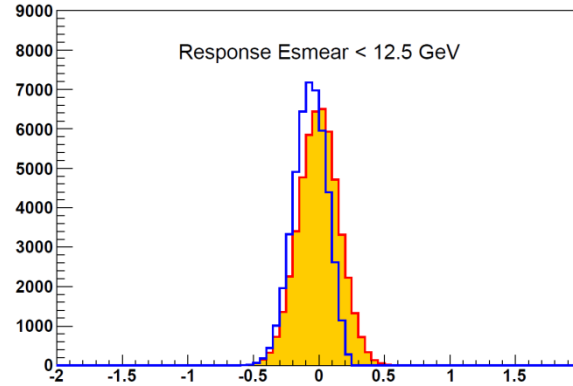
$$R = \frac{E_{\text{smear}} - E_{\text{true}}}{E_{\text{true}}} \text{ with } \langle R \rangle = 0$$





$$p(E_{\text{true}}) = \text{const}$$

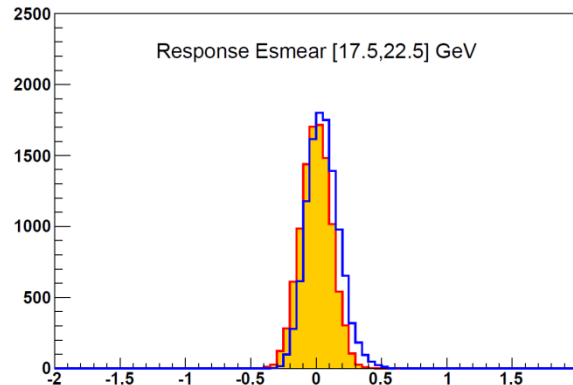
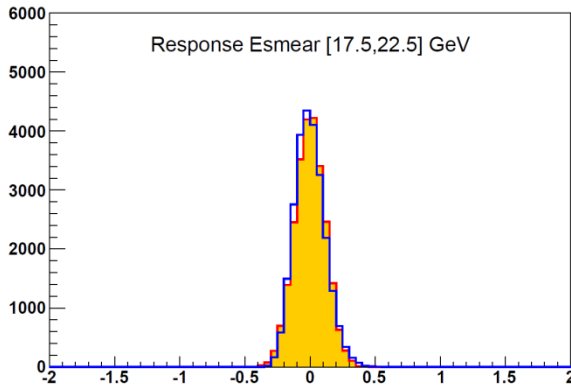


$$p(E_{\text{true}}) \propto E_{\text{true}}^{-4}$$

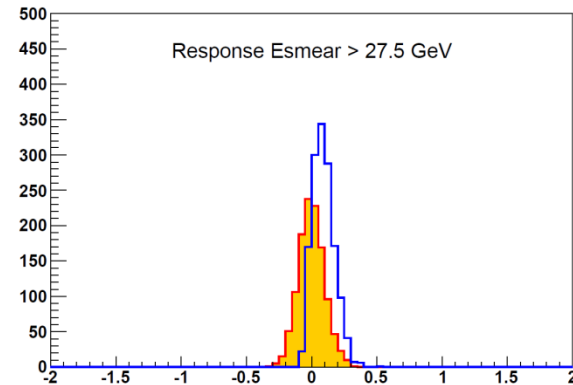
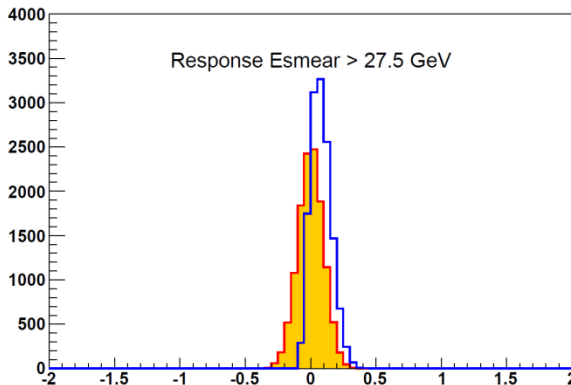


 binned in E_{true}
 binned in E_{smear}

Lower edge of E spectrum



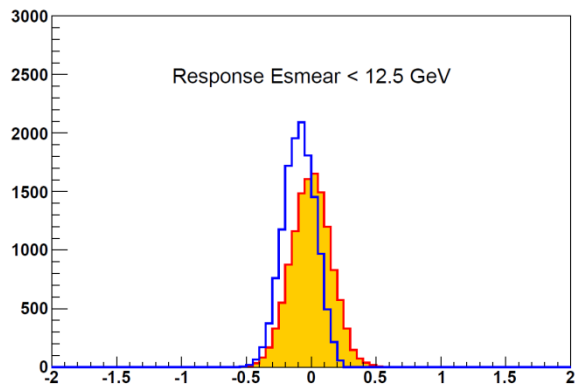
Center of E spectrum



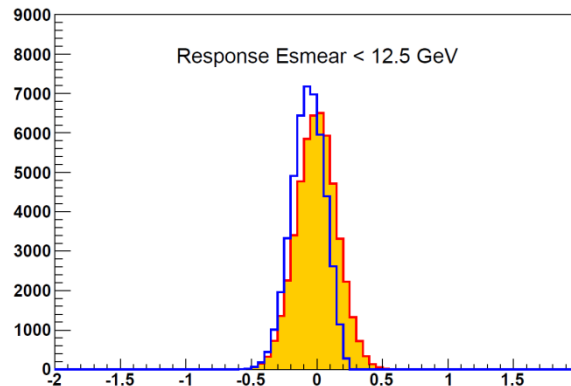
High end of E spectrum



$$p(E_{\text{true}}) = \text{const}$$



$$p(E_{\text{true}}) \propto E_{\text{true}}^{-4}$$

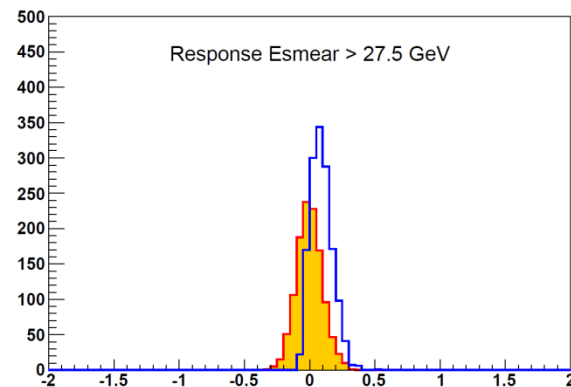
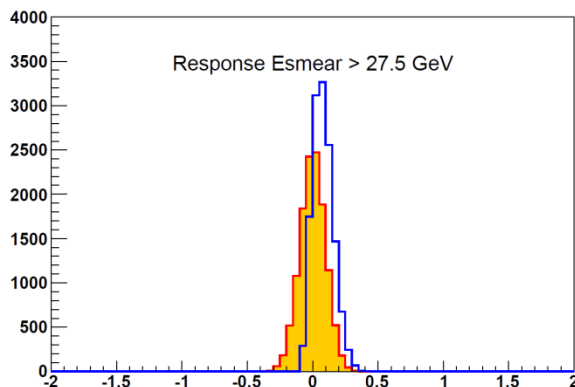


- binned in E_{true}
- binned in E_{smear}

Lower edge of E spectrum

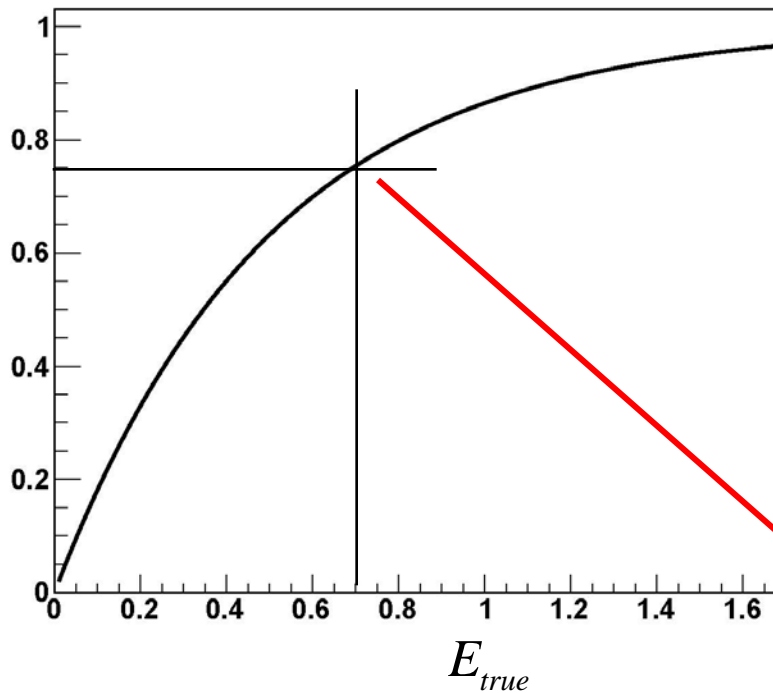
Response only Gaussian in
bins of true energy!

Center of E spectrum

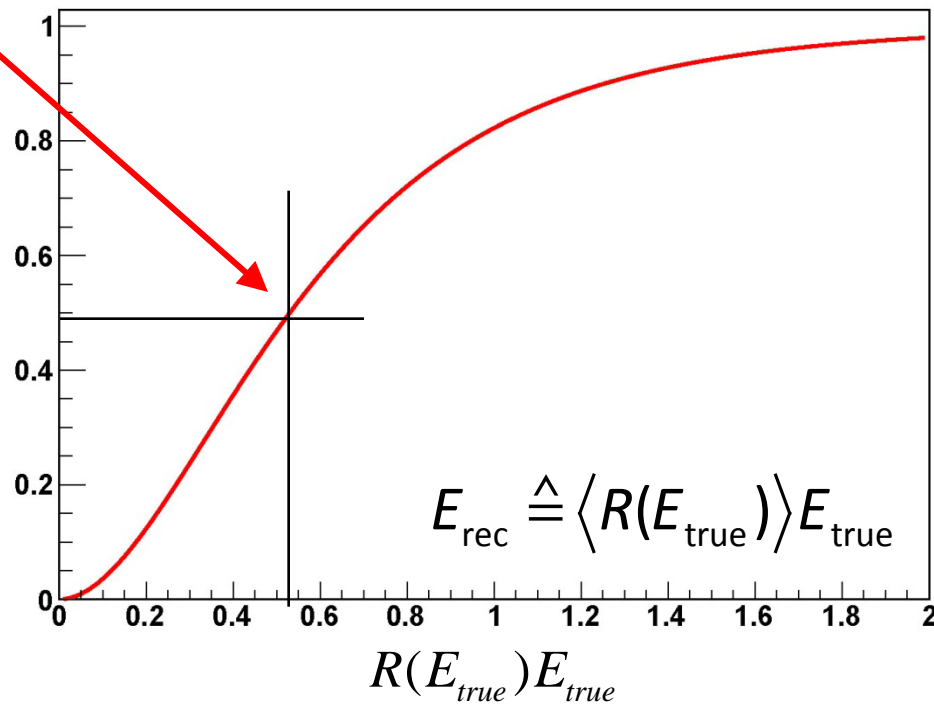


High end of E spectrum



$R(E_{true})$


Transfer of response function from dependence on true variable to dependence on measured variable

 $R(E_{rec})$


$$R(E_{rec}) = R(\langle R(E_{true}) \rangle \cdot E_{true})$$

$$E_{rec} \hat{=} \langle R(E_{true}) \rangle E_{true}$$



Often simple functions

Address residual energy (pT) and direction dependence of calorimeter jet response

Determine response functions R in bins of true jet pT and reconstructed pseudo-rapidity $\eta_{\text{rec,jet}}$

Apply numerical inversion to determine calibration functions in reconstructed variable space ($p_{\text{T,rec,jet}}$, $\eta_{\text{rec,jet}}$)

Use calibration functions to get jet energy scale

Technique can be applied to locally or globally calibrated jet response, with likely different calibration functions

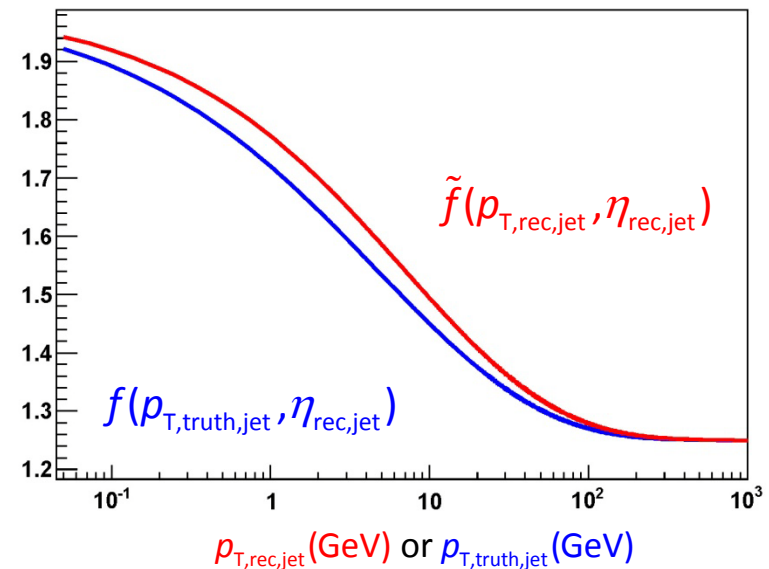
$$f(p_{\text{T,true,jet}}, \eta_{\text{reco,jet}}) = R^{-1}(p_{\text{T,true,jet}}, \eta_{\text{reco,jet}})$$

with $\eta_{\text{reco,jet}} \approx \eta_{\text{true,jet}}$ and

$$R^{-1}(p_{\text{T,true,jet}}, \eta_{\text{reco,jet}}) = \left\langle \frac{E_{\text{true,jet}}}{E_{\text{rec,jet}}} \right\rangle (p_{\text{T,true,jet}}, \eta_{\text{reco,jet}})$$

then apply numerical inversion

$$f(p_{\text{T,true,jet}}, \eta_{\text{reco,jet}}) \xrightarrow{\text{numerical inversion}} \tilde{f}(p_{\text{T,rec,jet}}, \eta_{\text{reco,jet}})$$



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Technique can be applied to locally or globally calibrated jet response, with likely different calibration functions

global calibration:

$$\begin{pmatrix} E_{\text{calib,jet}} \\ \vec{p}_{\text{calib,jet}} \end{pmatrix} =$$

$$\tilde{f}(p_{\text{T,rec,jet}}, \eta_{\text{rec,jet}}) \cdot \left(\begin{array}{l} \overbrace{\sum_{\text{cells in jet}} w(\rho_{\text{cell}}, \mathcal{R}_{\text{cell}}) \cdot E_{0,\text{cell}} + E_{\text{DM}}}^{E_{\text{reco,jet}}} \\ \underbrace{\sum_{\text{cells in jet}} w(\rho_{\text{cell}}, \mathcal{R}_{\text{cell}}) \cdot \vec{p}_{0,\text{cell}} + E_{\text{DM}} \frac{\vec{p}_{0,\text{jet}}}{|\vec{p}_{0,\text{jet}}|}}_{\vec{p}_{\text{reco,jet}}, \text{ with } p_{\text{T,rec,jet}} = |\vec{p}_{\text{reco,jet}}| \sqrt{1 - \tanh^2 \eta_{\text{reco,jet}}}} \end{array} \right)$$

local calibration:

$$\begin{pmatrix} E_{\text{calib,jet}} \\ \vec{p}_{\text{calib,jet}} \end{pmatrix} = \tilde{f}'(p_{\text{T,rec,jet}}, \eta_{\text{rec,jet}}) \cdot \left(\begin{array}{l} \sum_{\text{clusters in jet}} E_{\text{rec,cluster}} \\ \sum_{\text{towers in jet}} \vec{p}_{\text{rec,cluster}} \end{array} \right)$$



Why not use direct relation between reconstructed and true energy?

Same simulation data input

Has been used in some experiments

Dependence on truth energy spectrum

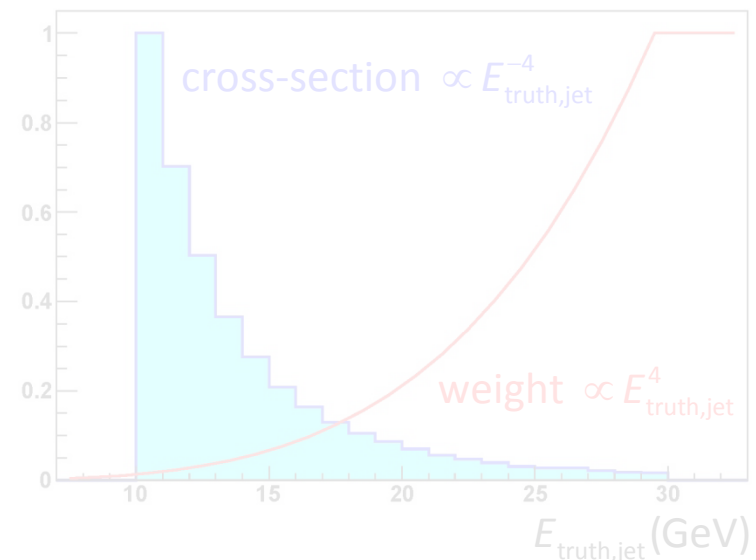
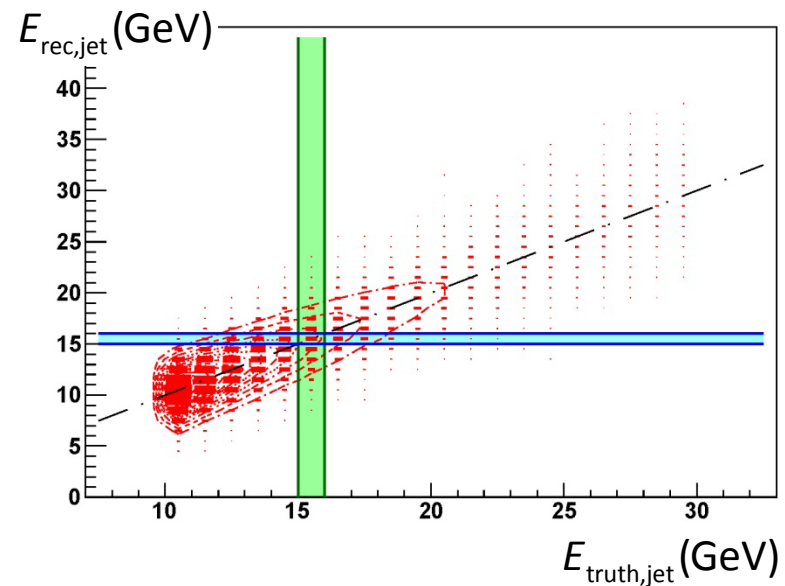
Need to make sure calibration sample is uniform in truth energy

Alternatively, unfold driving truth energy spectrum

Residual non-gaussian behaviour of truth energy distribution

Error on reconstructed energy hard to understand

Could still use response distribution \rightarrow same issues as discussed on previous slide!



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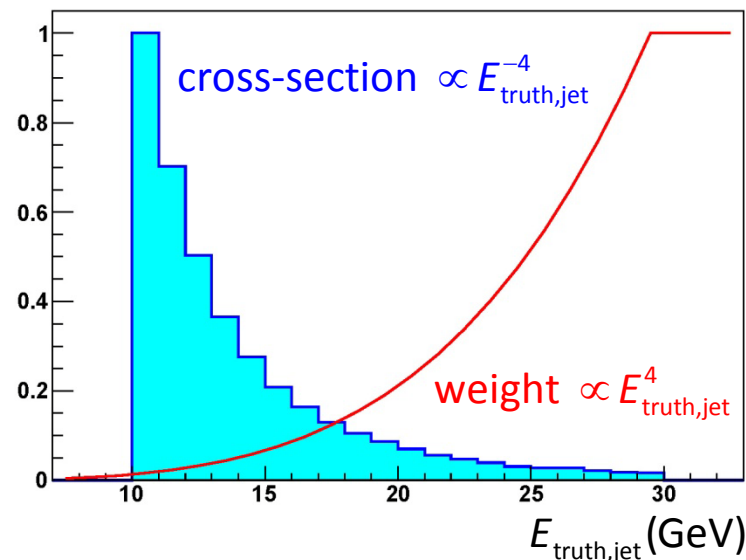
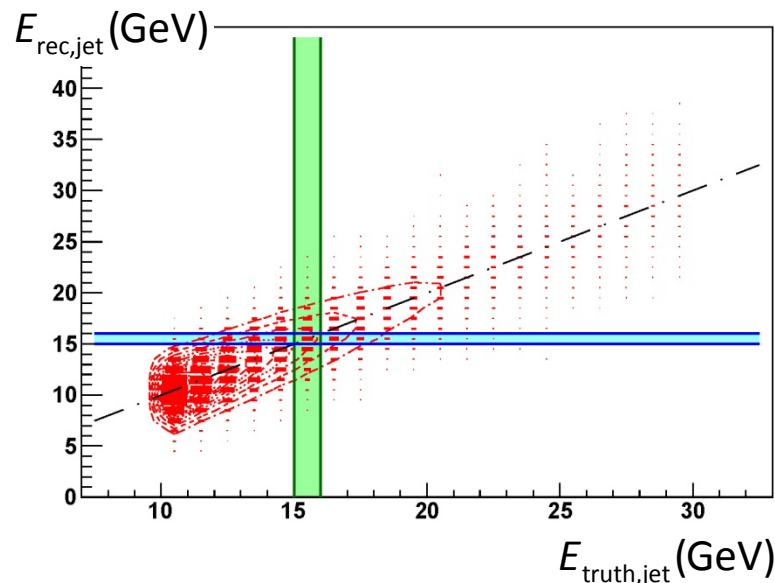
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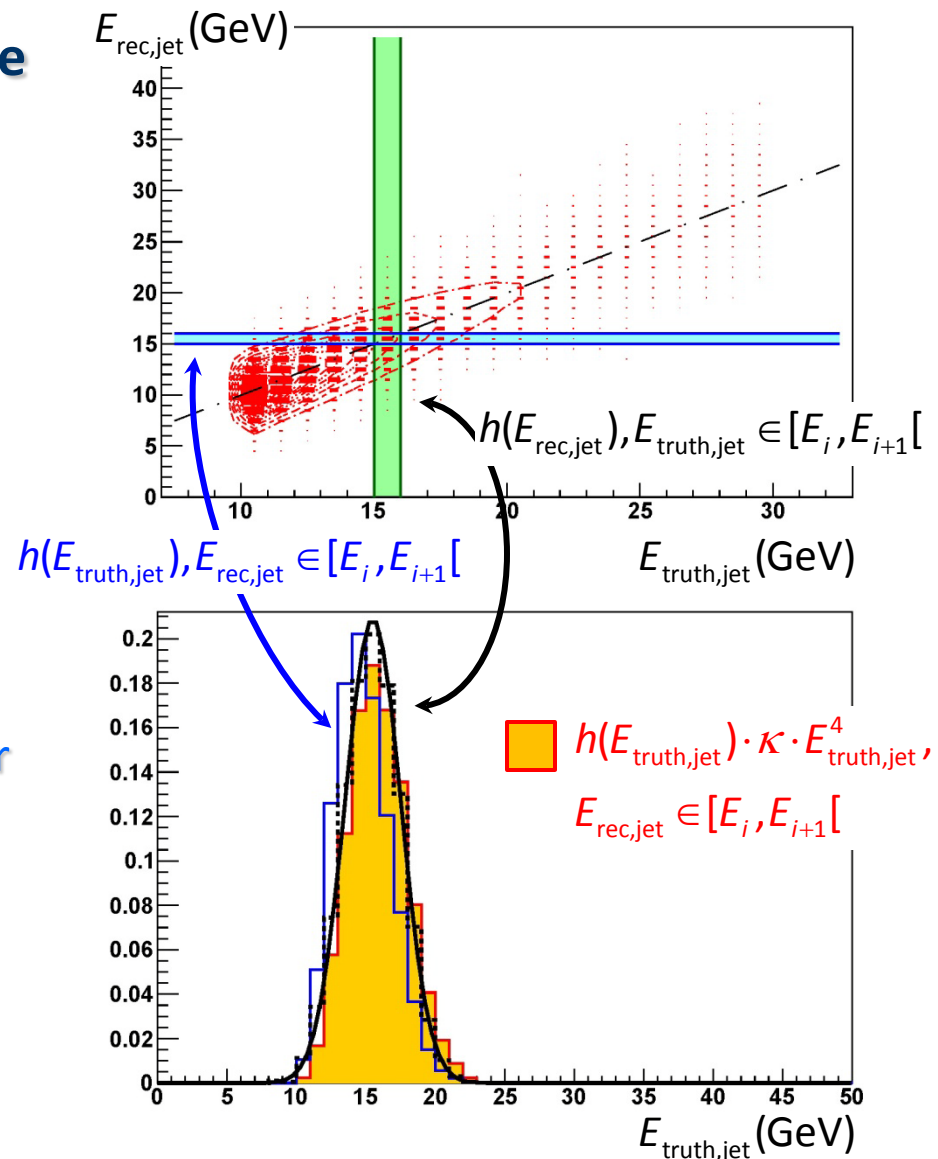
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Strategy from simulations

Determine all calibrations with fixed conditions

- Ideal detector model – everything is aligned
- Fixed (best) GEANT4 shower model – from testbeam evaluations
- Fixed calorimeter signal definition – e.g., towers
- Fixed jet definition – like seeded cone with size 0.7
- Fixed final state – QCD di-jets preferred

Study change in performance for changing conditions with ideal calibration applied

- Detector misalignment and changes in material budgets
- Different shower GEANT4 model
- Different calorimeter signal definitions – e.g., clusters
- Different jet definitions – e.g., kT, AntikT, different cone or cone sizes...
- Different physics final state – preferably more busy ones like SUSY, ttbar,...

Use observed differences as systematic error estimates

Use of collision data

Compare triggered final states with simulations

- Level of comparison represents understanding of measurement – systematic error (at least for standard final states)

Use in-situ final states to validate calibration

- Careful about biases and reference levels (see session 9)



Calibration functions determined with “perfect” detector description and one reference jet definition

Validate performance in perfect detector

Signal linearity & resolution

Quality of calibration for a real detector

A priori unknown real detector

Absolute and relative alignments, inactive material distributions

Estimate effect of distorted (real) detector

Implement realistic assumptions for misalignment in simulations

Small variations of inactive material thicknesses and locations

But use “perfect” calibration for reconstruction

Change jet signals

Tower or clusters

E.g, change from reference calorimeter signal

Different jet finder

E.g., use kT instead of cone

Different configuration

E.g., use narrow jets (cone size 0.4) instead of wide jets (0.7)



Response

Linear within +/-1% after calibration applied for $p_T > 100$ GeV

Clear improvement compared to basic signal scale

Problems with low p_T regime

ATLAS limit $p_T > 20-40$ GeV, depending on luminosity

May be resolution bias – under study

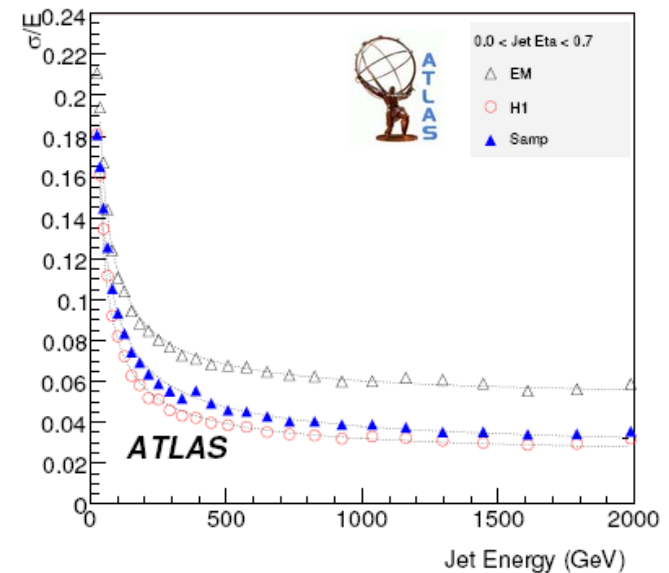
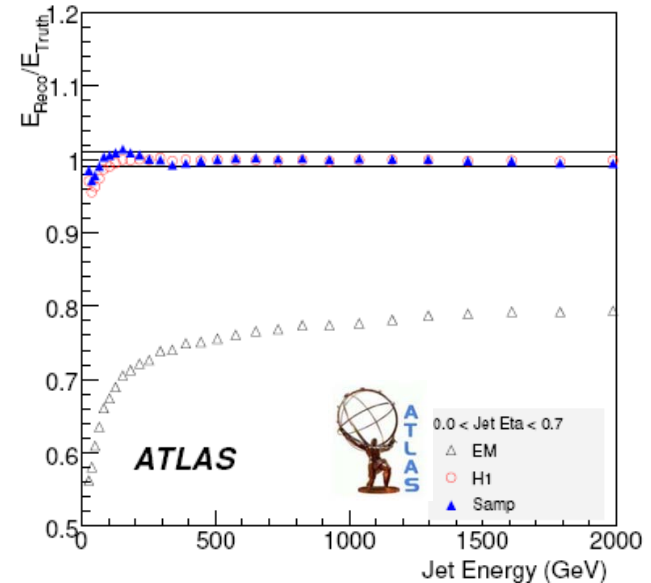
Resolution

Jet energy resolution clearly improved by calibration as well

Slight dependence on calibration strategy

Close to required performance

$$\frac{\sigma}{E} \approx \frac{65\%}{\sqrt{E}} \oplus 3\%$$

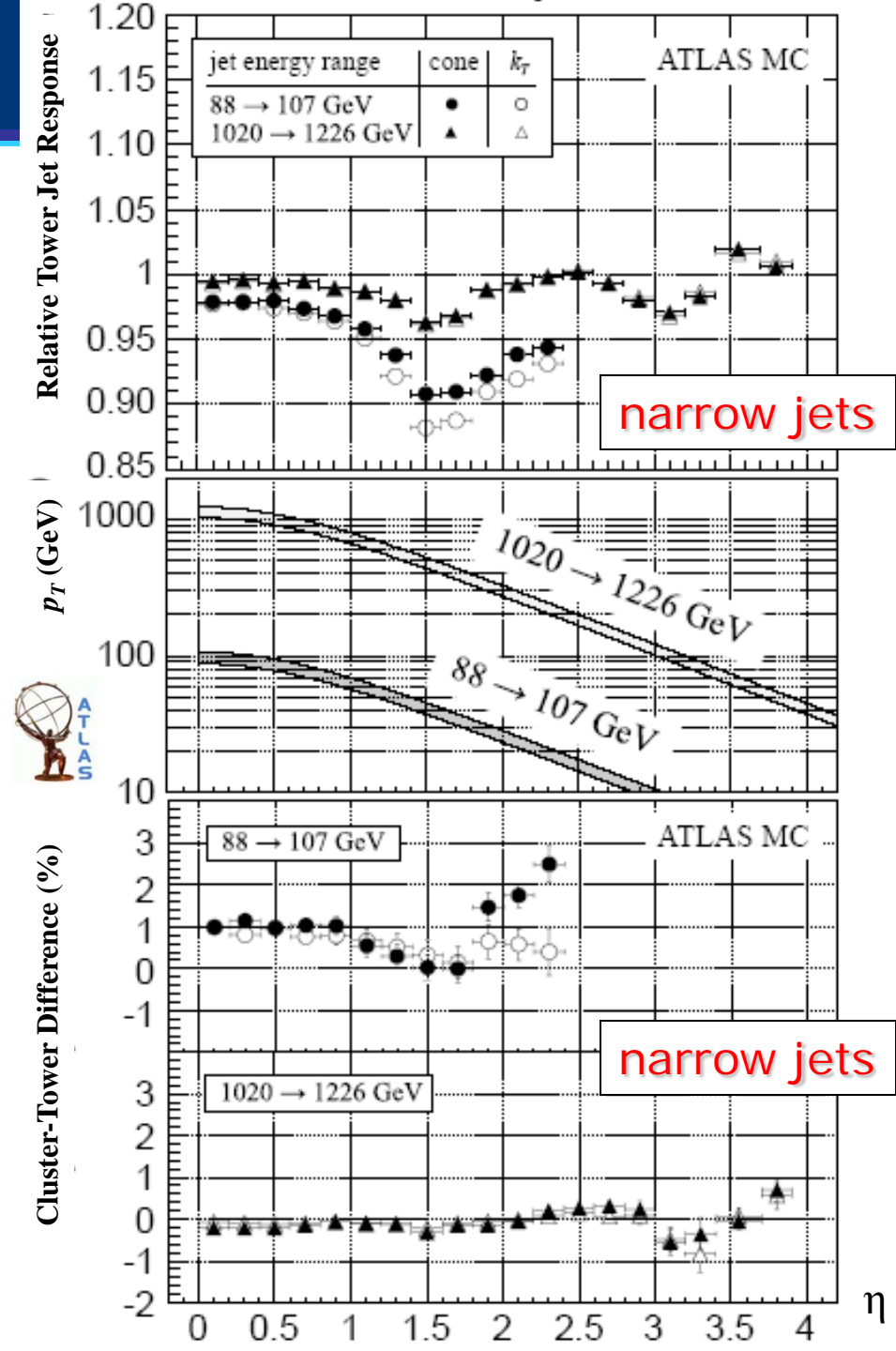


Characterizes “real” detector jet response

- Variation of response with direction
- Changing inactive material distribution
- Cracks between calorimeter modules

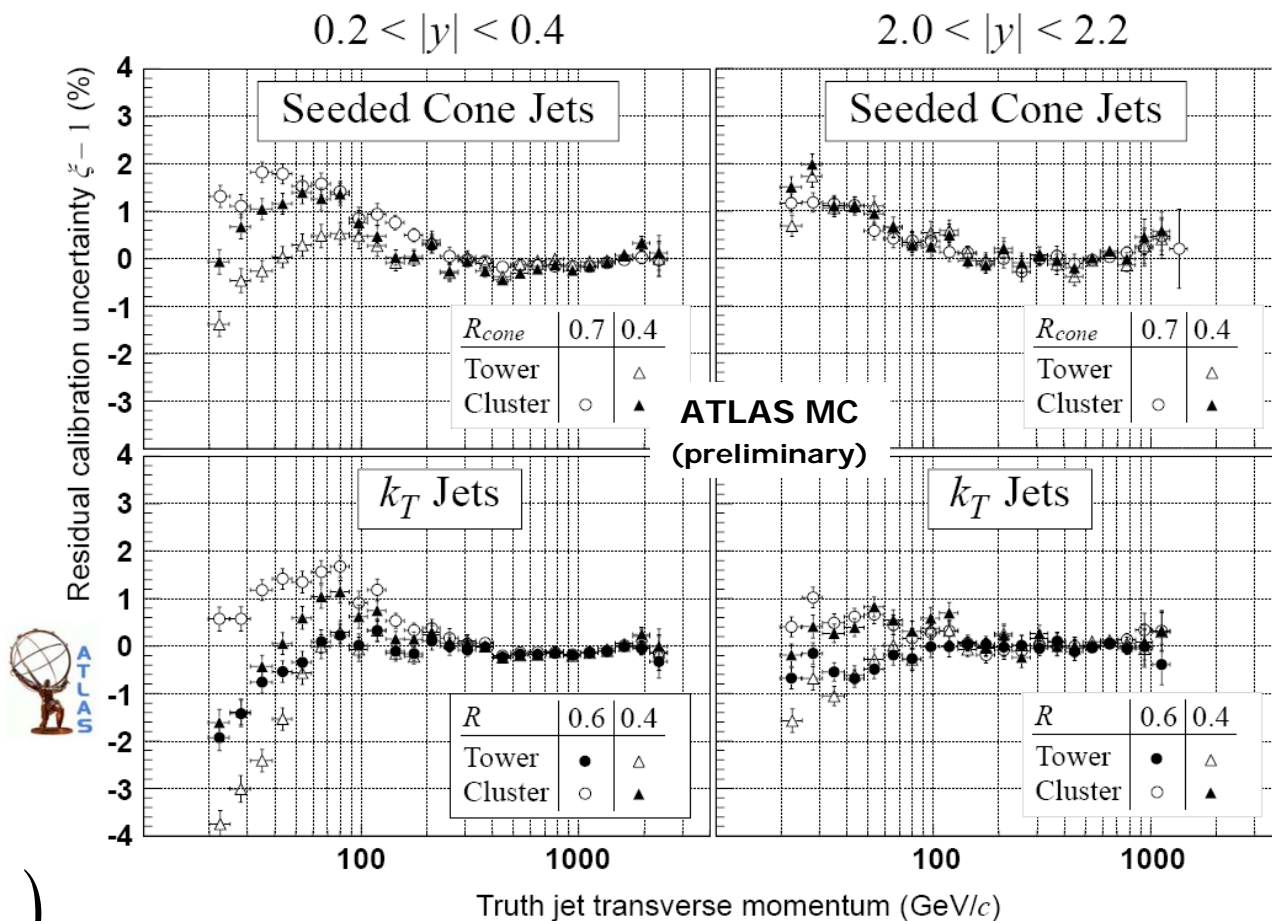
Variations

- No strong dependence on calorimeter signal definition
 - Towers/clusters
- ATLAS cone jet performs better in crack region at low p_T



Estimated effect of a distorted detector

Effect of detector distortion depends on jet size, calo signal choice, and kinematic domain



$$\xi = \frac{\left(E_{\text{rec,jet}} / E_{\text{truth,jet}} \right)_{\text{distorted}}}{\left(E_{\text{rec,jet}} / E_{\text{truth,jet}} \right)_{\text{ideal}}}$$



Larger fluctuations for kT jets at low pT

Vacuum effect for tower jets?

Less pronounced for cluster jets

Noise suppression important in this domain

Very similar resolutions at high pT

No strong dependence on jet definition

No strong dependence on calorimeter signal definition

No significant noise contribution anymore

$$\Delta_\sigma = \left(\frac{\sigma}{E}\right)_{\text{cluster}}^2 - \left(\frac{\sigma}{E}\right)_{\text{tower}}^2$$

$$\psi_\sigma = \begin{cases} \sqrt{\Delta_\sigma} & \text{for } \Delta_\sigma > 0 \\ -\sqrt{-\Delta_\sigma} & \text{for } \Delta_\sigma < 0 \end{cases}$$

