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General cluster features

Motivated by shower reconstruction

No bias in signal definition towards reconstruction of a certain, possibly very specific, physics signal object like a jet

Clusters have shapes and location information

Spatial cell energy distributions and their correlations drive longitudinal and lateral extensions

Density and energy sharing measures

Signal center of gravity and (directional) barycenter

Shapes are sensitive to shower nature

At least for a reasonable clustering algorithm

Local (cluster) calibration strategy

First reconstruct truly deposited energy at cluster location...

e/h, mostly

...then correct for other energy losses in the vicinity of signal cluster

Dead material energy losses and signal losses due to noise suppression

Calibration input

Reconstructed cluster shapes represent shower shapes

E.g., dense and compact clusters indicate electromagnetic shower activity anywhere in the calorimeter

Can be intrinsic to a hadronic shower!

Calibration functions can exploit the cluster shapes to apply the corrections for e/h ≠ 1 dynamically

Location of cluster together with shape

E.g., dense and compact clusters located in electromagnetic calorimeter indicate electron or photon as particle originating the signal

Cluster not (part of) hadronic shower signal!

Clusters can classified before calibration

Electron/photon clusters need different calibration than dense clusters from hadronic showers!

Cluster calibration extensions

Shapes, location and size also indicate possible energy losses around the cluster

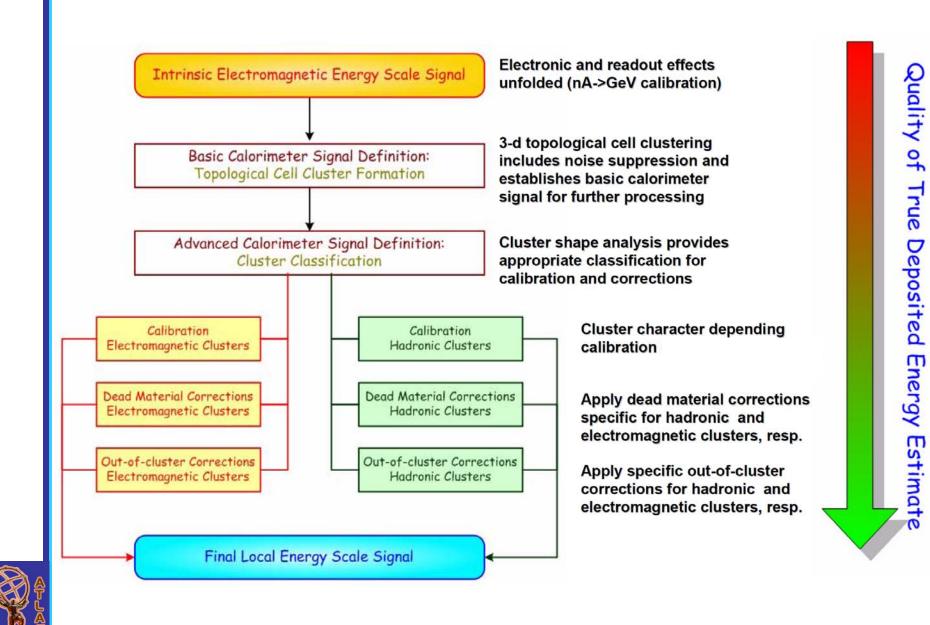
Some correlations between energy losses in inactive material in front or inside of clusters

Cluster size and signal neighbourhood sensitive to lost true signal in noise suppression algorithm Out-of-cluster corrections



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Phase-space pion counting method

Classify clusters using the correlation of Shower shape variables in single $\pi \pm$ MC events

 $\lambda_{cluster} = cluster center of gravity depth in calorimeter$

$$\overline{\rho}_{\text{cluster}} = \frac{1}{E_{0,\text{cluster}}} \sum_{\text{cells in cluster}} E_{0,\text{cell}} \cdot \rho_{\text{cell}}$$

Electromagnetic fraction estimator in bin of shower shape variables:

$$F \equiv \frac{\varepsilon(\pi^0)}{\varepsilon(\pi^0) + 2\varepsilon(\pi^-)}$$

 $\mathcal{E}(X) = \frac{N(X) \text{ producing a cluster in a given } (\eta, E_{0, \text{cluster}}, \lambda_{\text{cluster}}, \overline{\rho}_{\text{cluster}})}{N(X) \text{ total}}$

Implementation

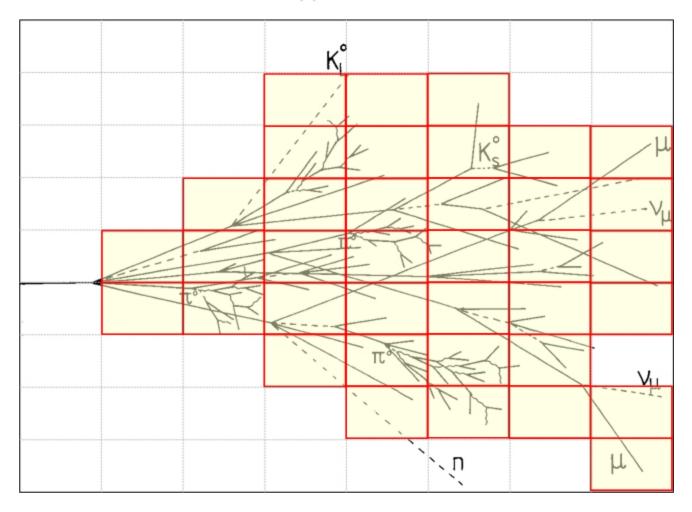
keep F in bins of η , E, λ , ρ of clusters for a given cluster

If E < 0, then classify as unknown Lookup F from the observables $|\eta|$, E, λ , ρ Cluster is EM if F > 50%, hadronic otherwise



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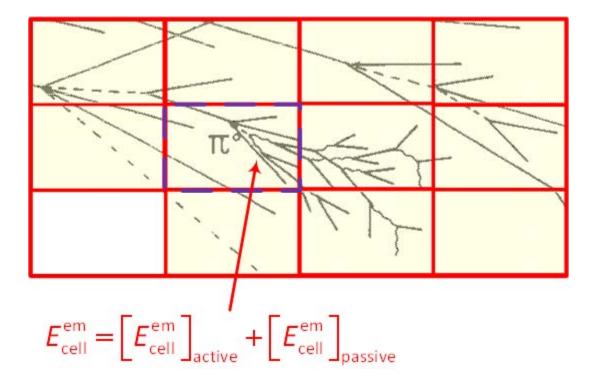






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Calibration with cell signal weights

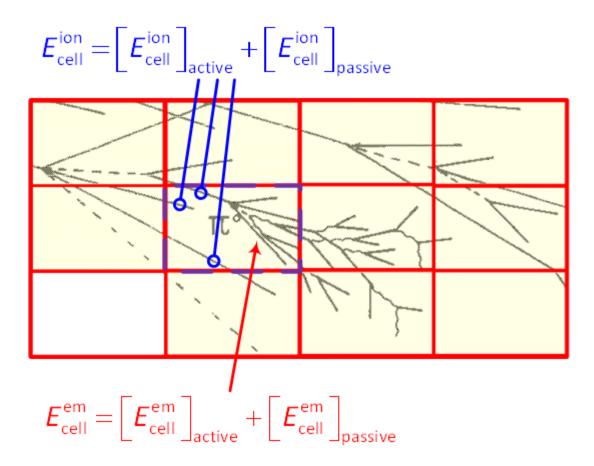






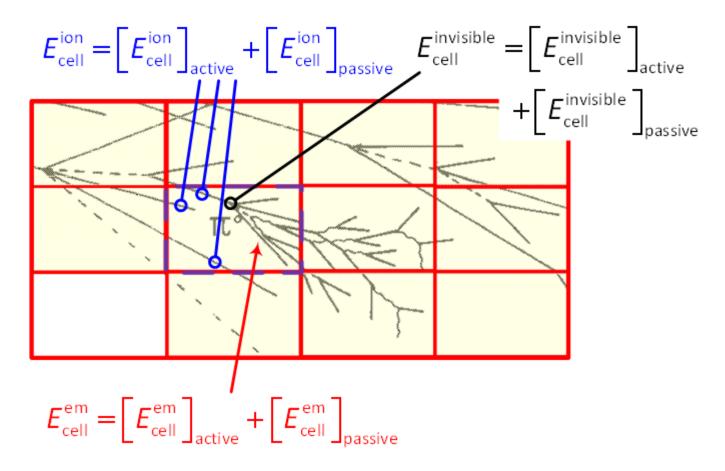
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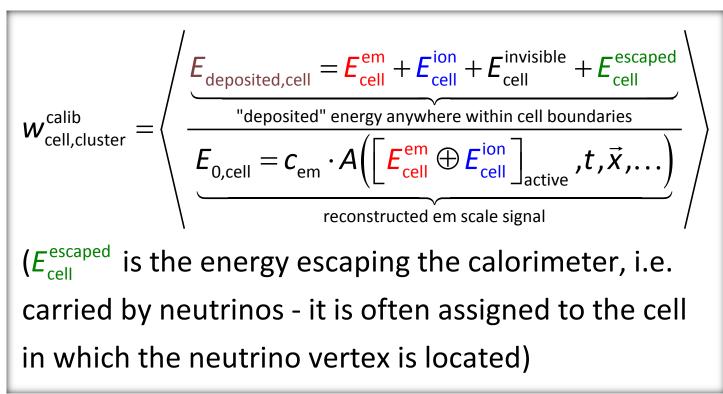


Idea is to compensate for lack of pion response in each cell

Pioneered in CDHS and applied in H1

Uses deposited energies in cells

Deposit can be in active or passive medium of calorimeter!





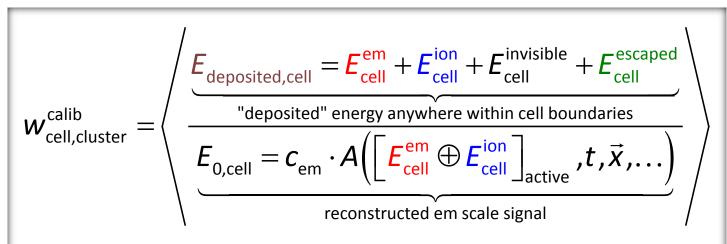


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Only signal contribution from energy deposited by electromagnetic sub-showers and through ionization by charged particles!



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Deposit can be in active or passive medium of calorimeter!

Energy deposited in cell not available in experiment

- Use of detector simulations
 - Deposited energy and signal available
 - Use "unit cell" volume concept to collect invisible energies
 - Shower model dependent!
- Use single pion testbeam data
 - Develop model for weights in cells
 - Fit parameters of model using cells testbeam

Minimize resolution with beam energy constraint

Statistical – does not necessarily produce the correct weights!



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Use a dynamically self-adjusting calibration weight

High cell signal density → electromagnetic deposit Low cell signal density → hadronic deposit

Principal weighting function characteristics

Depends on cell energy density Depends on cell location Accidental application to electron signals should yield correct energy as well

Extraction of weighting functions

Minimize resolution in (pion) testbeam data

Fitting function model

May not produce the correct weights - may even be unphysical!

Use simulation

Deterministic approach relates signal to deposited energy within cell volume – no fitting!

May depend on details of (hadronic) shower modeling

$$E_{\text{rec,cell}} = w_{\text{cell}}(\ldots) \cdot E_{0,\text{cell}} = E_{\text{deposited,cell}}$$



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Use a dynamically self-adjusting calibration weight

High cell signal density → electromagnetic deposit

Low cell signal density \rightarrow hadronic deposit

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e.g. in H1:

$$E_{\text{rec,cell}} = \underbrace{\left(1 + \alpha(\vec{x}, \ldots) \cdot e^{-\beta(\vec{x}, \ldots) \cdot \rho_{\text{cell}}}\right)}_{= w(\rho_{\text{cell}}, \vec{x}, \ldots)} \cdot E_{0,\text{cell}},$$

with $\lim_{\rho_{\text{cell}} \to \infty} E_{\text{rec,cell}} = E_{0,\text{cell}}$



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Basic idea

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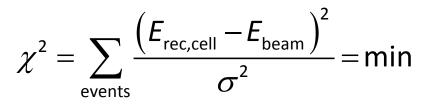
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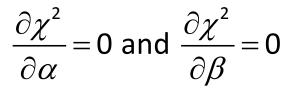
Deterministic approach relates signal to deposited energy within cell volume - no

shower modeling

Fit
$$\alpha(\vec{x},...), \beta(\vec{x},...)$$
 with



i.e.:





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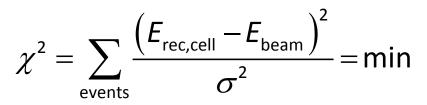
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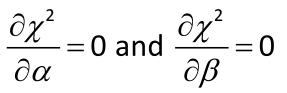
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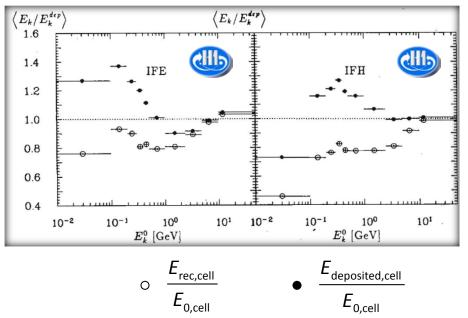
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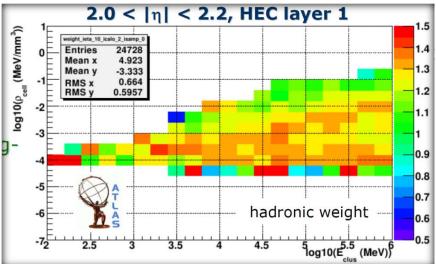
Use simulation

Deterministic approach relates signal to deposited energy within cell volume – no fitting!

May depend on details of (hadronic) shower modeling

ATLAS cluster-based approach:

- 1. Use only cells in hadronic clusters
- 2. Cluster sets global energy scale as a reference for densities
- 3. Calculate $E_{deposited,cell}/E_{0,cell}$ from single pion simulations in bins of cluster energy, cell energy density, cluster direction, and calorimeter sampling layer
- 4. Store $[E_{deposited,cell}/E_{0,cell}]^{-1}$ in look-up tables





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- 4. Store $[E_{deposited,cell}/E_{0,cell}]^{-1}$ in look-up tables
- Retrieve weights for any cell in any cluster from look-up table to reconstruct cell and cluster energies

$$E_{\text{rec,cluster}}^{\text{calib}} = \sum_{\text{cells in cluster}} E_{\text{rec,cell}} = \sum_{\text{cells in cluster}} W_{\text{cell,cluster}}^{\text{calib}} (E_{0,\text{cluster}}, \eta_{\text{cluster}}, S_{\text{cell}}, \rho_{\text{cell}}) \cdot E_{0,\text{cell}}$$

cells in cluster



Dead material

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Energy losses not directly measurable Signal distribution in vicinity can help Introduces need for signal corrections up to O(10%)

> Exclusive use of signal features Corrections depend on electromagnetic or hadronic energy deposit

Major contributions

Upstream materials Material between LArG and Tile (central)

Cracks

dominant sources for signal losses

|η|≈1.4-1.5 |η|≈3.2

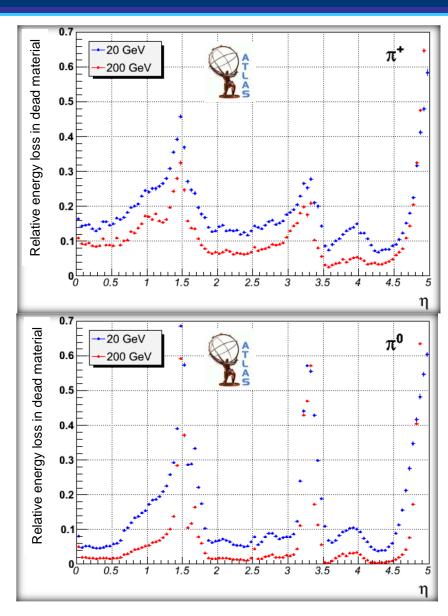
Clearly affects detection efficiency for particles and jets

Already in trigger! Hard to recover jet reconstruction inefficiencies

Generate fake missing Et contribution Topology dependence of missing Et reconstruction quality

Additive correction:

$$E_{\text{rec,cluster}}^{\text{calib+DM}} = E_{\text{rec,cluster}}^{\text{calib}} + E_{\text{rec,cluster}}^{\text{DM}} (\vec{x}_{\text{cluster}}, \ldots)$$





Compensate loss of true signal

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Limited efficiency of noise suppression scheme

Discard cells with small true energy not close to a primary or secondary seed

Accidental acceptance of a pure noise cell

Can be significant for isolated pions

10% at low energy

Correction derived from single pions

Compensates the isolated particle loss

But in jets neighboring clusters can pick up lost energy

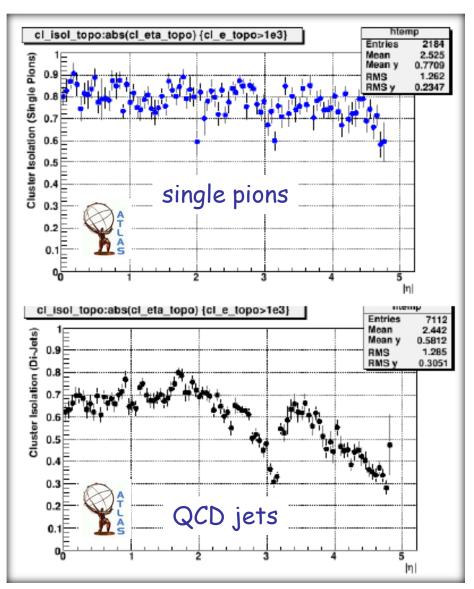
Use isolation moment to measure effective "free surface" of each cluster

Scale single pion correction with this moment (0...1)

Additive correction:

 $E_{\rm rec, cluster}^{\rm calib+DM+OOC} = E_{\rm rec, cluster}$

 $= E_{\text{rec,cluster}}^{\text{calib+DM}} + E_{\text{rec,cluster}}^{\text{OOC}} (\vec{x}_{\text{cluster}}, m_{\text{isol}}, E_{0,\text{cluster}}, \ldots)$



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Attempt to calibrate hadronic calorimeter signals in smallest possible signal context

Topological clustering implements noise suppression with least bias signal feature extraction

Residual concerns about infrared safety!

No bias towards a certain physics analysis

Calibration driven by calorimeter signal features without further assumption

Good common signal base for all hadronic final state objects

Jets, missing Et, taus

Factorization of cluster calibration

Cluster classification largely avoids application of hadronic calibration to Electromagnetic signal objects

Low energy regime challenging

Signal weights for hadronic calibration are functions of cluster and cell parameters and variables

 $\begin{array}{c} \text{Cluster energy and} \\ \text{direction} \\ \text{Cell signal density and} \\ \text{location (sampling} \\ \text{layer}) \\ \begin{array}{c} \text{Dead material and out of} \\ \text{cluster corrections are} \\ \text{independently applicable} \\ \text{Factorized calibration} \\ \text{scheme} \end{array} \\ \begin{array}{c} E_{\text{loc}}^{\text{loc}} \\ E_{\text{loc}}^{$

$E_{true}^{jet} =$	$\frac{E_{dep}^{calo}}{E_{dep}} + E_{mag}^{loss} + E_{low}^{loss} + E_{leak}^{loss} + E_{out}^{loss} - E_{UE\otimes PU}^{gain} - E_{env}^{gain}$	
$E_{\scriptscriptstyle dep}^{\scriptscriptstyle calo}$	energy deposited in the calorimeter within signal	definition
E_{mag}^{loss}	charged particle energy lost in solenoid field	only source
E_{low}^{loss}	particle energy lost in dead material	of signal!
E_{leak}^{loss}	energy lost due to longitudinal leakage	er ergræn
E_{out}^{loss}	energy lost due to jet algorithm/calorimeter signa	l definition
$E^{gain}_{UE\otimes PU}$	energy added by underlying event and/or pile-up	
E_{env}^{gain}	energy added by response from other nearby part	icles/jets

Local calibration does not

Energy losses not correlated with

Needs additional jet energy scale

Use specific jet context to derive

Magnetic field losses

Dead material losses

Only applicable to cluster jets!

cluster signals can not be corrected

reproduce jet energy

corrections

those

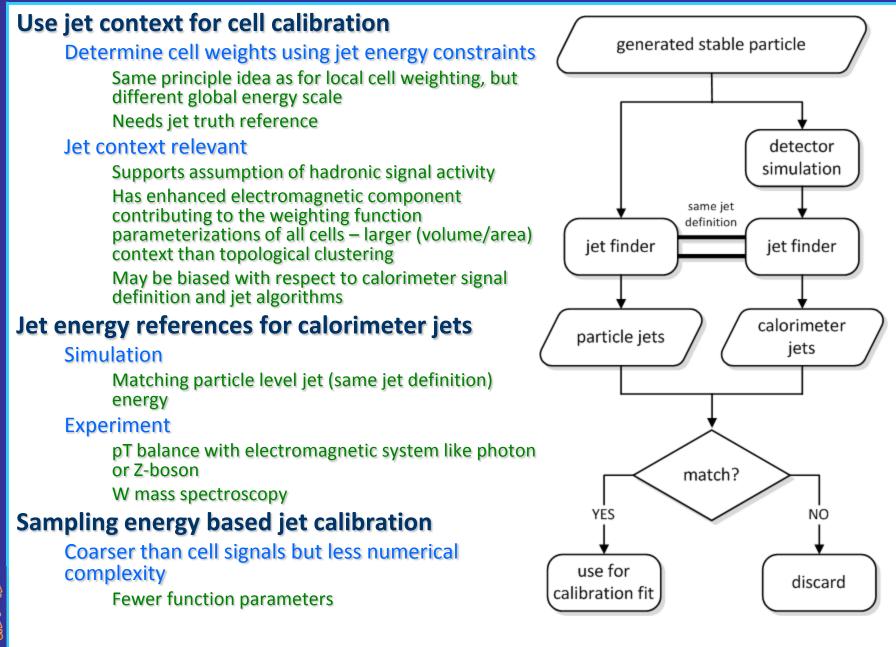


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Global Calibration Techniques



 $\Lambda R =$

 $\sqrt{(\eta_{\text{particle,jet}} - \eta_{\text{rec,jet}})^2 + (\varphi_{\text{particle,jet}} - \varphi_{\text{rec,jet}})^2}$

Simulated particle jets

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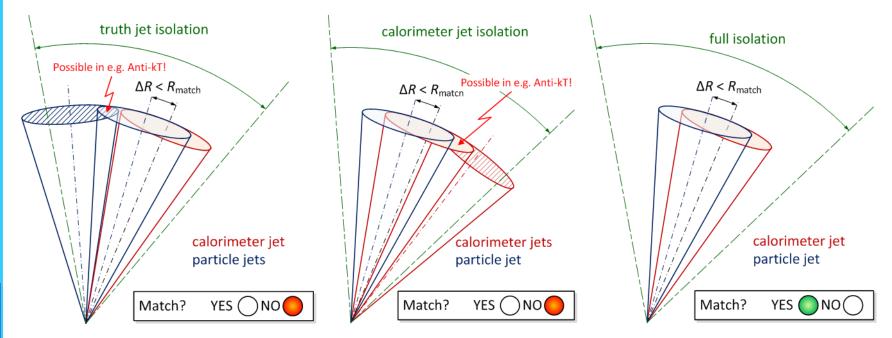
Establish "true" energy reference to constrain calibration function fits for calorimeter jets

Attempt to reconstruct true jet energy

Need matching definition

Geometrical distance

Isolation and unique 1-to-1 jet matching





Select matched jet pair

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> Typically small matching radius $R_{match} = 0.2 - 0.3$ Restrict jet directions to regions with good calorimeter response No excessive dead material Away from cracks and complex transition geometries

Calibration functions

Cell signal weighting

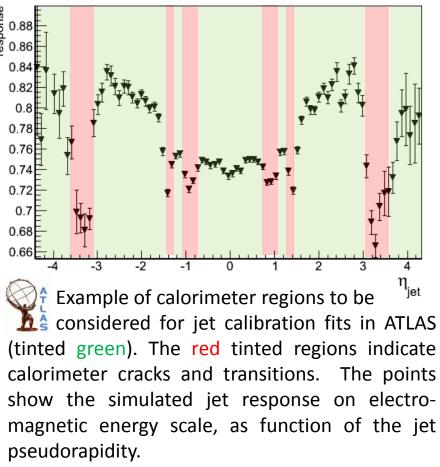
Large weights for low density signals

Small weights for high density signals

Sampling layer signal weighting Weights determined by longitudinal energy sharing in calorimeter jet

Functions can be complex

Often highly non-linear systems



(figure for illustration purposes only!)

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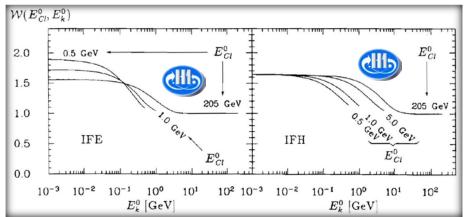
$$E_{\text{rec,cell}} = W_{\text{cell}}(\rho_{\text{cell}}, \ldots) \cdot E_{0,\text{cell}}$$

$$w_{\text{cell}}(\rho_{\text{cell}},\ldots) \searrow \begin{array}{c} \text{for } \rho_{\text{cell}} \uparrow \\ \text{for } \rho_{\text{cell}} \downarrow \end{array}$$

Typical boundary conditions: max($w_{cell}(\rho_{cell},...)$) $\approx 1.5 - 3.0$ (avoid boosting noise!)

 $\min(w_{cell}(\rho_{cell},\ldots))=1.0$

(avoid suppressing em response!)



Example: cell signal weights \mathcal{W} , parameterized as function of the cell energy E_k^0 and the cluster energy E_{cl}^0



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$$E_{\text{rec,cell}} = W_{\text{cell}}(\rho_{\text{cell}}, \dots) \cdot E_{0,\text{cell}}$$

$$w_{\text{cell}}(\rho_{\text{cell}},...)$$
 for ρ_{cell} for ρ_{cell}

Typical boundary conditions: $\max(w_{cell}(\rho_{cell},...)) \approx 1.5 - 3.0$ (avoid boosting noise!) $\min(w_{cell}(\rho_{cell},...)) = 1.0$ (avoid suppressing em response!)

Example for non-algebraic functional form:

(similar in ATLAS)

$$\boldsymbol{w}_{\text{cell}}(\boldsymbol{\rho}_{\text{cell}}, \boldsymbol{\Re}_{\text{cell}}) \!=\! \boldsymbol{\omega}_{ij} \text{ for } \begin{cases} \log(\boldsymbol{\rho})_i \leq \log(\boldsymbol{\rho}_{\text{cell}}) < \log(\boldsymbol{\rho})_{i+1} \\ \boldsymbol{\Re}_{\text{cell}} \in \boldsymbol{\Re}_j \end{cases}$$

 $\boldsymbol{\mathfrak{R}}_{\mbox{\tiny cell}}$ is a region descriptor for a given cell,

calorimeter module id, sampling id

like
$$\Re_{\text{cell}} = \{M_{\text{cell}}, S_{\text{cell}}\}$$



Global Calibration Fits Using Simulations THE UNIVERSITY

Select matched jet pair

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 $E_{\text{rec},S} = w_s E_{0,S} = w_s \cdot \sum_{o,cell} E_{o,cell}$ sampling S

Possible parameterizations:

$$w_{s} = w_{s}(f_{\text{EMC}})$$
, with $f_{\text{EMC}} = \frac{\sum_{\substack{\text{jet cells in} \\ EMC}} E_{0,\text{cell}}}{\sum_{\text{all jet cells}} E_{0,\text{cell}}}$

Example for non-algebraic functional form:

 $w_{s}(f_{EMC}) = \omega_{s,i}$ for $F_{EMC,i} \leq f_{EMC} < F_{EMC,i+1}$

Fitting

- **Possible constraints**
 - **Resolution optimization**
 - Signal linearity
 - Combination of both
- Regularization of calibration functions
 - Try to linearize function ansatz
 - Use polynomials
 - Can reduce fits to solving system of linear equations
- Non-linear function fitting
 - Use numerical approaches to find (local) minimum for multidimensional test functions (e.g., software like MINUIT etc.)

Reconstructed jet energy with cell calibration:

$$E_{\text{rec,jet}} = \sum_{\text{cells in jet}} w_{\text{cell}}(\rho_{\text{cell}}, \Re_{\text{cell}}) \cdot E_{0,\text{cell}}$$

Fit $\{\omega_{ij}\}$ such that...
 $\chi^{2} = \sum_{\substack{\text{matching} \\ \text{jet pairs}}} \frac{\left(E_{\text{rec,jet}} - E_{\text{particle,jet}}\right)^{2}}{\sigma_{\text{rec,jet}}^{2} + \sigma_{\text{particle,jet}}^{2}} = \min$

Reconstructed jet energy with sampling calibration:

$$E_{\text{rec,jet}} = \sum_{s \text{ in jet}} w_s(f_{\text{EMC}}) \cdot E_{0,s}$$

Fit $\{\omega_{i,s}\}$ using the same χ^2 test function!
Note that $\sigma_{\text{rec,jet}}^2 \sim E_{\text{rec,jet}}^{-1}$!



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Attempted de-convolution of signal contributions

Normalization choice convolutes various jet response features

E.g., cell weights correct for dead material and magnetic field induced energy losses, etc.

Limited de-convolution

Fit corrections for energy losses in material between calorimeter modules with different functional form Separation in terms, but still a correlated parameter fit

Reconstructed jet energy with cell calibration:

$$E_{\text{rec,jet}} = \sum_{\text{cells in jet}} W_{\text{cell}}(\rho_{\text{cell}}, \Re_{\text{cell}}) \cdot E_{0,\text{cell}} + E_{\text{DM,jet}}$$

Use χ^2 test function such that...

$$\chi^{2} = \sum_{\substack{\text{matching}\\\text{jet pairs}}} \frac{\left(E_{\text{rec,jet}} - E_{\text{particle,jet}}\right)^{2}}{\sigma_{\text{rec,jet}}^{2} + \sigma_{\text{particle,jet}}^{2}}$$
$$= \sum_{\substack{\text{matching}\\\text{jet pairs}}} \frac{\left(\left[\sum_{\substack{\text{cells in jet}}} W_{cell}(\rho_{cell}, \Re_{cell}) \cdot E_{0,cell} + \alpha \cdot \sqrt{E_{0,S=before}} \cdot E_{0,S=behind}\right] - E_{\text{particle,jet}}\right)^{2}}{\sigma_{\text{rec,jet}}^{2} + \sigma_{\text{particle,jet}}^{2}}$$

= min



with empirically motivated ansatz for $E_{\text{DM,jet}}$ for dead material between sampling layers S = before and S = behind, in a combined fit of $(\{W_{\text{cell}}\}, \alpha)$

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 $\chi^{2} = \sum_{\text{matching}} \frac{\left(E_{\text{rec,jet}} - E_{\text{particle,jet}}\right)^{2}}{\sigma_{\text{rec,jet}}^{2} + \sigma_{\text{particle,jet}}^{2}}$

Relatively low level of factorization in this particular approach with correlated (by combined fit) parameters!

$$= \sum_{\substack{\text{matching}\\\text{jet pairs}}} \frac{\left(\left[\sum_{cells \text{ in jet}} w_{cell}(\rho_{cell}, \Re_{cell}) \cdot E_{0,cell} + \alpha \cdot \sqrt{E_{0,S=before}} \cdot E_{0,S=behind}\right] - E_{\text{particle,jet}}\right)^2}{\sigma_{\text{rec,jet}}^2 + \sigma_{\text{particle,jet}}^2}$$

=min



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