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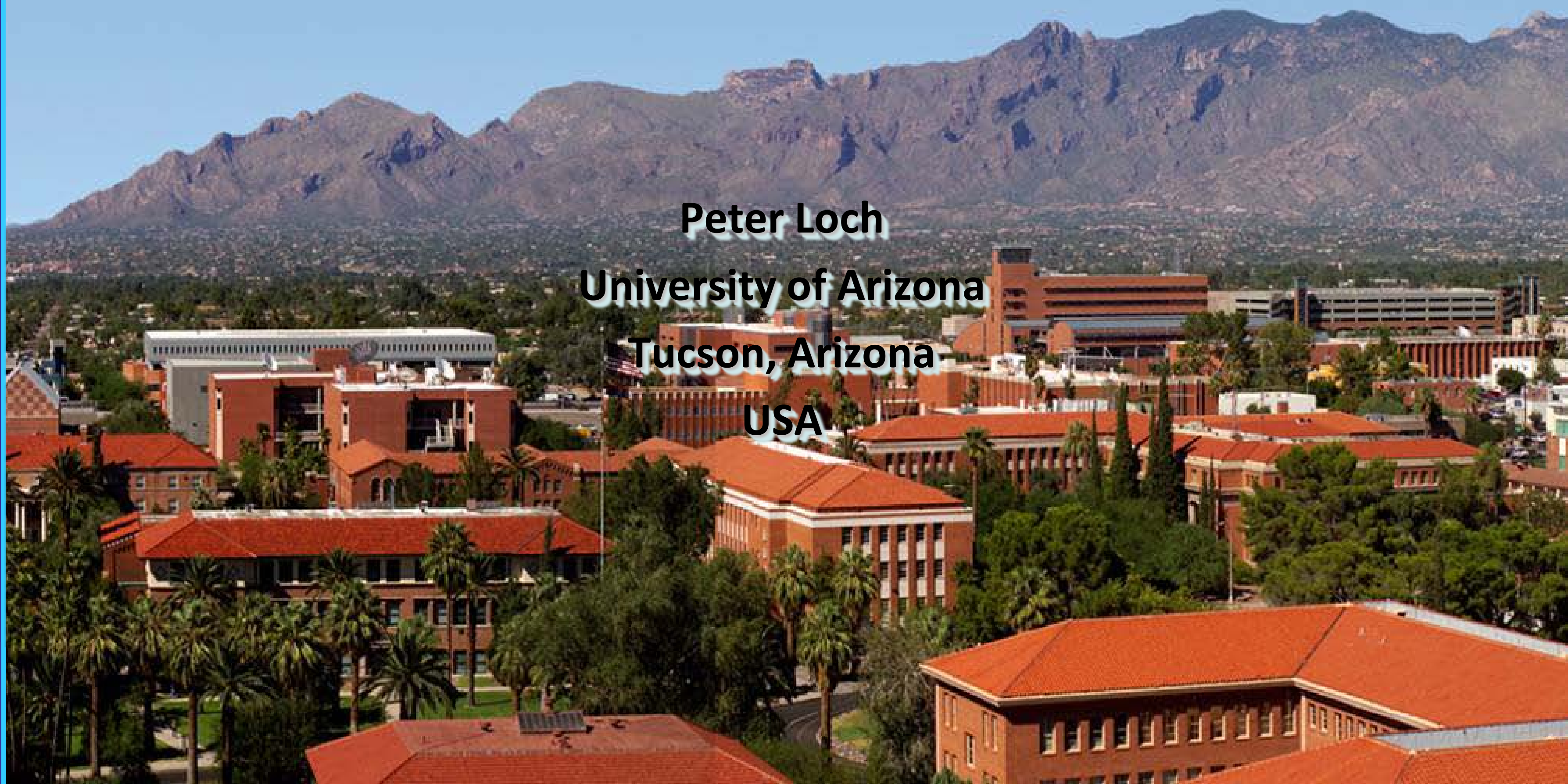
Introduction to Hadronic Final State Reconstruction in Collider Experiments (Part VI)



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Theoretical Requirements For Jet Finders

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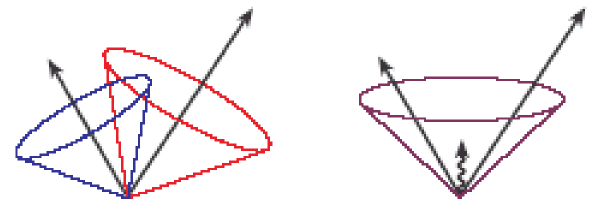
Very important at LHC

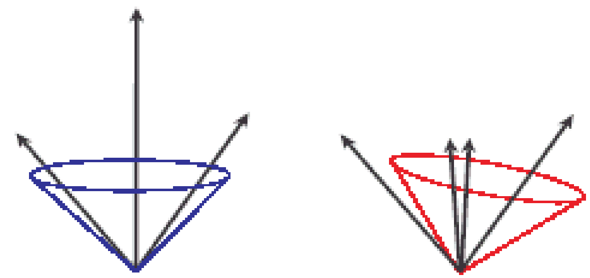
- Often LO (or even NLO) not sufficient to understand final states
  - Potentially significant K-factors can only be applied to jet driven spectra if jet finding follows theoretical rules
    - E.g., jet cross-section shapes
- Need to be able to compare experiments and theory
  - Comparison at the level of distributions
    - ATLAS and CMS will unfold experimental effects and limitations independently – different detector systems

Theoretical guidelines

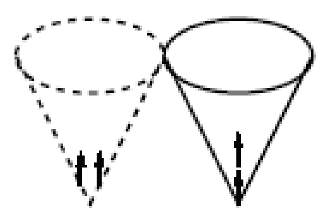
- Infrared safety
  - Adding or removing soft particles should not change the result of jet clustering
- Collinear safety
  - Splitting of large pT particle into two collinear particles should not affect the jet finding



infrared sensitivity
(soft gluon radiation merges jets)



collinear sensitivity (1)
(sensitive to E_t ordering of seeds)



collinear sensitivity (2)
(signal split into two towers below threshold)

ATLAS

Iterative Seeded Fixed Cone

Use following jet finder rules:

- Find particle with largest pT above a seed threshold
  - Create an ordered list of particles descending in pT and pick first particle
- Draw a cone of fixed size around this particle
  - Resolution parameter of algorithm
- Collect all other particles in cone and re-calculate cone directions from those
  - Use four-momentum re-summation
- Collect particles in new cone of same size and find new direction as above
  - Repeat until direction does not change → cone becomes stable
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- Repeat procedure and find next proto-jet
  - Note that this is done with all particles, including the ones found in previous cones
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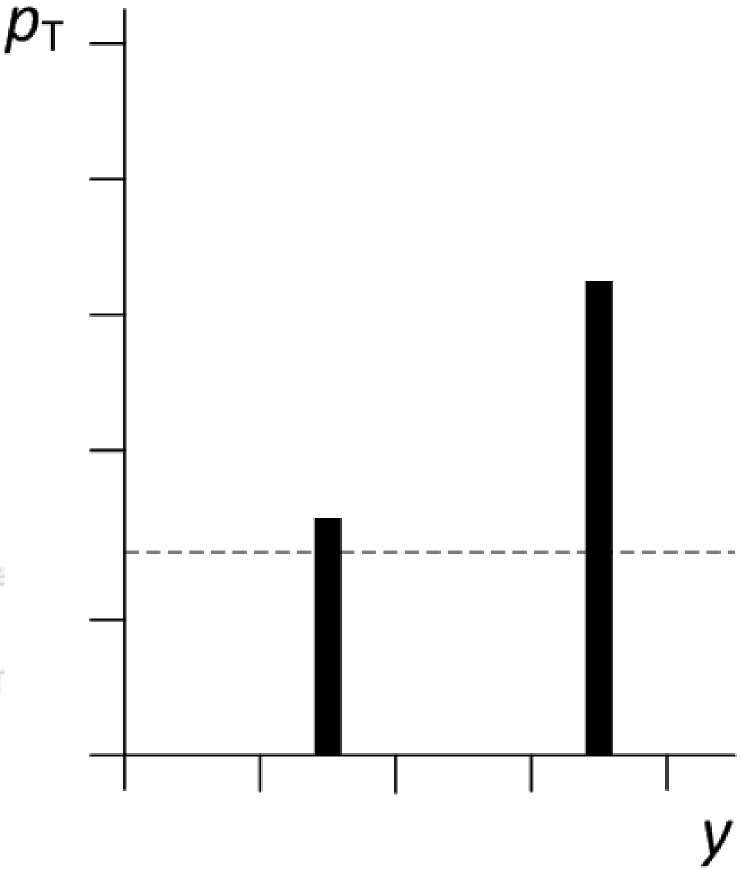

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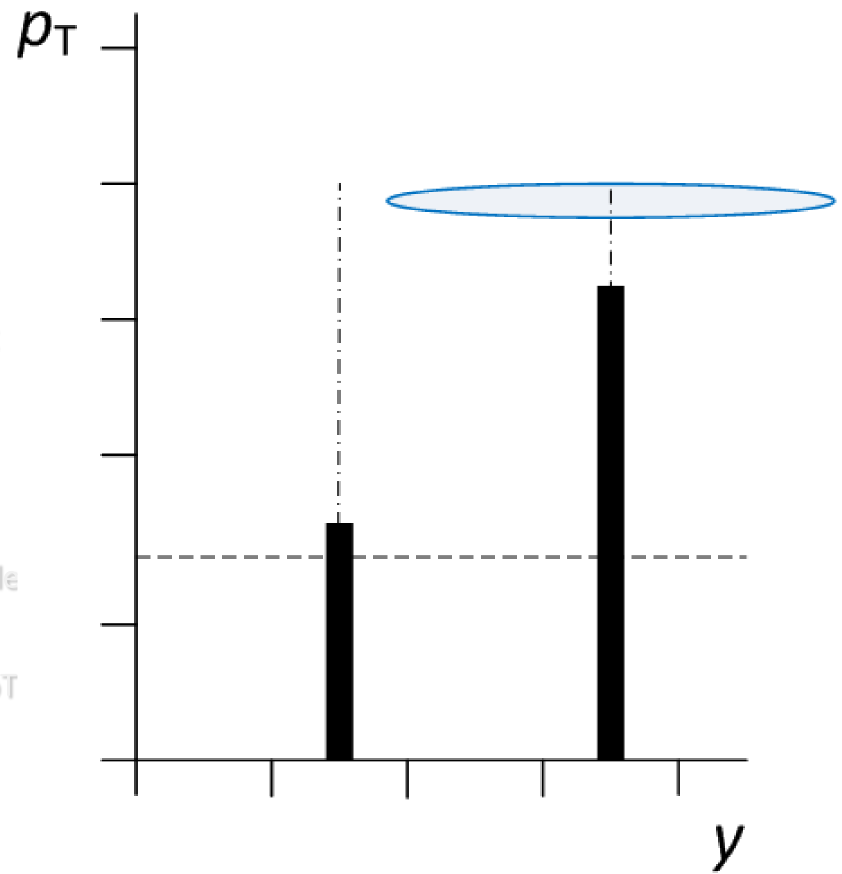

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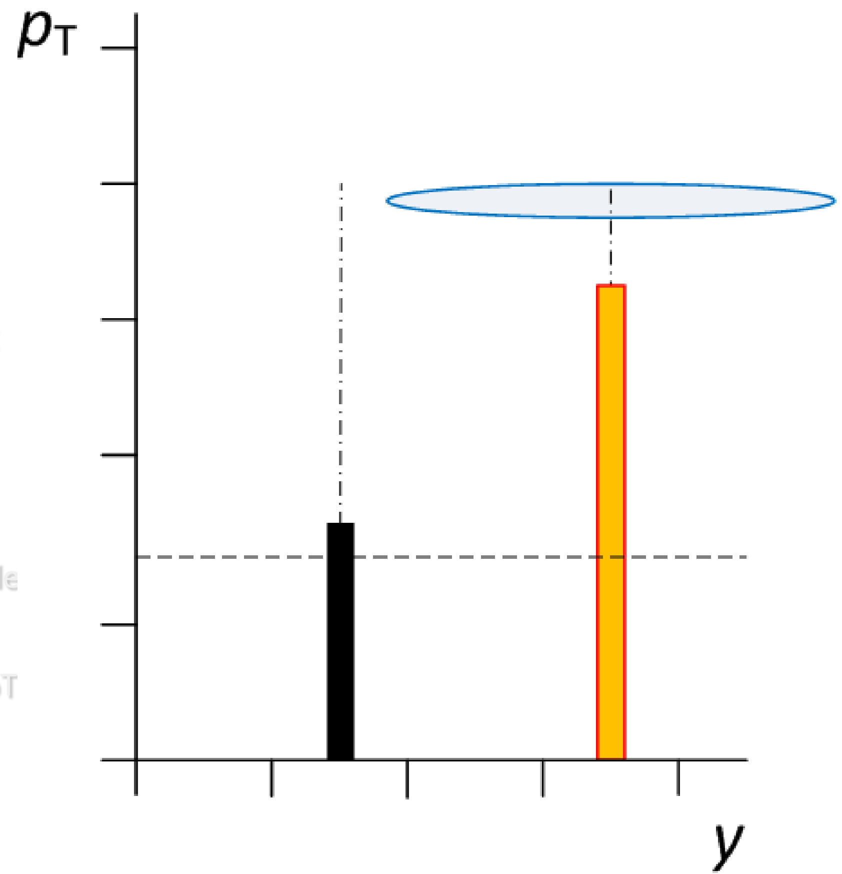
(first protojet)



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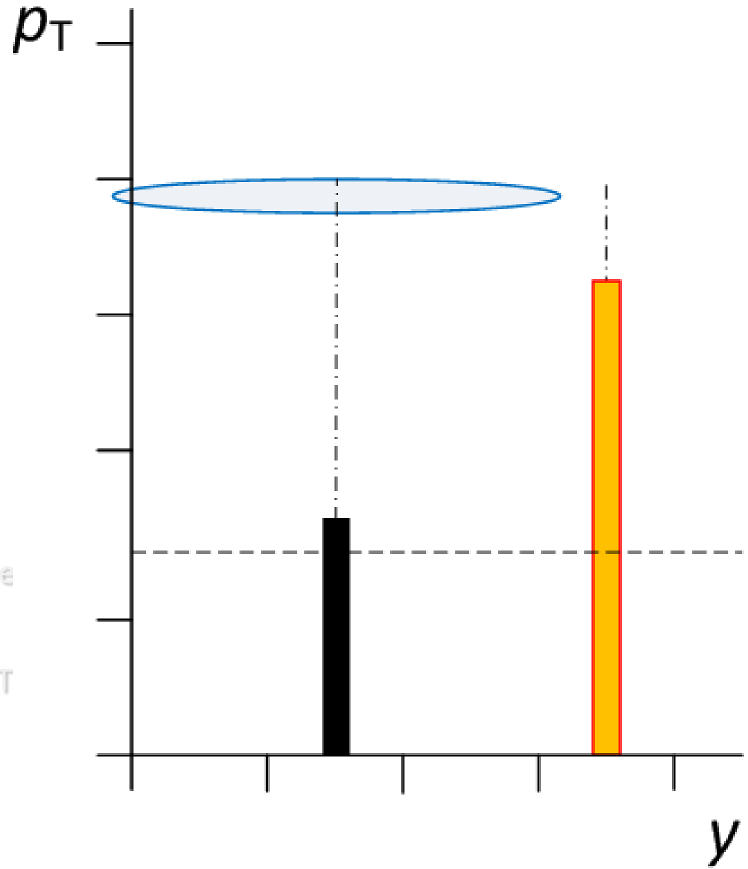

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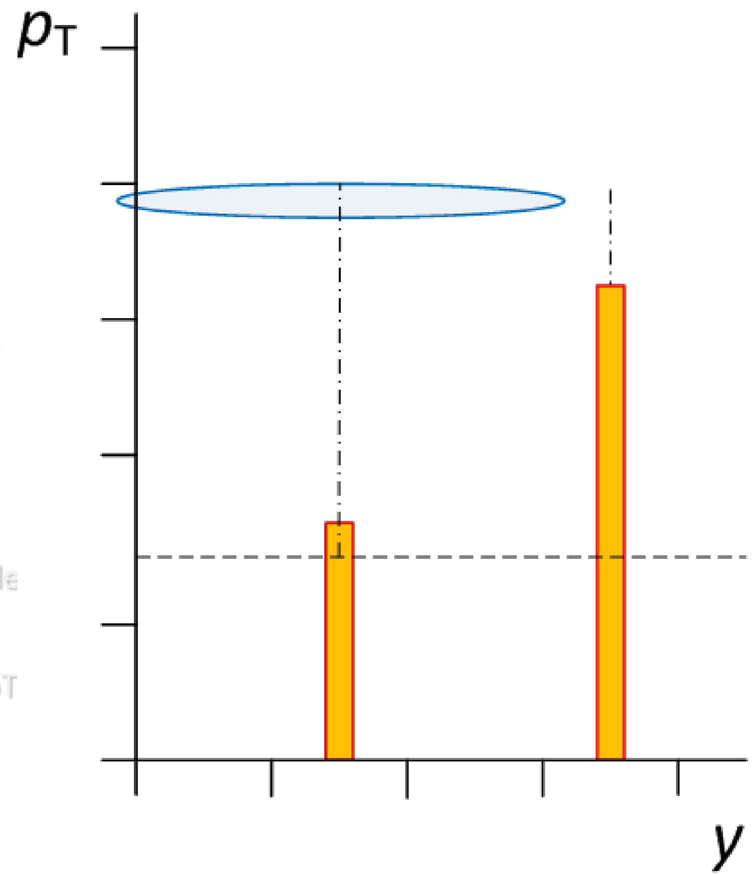
(second protojet)



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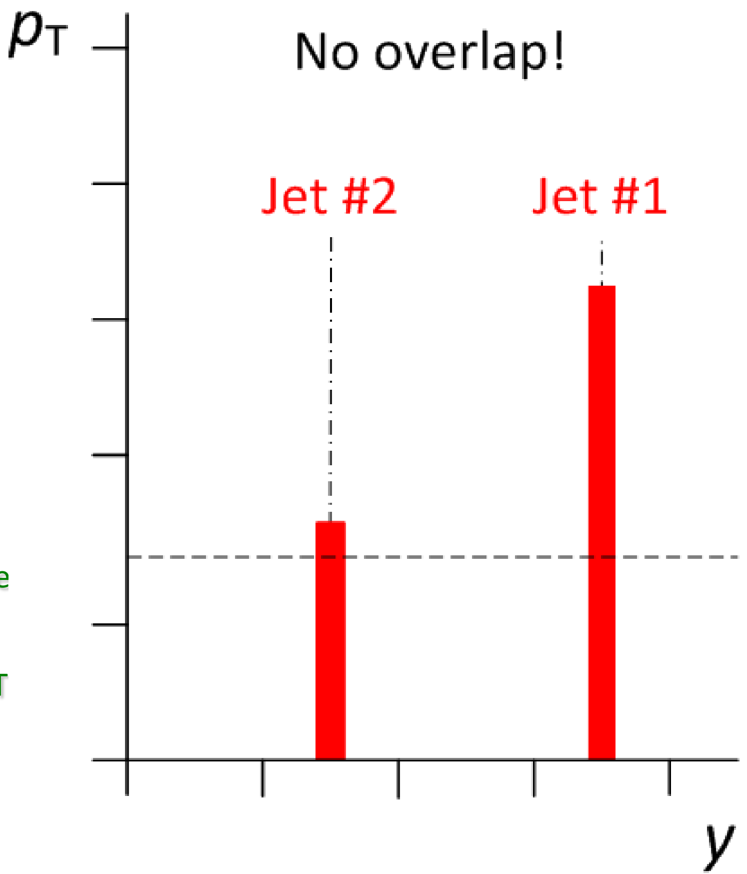
(two jets)



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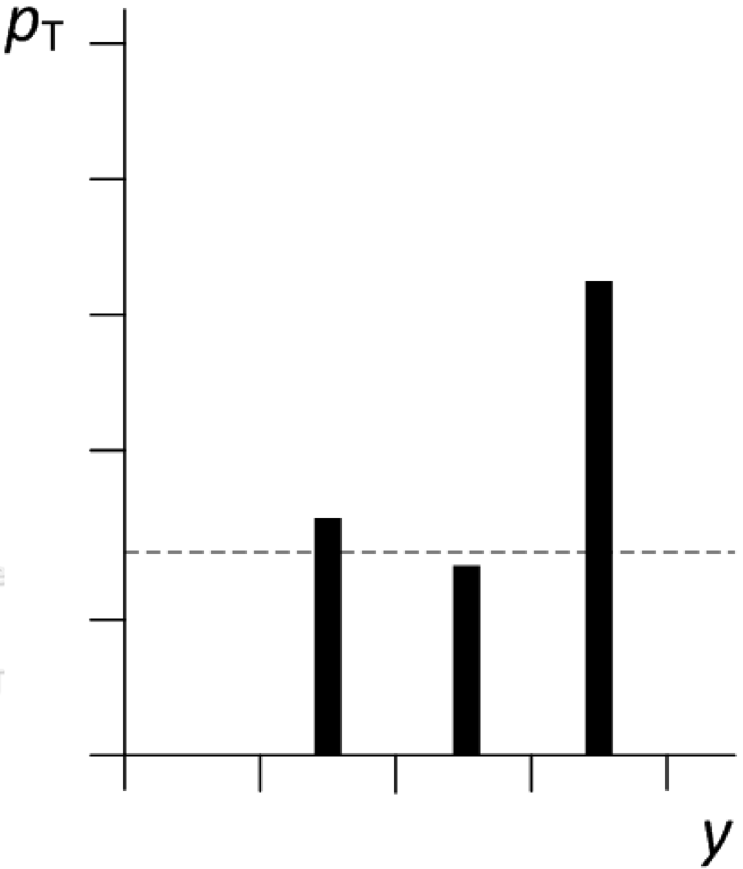

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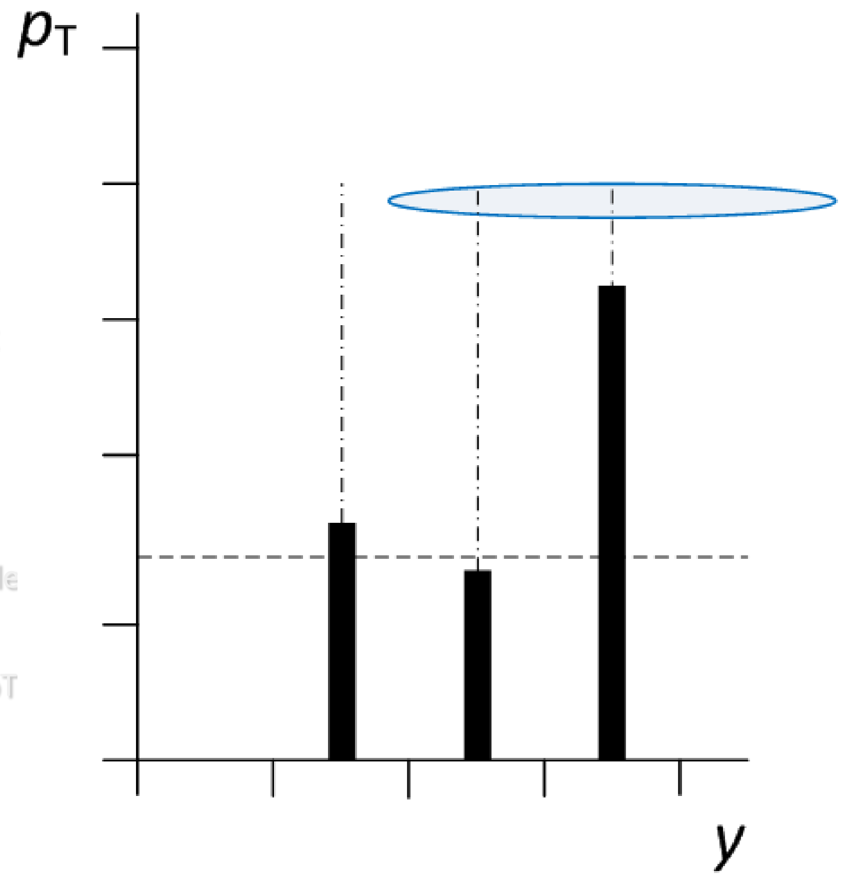

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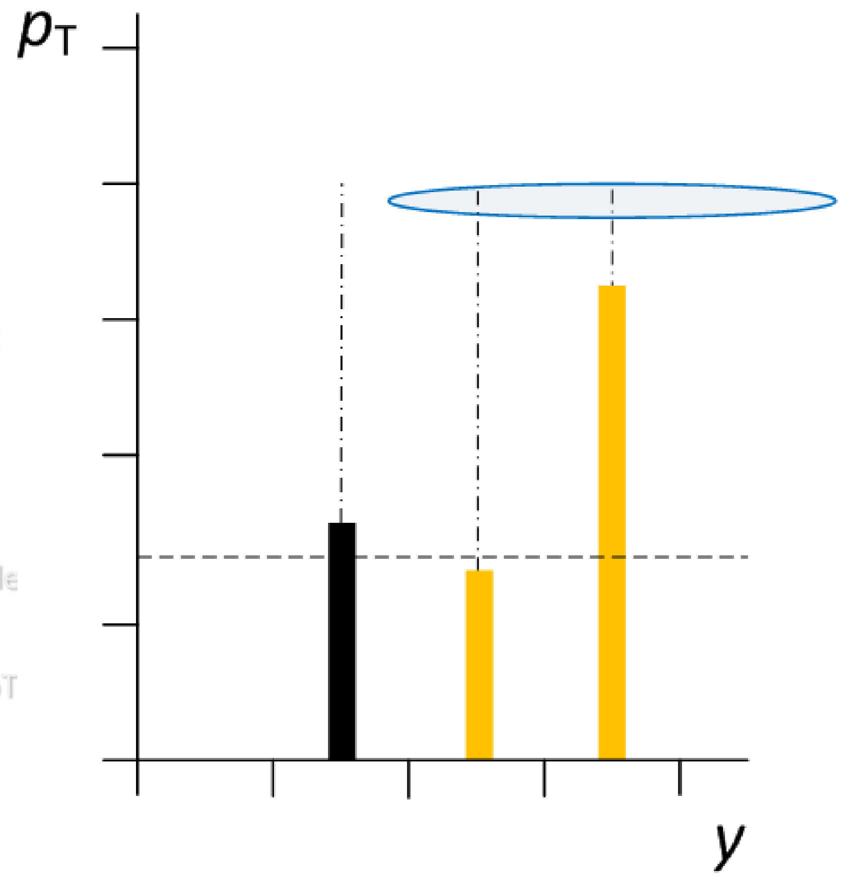

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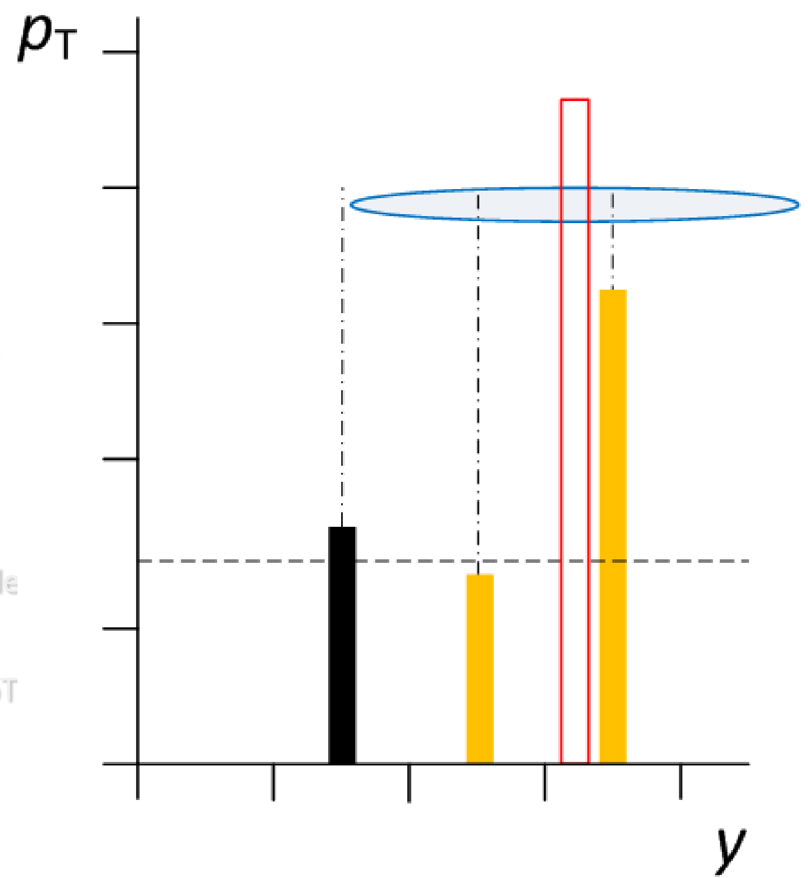

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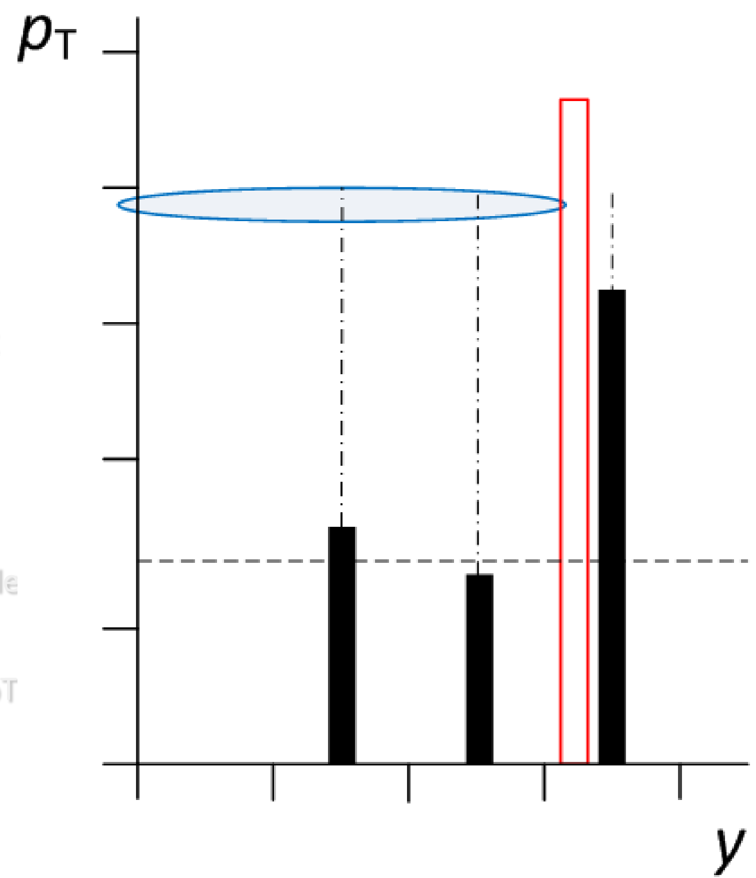

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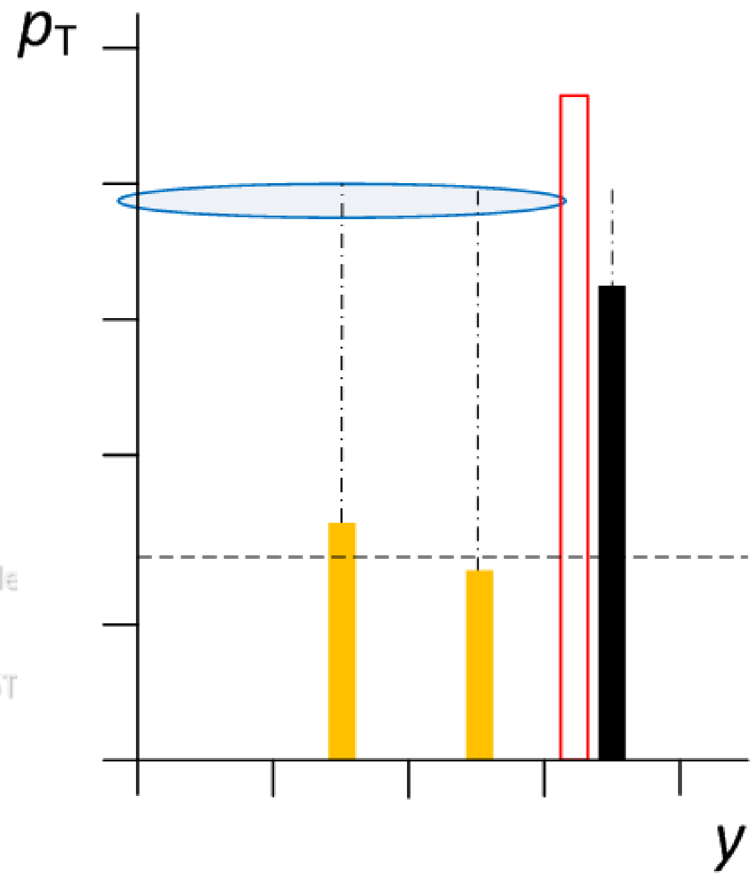

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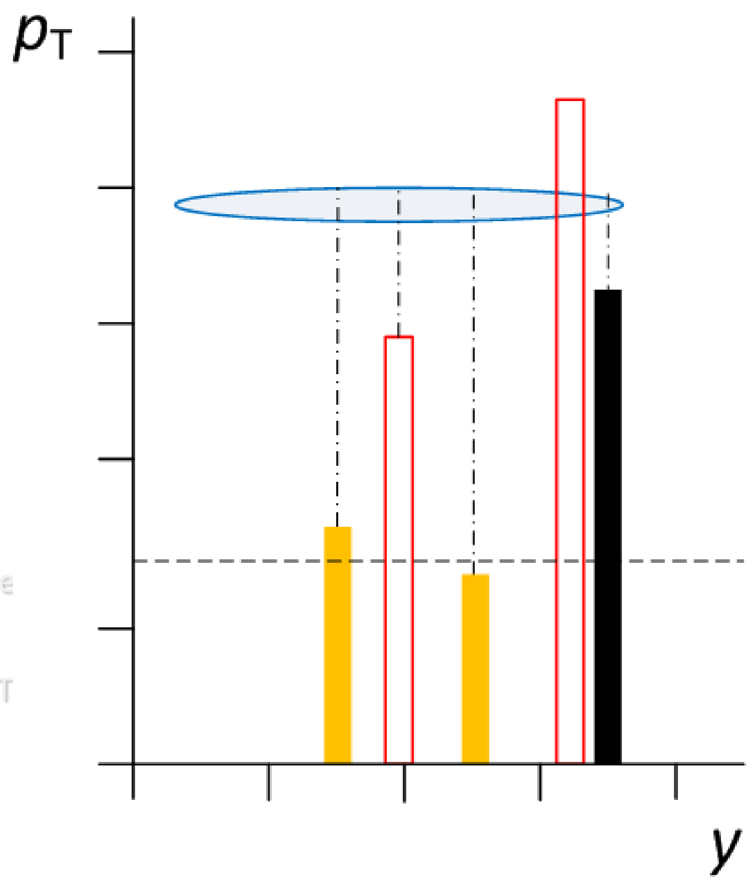

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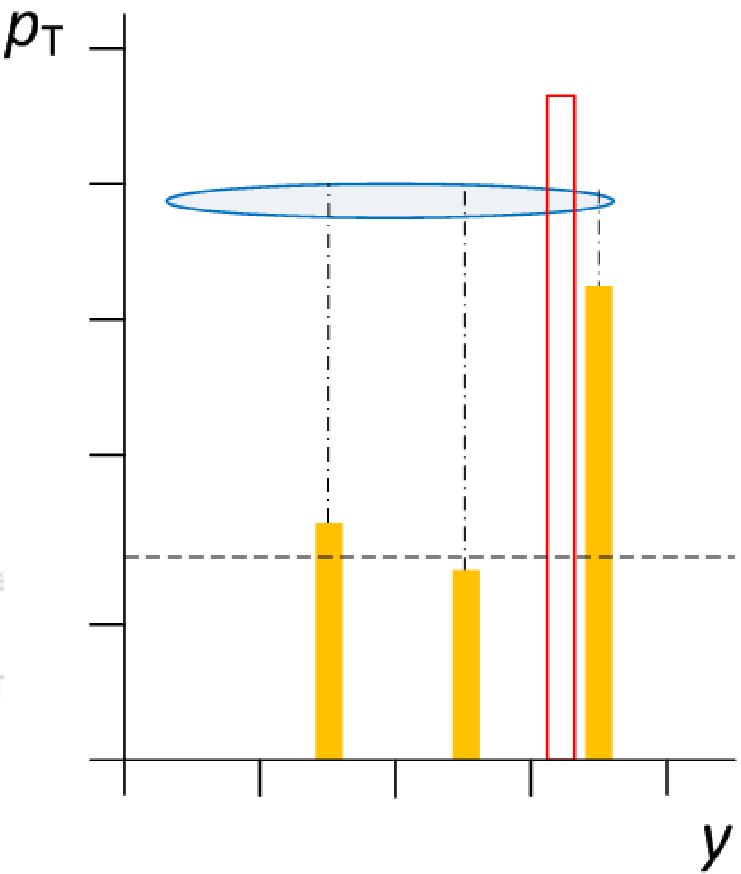

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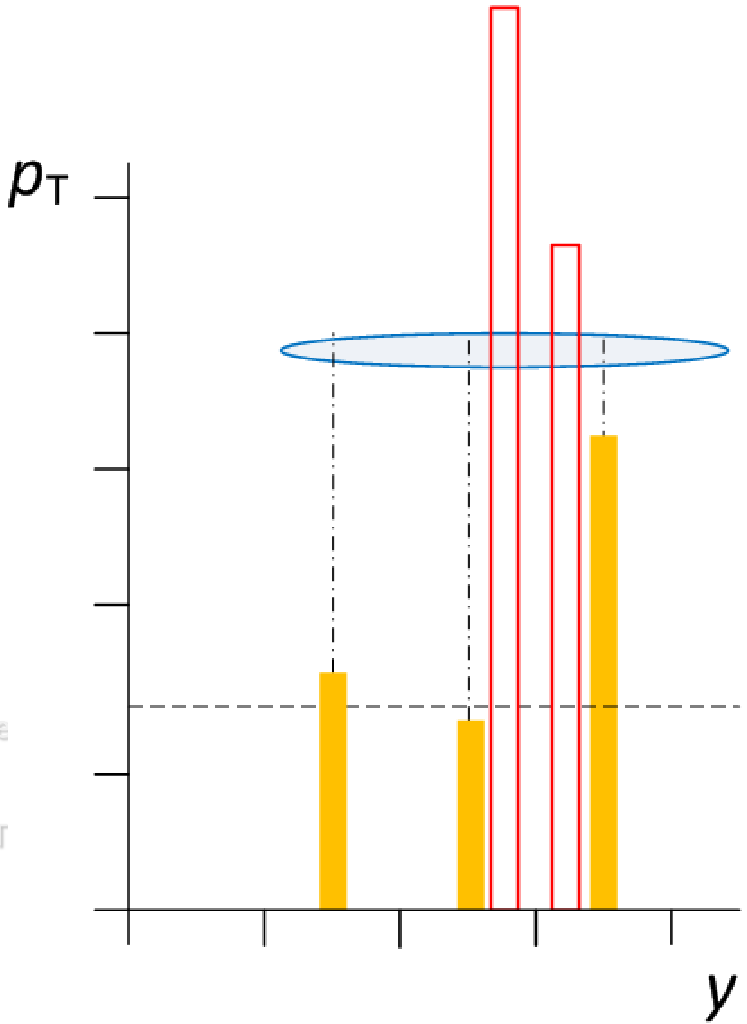
(second protojet)



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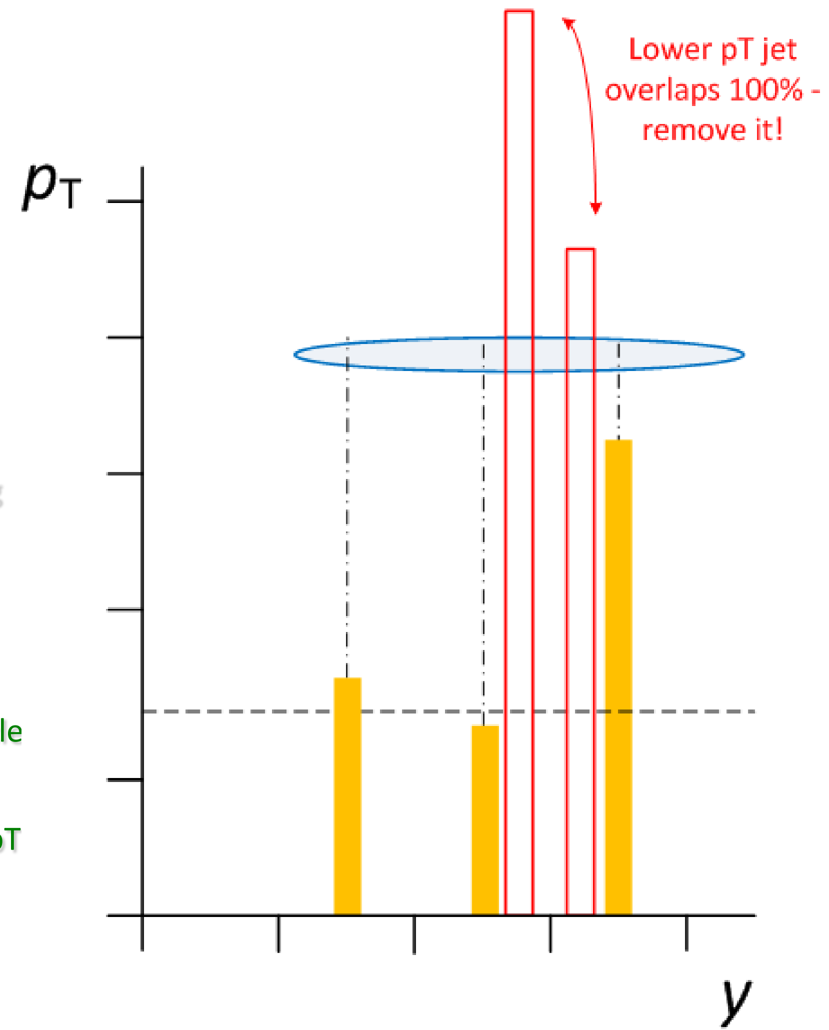

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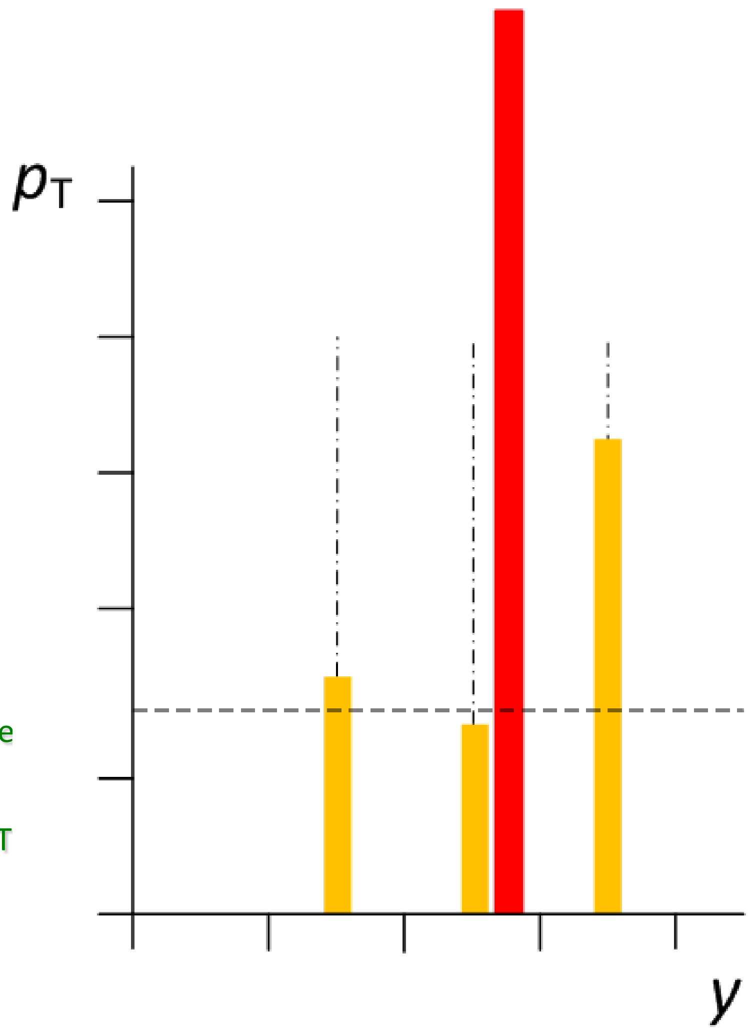

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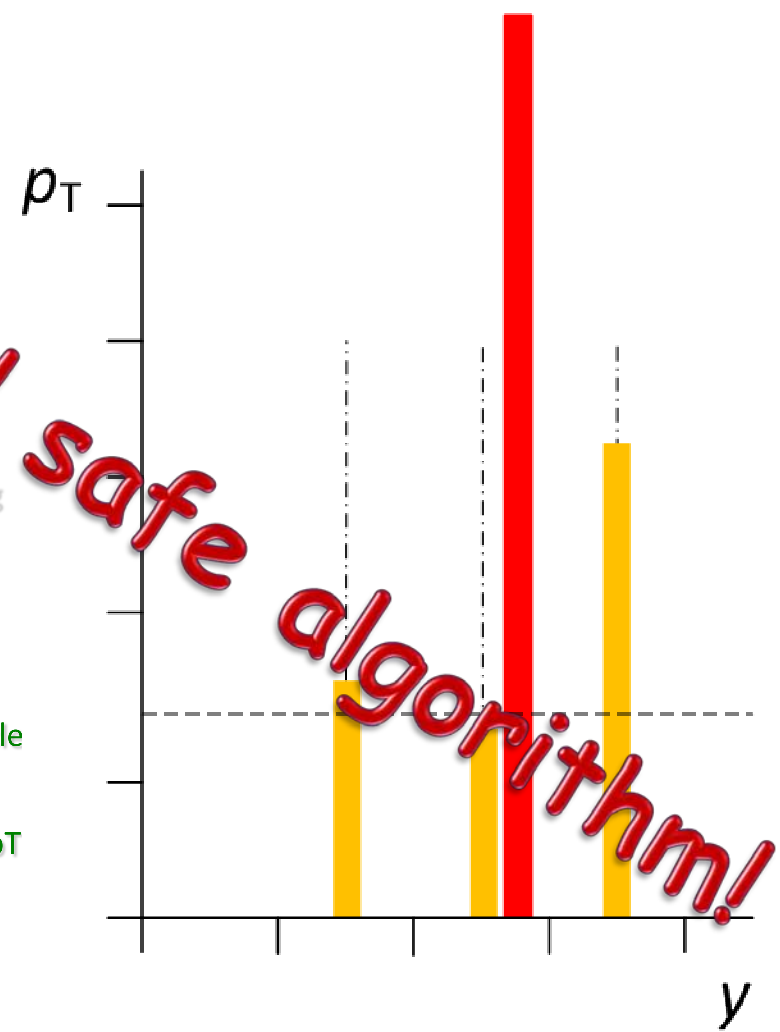
- Draw a cone of fixed size around this particle
  - Resolution parameter of algorithm
- Collect all other particles in cone and re-calculate cone directions from those
  - Use four-momentum recombination
- Collect particles in new cone of same size and find new direction as above
  - Repeat until direction does not change → cone becomes stable
- Take next particle from list if above pT seed threshold
- Repeat procedure and find next proto-jet
  - Note that this is done with all particles, including the ones found in previous cones
- Continue until no more proto-jets above threshold can be constructed
  - The same particle can be used by 2 or more jets
- Check for overlap between proto-jets
  - Add lower pT jet to higher pT jet if sum of particle pT in overlap is above a certain fraction of the lower pT jet (**merge**)
  - Else remove overlapping particles from higher pT jet and add to lower pT jet (**split**)
- All surviving proto-jets are the final jets

$$\Delta R = \sqrt{\Delta\eta^2 + \Delta\varphi^2} < R_{\text{cone}}$$

(one jet)

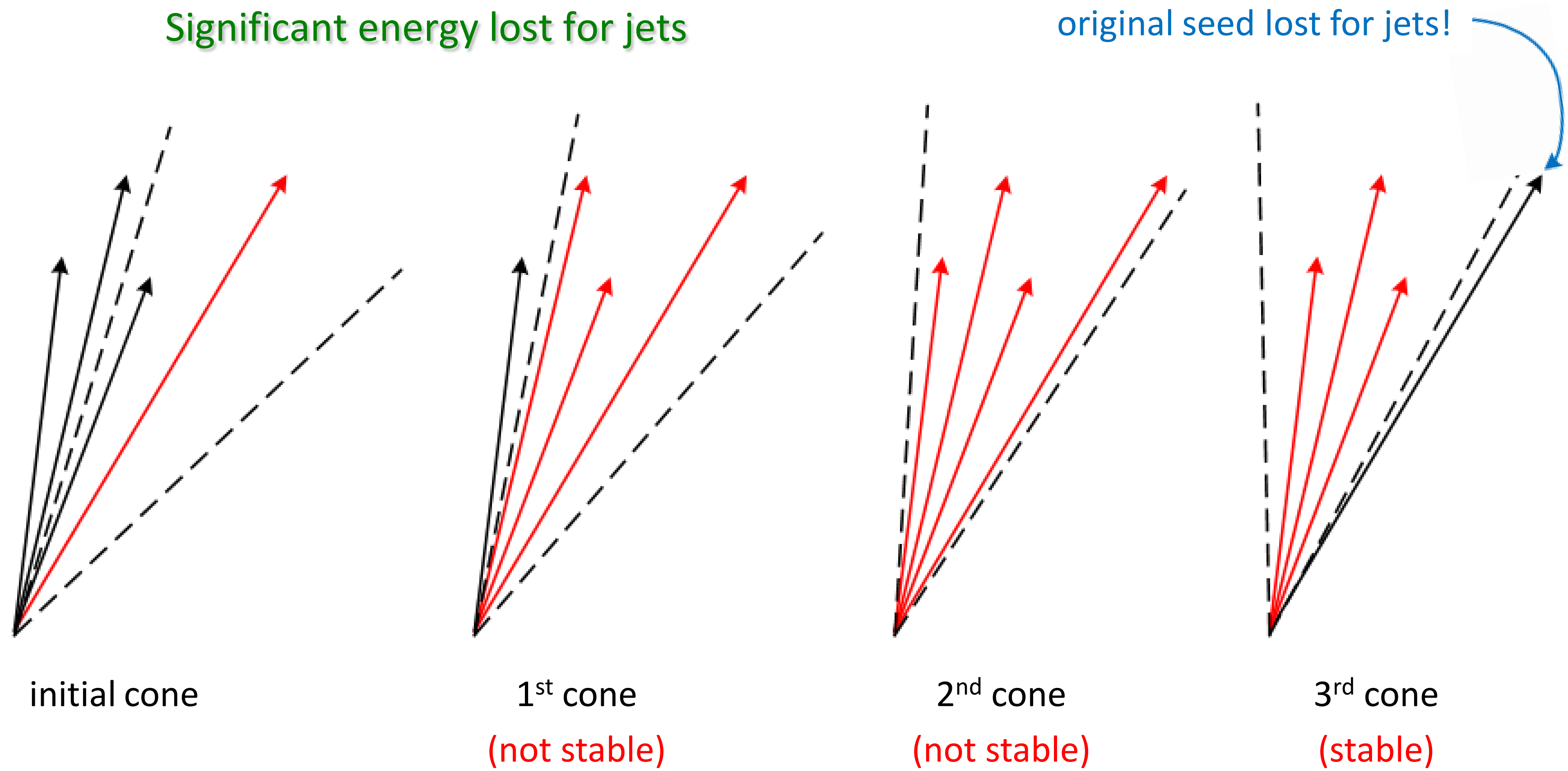


not an infrared safe algorithm!

Iterative Seeded Fixed Cone

Other problems with iterative cone finders:

- "Dark" tower problem
  - Original seed moves out of cone
  - Significant energy lost for jets



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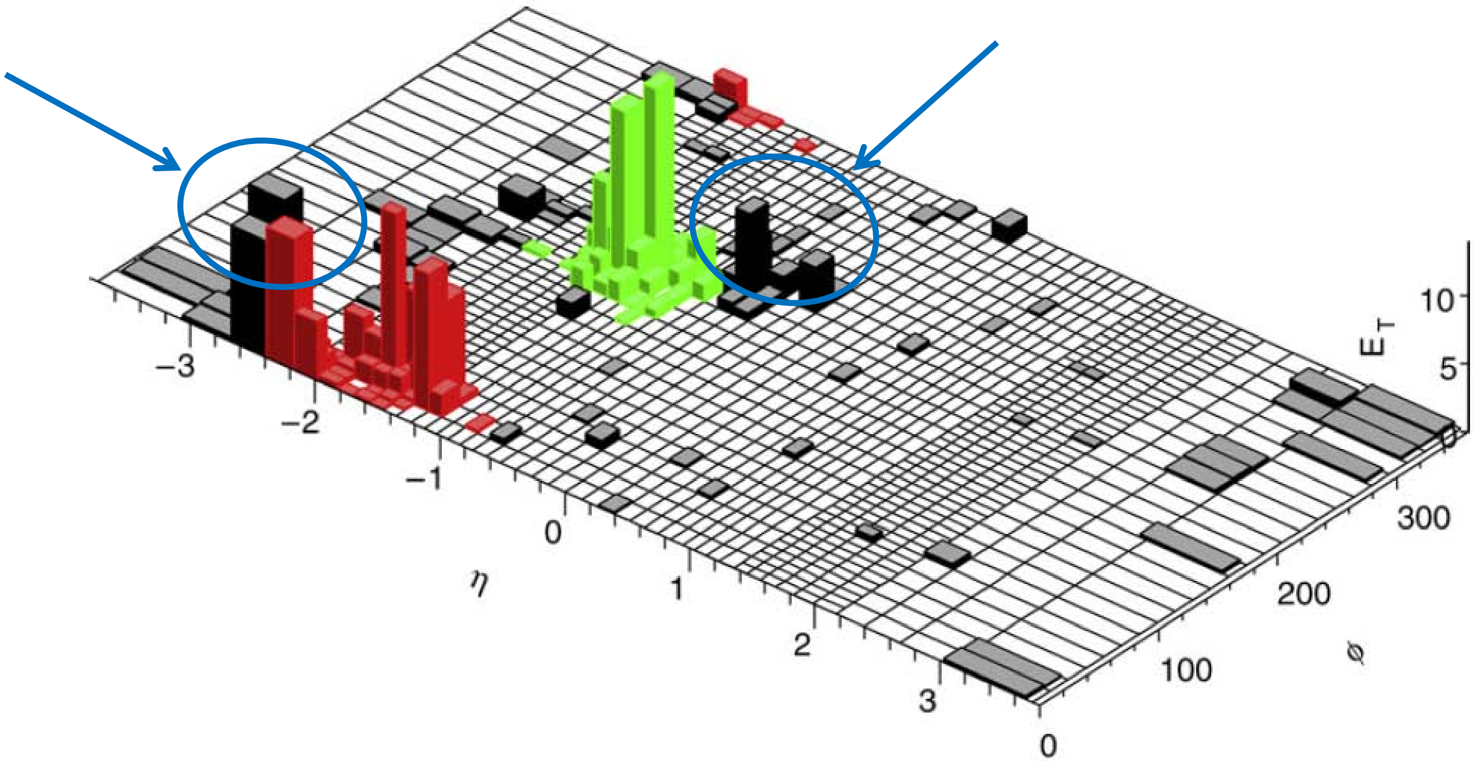

Iterative Seeded Fixed Cone

Advantages

- Simple geometry based algorithm
  - Easy to implement
- Fast algorithm
  - Ideal for online application in experiment

Disadvantages

- Not infrared safe
  - Can partially be recovered by splitting & merging
    - Introduces split/merge pT fraction *f* (typically 0.50 - 0.75)
  - Kills "trace" of pertubative infinities in experiment
    - Hard to confirm higher order calculations in "real life" without infinities!
- Not collinear safe
  - Used pT seeds (thresholds)
- Jets not cone shaped
  - Splitting and merging potentially makes jets bigger than original cone size and changes jet boundaries

Recursive Recombination

Motivated by gluon splitting function

- QCD branching happens all the time
  - Attempt to undo parton fragmentation
- Pair with strongest divergence likely belongs together
  - kT/Durham, first used in e^+e^-
  - Catani, Dokshitzer, Olsson, Turnock & Webber 1991

$$\left[dk_j\right]\left|M^2_{g\to g_i g_j}(k_j)\right| \simeq \frac{2\alpha_s C_A}{\pi}\frac{dE_j}{\min(E_i,E_j)}\frac{d\Theta_{ij}}{\Theta_{ij}}$$

$$(E_j \ll E_i, \Theta_{ij} \ll 1)$$

Distance between all particles *i* and *j*

$$y_{ij} = \frac{2\min(E_i^2,E_j^2)(1-\cos\Theta_{ij})}{Q^2}$$

$y_{ij} < y_{cut} \rightarrow$ combine *i* and *j*, else stop

Longitudinal invariant version for hadron colliders

- Transverse momentum instead of energy
  - Catani, Dokshitzer, Seymour & Webber 1993
  - S.D. Ellis & D. Soper 1993
- Valid at all orders!

Drop normalization to Q^2 (not fixed in pp)

$$y_{ij} \rightarrow d_{ij} = \min(d_i,d_j)\Delta R_{ij}^2,\ d_{i,j} = p_{\mathrm{T}i,j}^2$$

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(exclusive kT)

Inclusive longitudinal invariant clustering

$$d_{ij} = \min(d_i,d_j)\Delta R_{ij}^2 / R^2$$

Recursive Recombination

P. Loch
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March 09, 2010

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kT –like Algorithms

P. Loch
U of Arizona
March 09, 2010

Classic procedure

- Calculate all distances d_{ji} for list of particles
  - Uses distance parameter
- Calculate d_i for all particles
  - Uses pT
- If minimum of both lists is a d_{ij}, combine *i* and *j* and add to list
  - Remove *i* and *j*, of course
- If minimum is a d_i, call *i* a jet and remove from list
- Recalculate all distances and continue all particles are removed or called a jet

Features

- Clustering sequence is ordered in kT
  - Follows jet structure

Inclusive longitudinal invariant clustering

$$d_{ij} = \min(d_i, d_j)\Delta R_{ij}^2 / R^2$$

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Alternatives

- Cambridge/Aachen clustering
  - Uses angular distances only
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  - No particular ordering, sequence not meaningful

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$$d_i = p_{Ti}^{2n}$$

Cambridge/Aachen $(n = 0)$

cluster smallest d_{ij} first until d_{ij}>1

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cluster smallest d_{ij} first until $d_{ij} > 1$

Anti-kT $(n = -1)$

follow classic algorithm

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kT Examples

P. Loch
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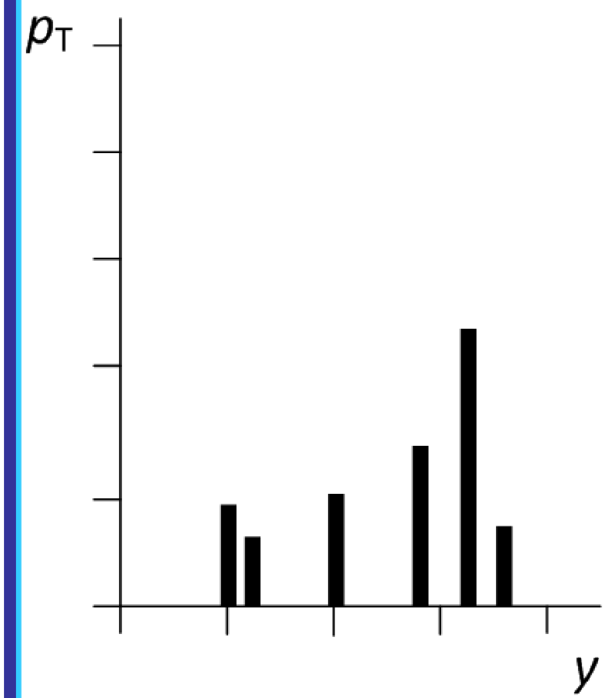
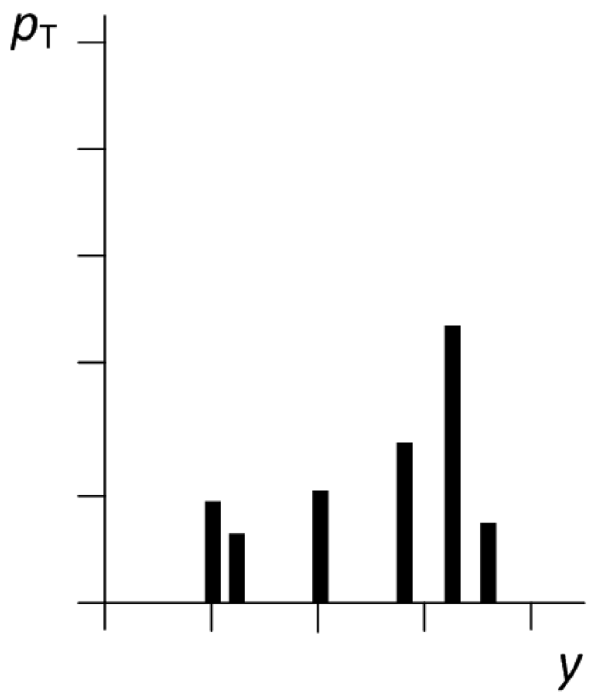
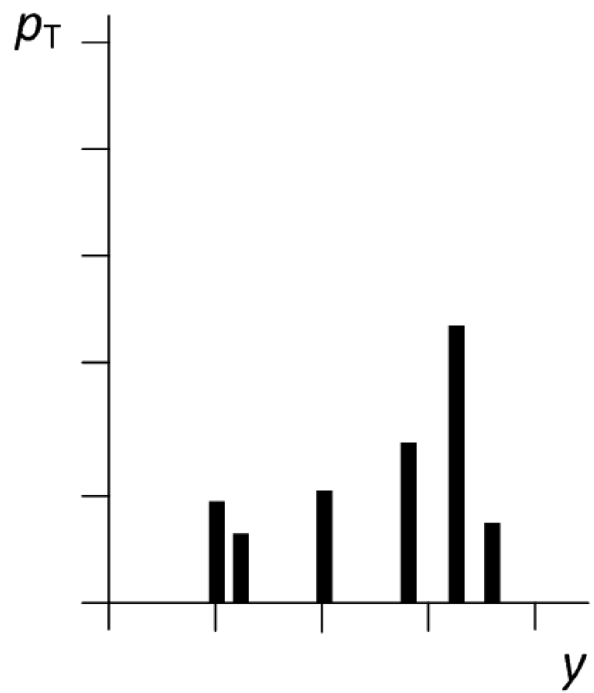
kT, *n*=1



Anti-kT, *n*=-1



Cambridge/Aachen, *n*=0



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kT Examples

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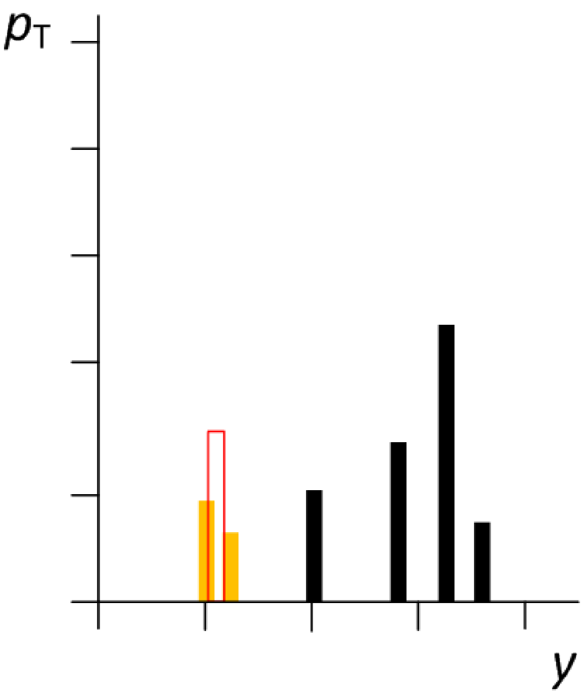

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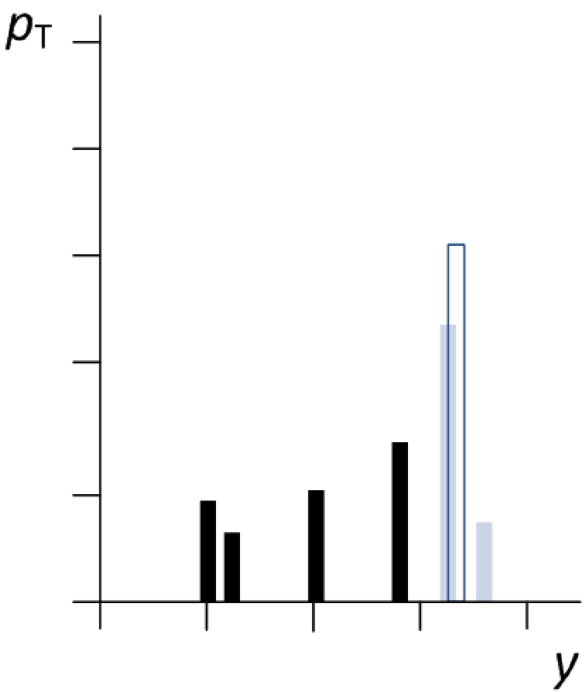
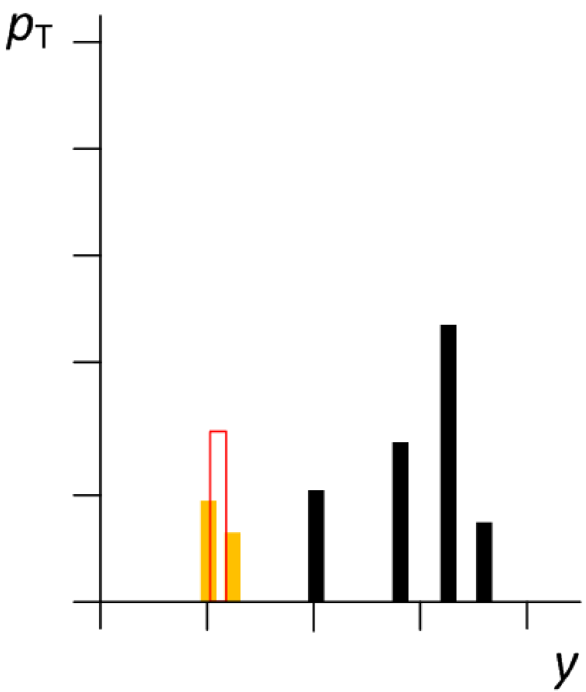


Cambridge/Aachen, *n*=0



kT Examples

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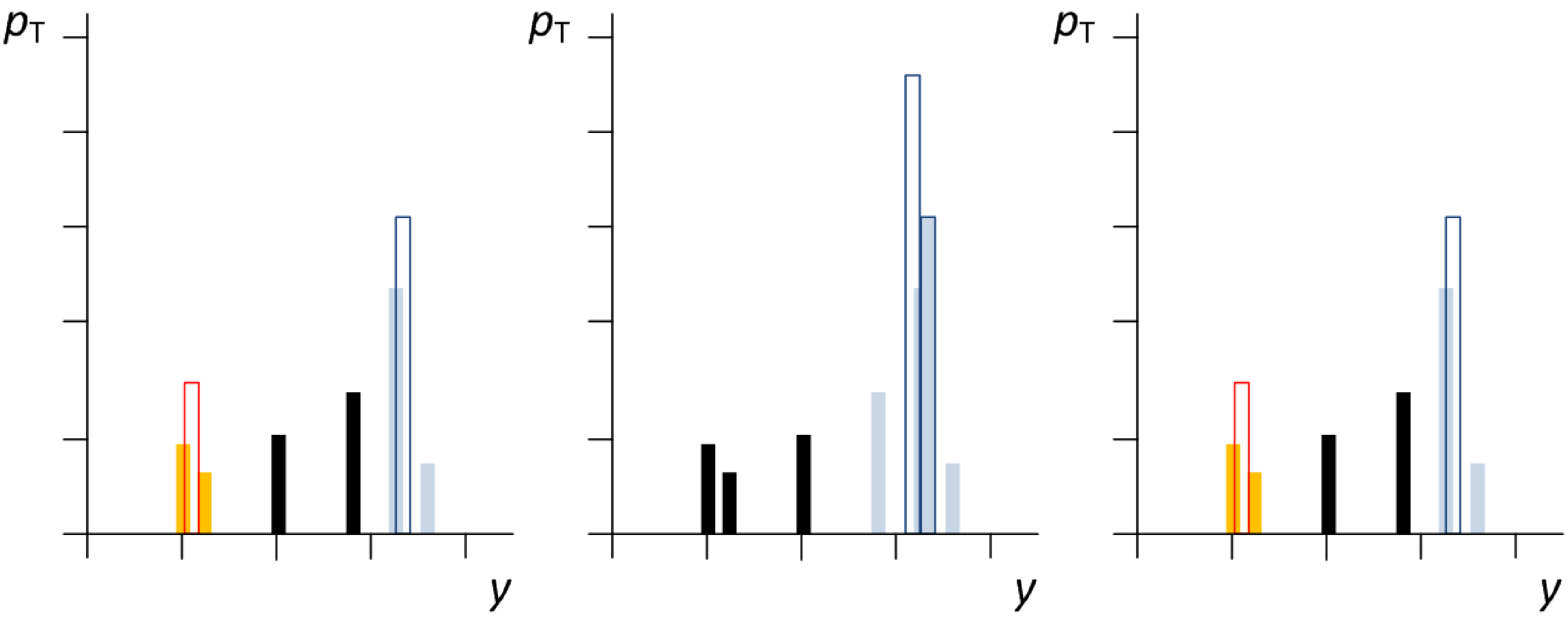
kT, n=1 | Anti-kT, n=-1 | Cambridge/Aachen, n=0



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kT Examples

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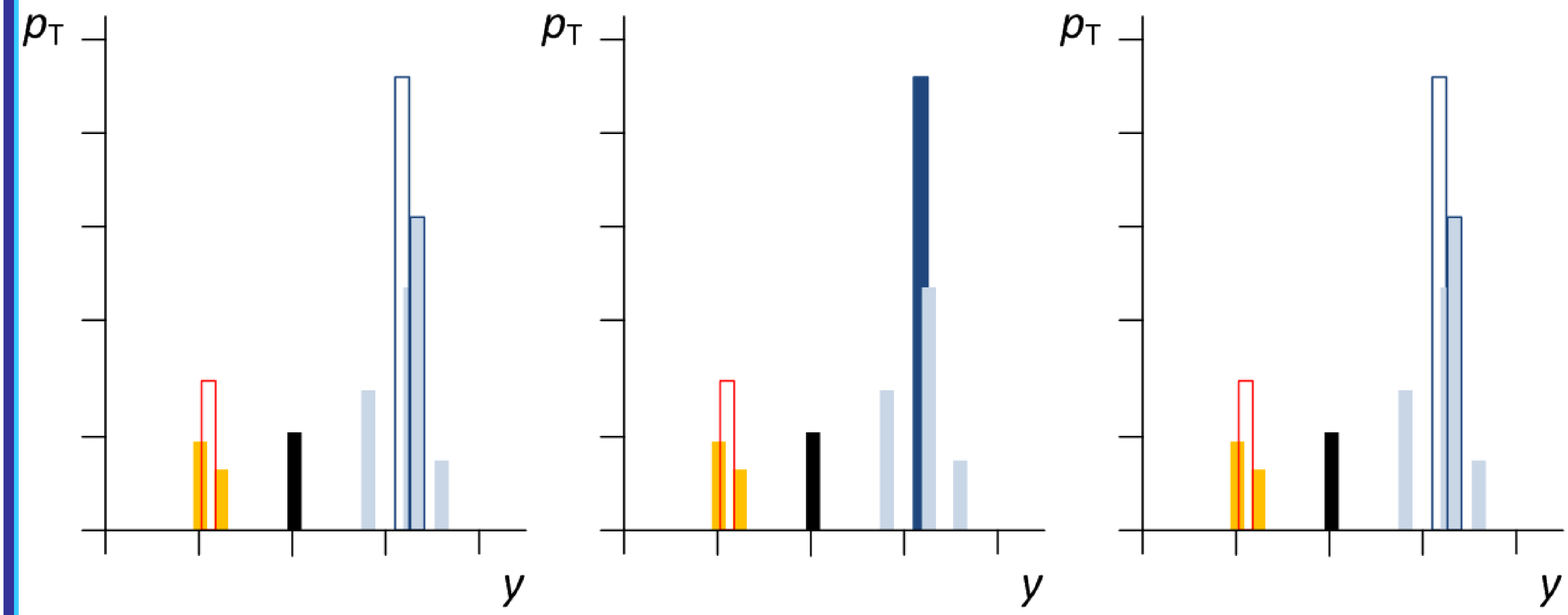
kT, n=1 — Anti-kT, n=-1 — Cambridge/Aachen, n=0



kT Examples

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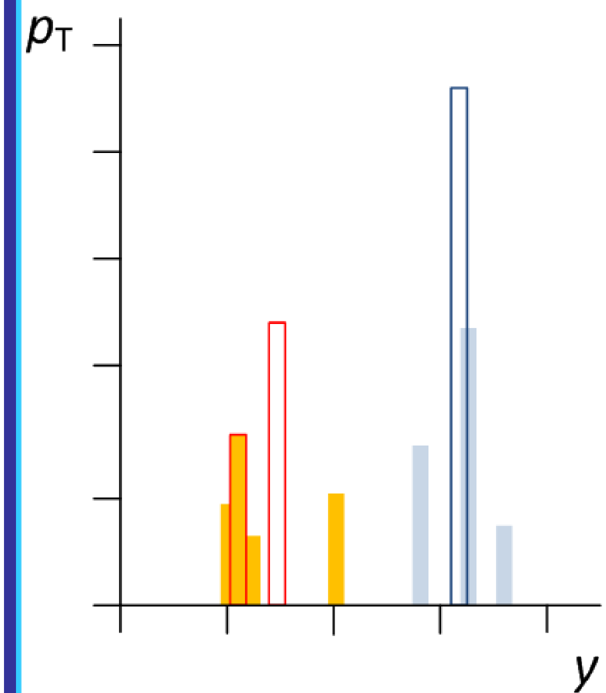
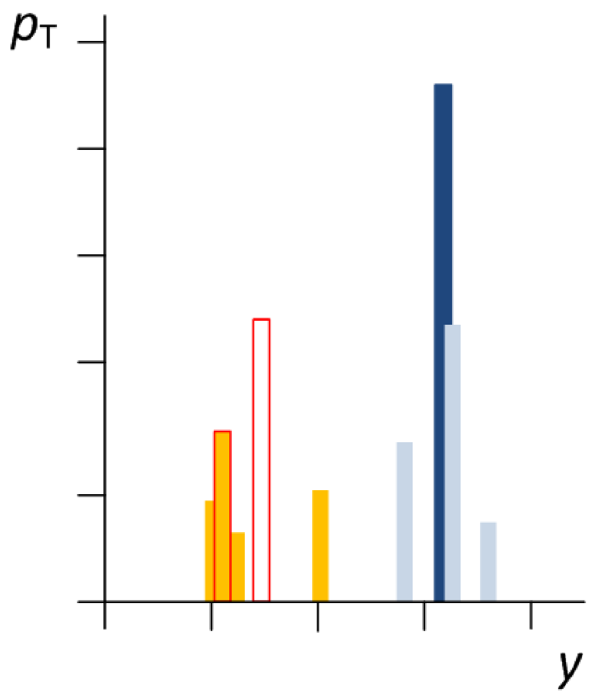
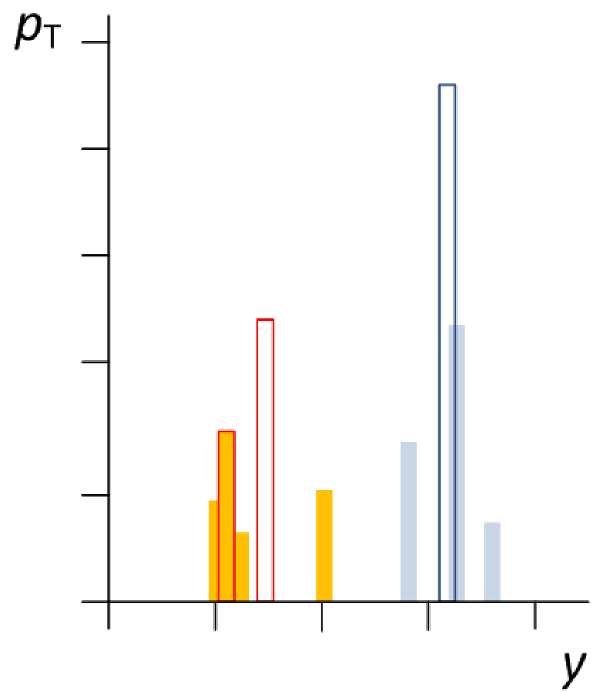
kT, n=1



Anti-kT, n=-1



Cambridge/Aachen, n=0



kT Examples

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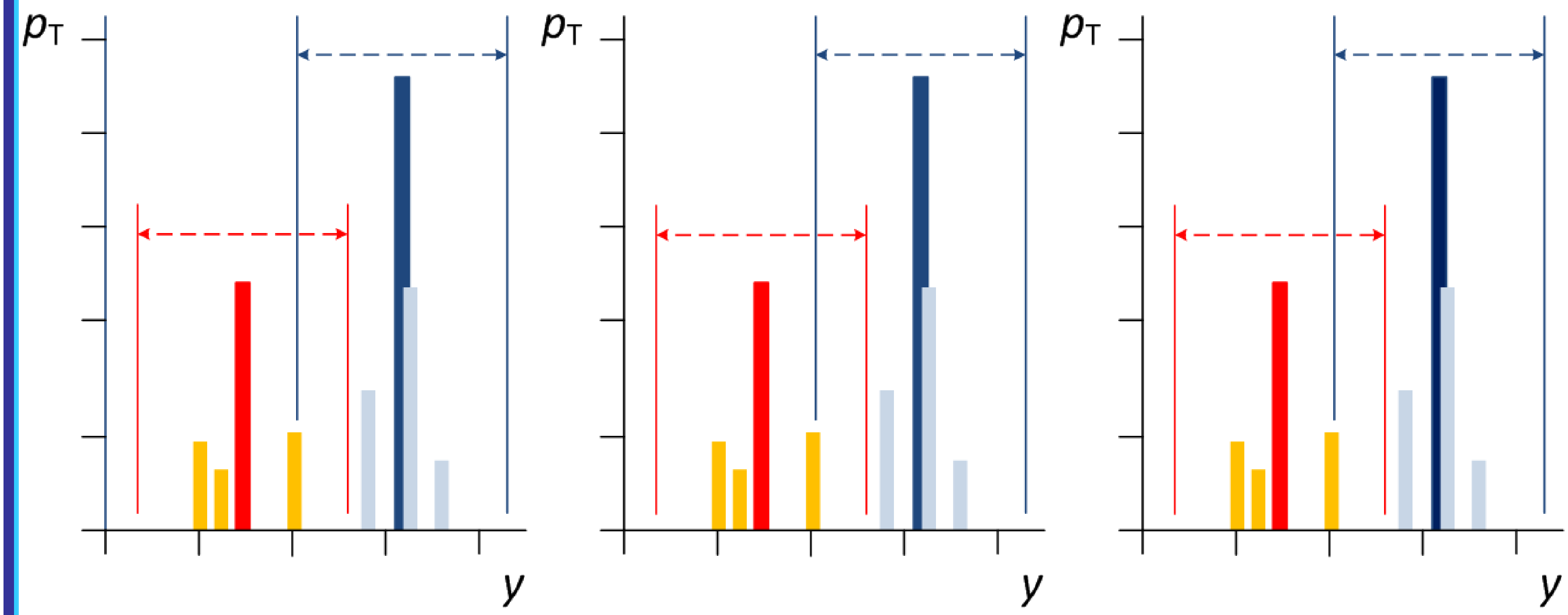
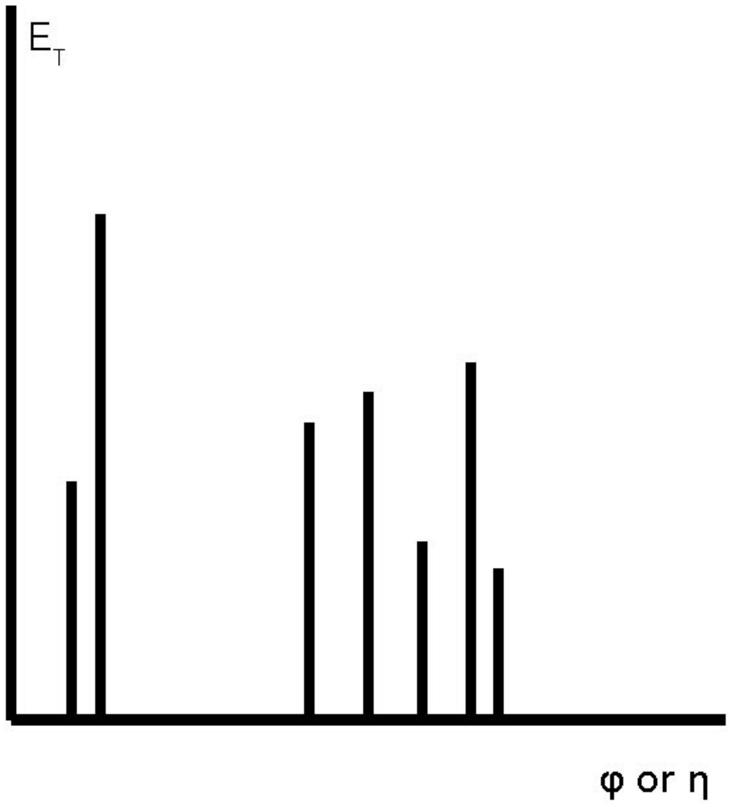


Recursive Recombination Algorithms

Clustering Algorithms

CTEQ-MCnet school 2008
Gavin Salam Lectures on Jets



Clustering Algorithms:

- Define a distance d_{ij} between two objects i, j:

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

$$d_{ij} = \min(k_{ti}^2, k_{tj}^2)\Delta R_{ij}^2 / R^2$$

and a distance d_{iB} between one object i and the beam direction B:

$$d_{iB} = k_{ti}^2$$

- Find the smallest of d_{ij}, d_{iB} .

  If d_{ij} recombine i, j ;

  If d_{iB} , i is a jet.

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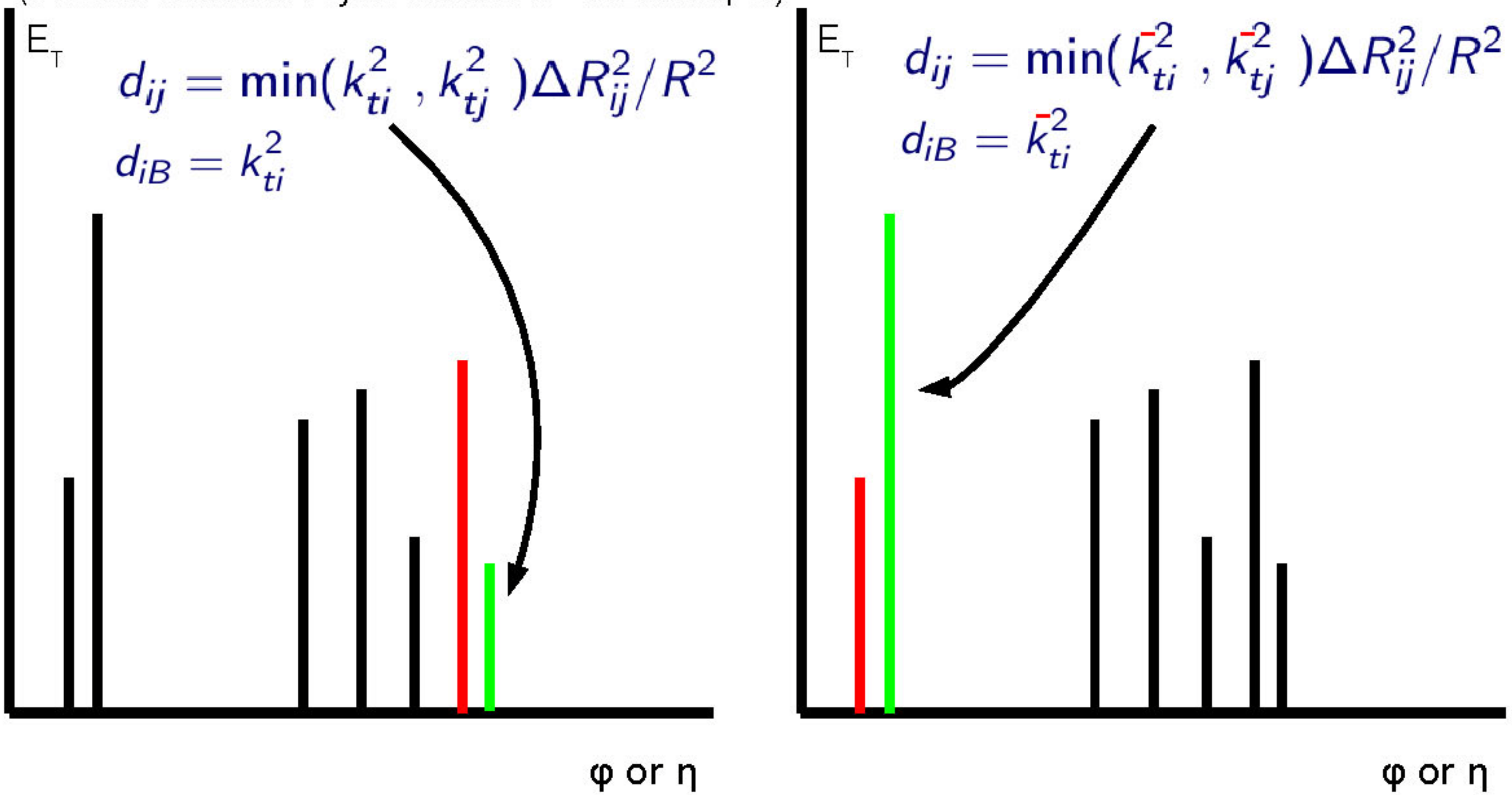
Kt (Catani/Dokshitzer/Seymour/Webber - S.Ellis/Soper)

AntiKt (Cacciari/Salam/Sovez)



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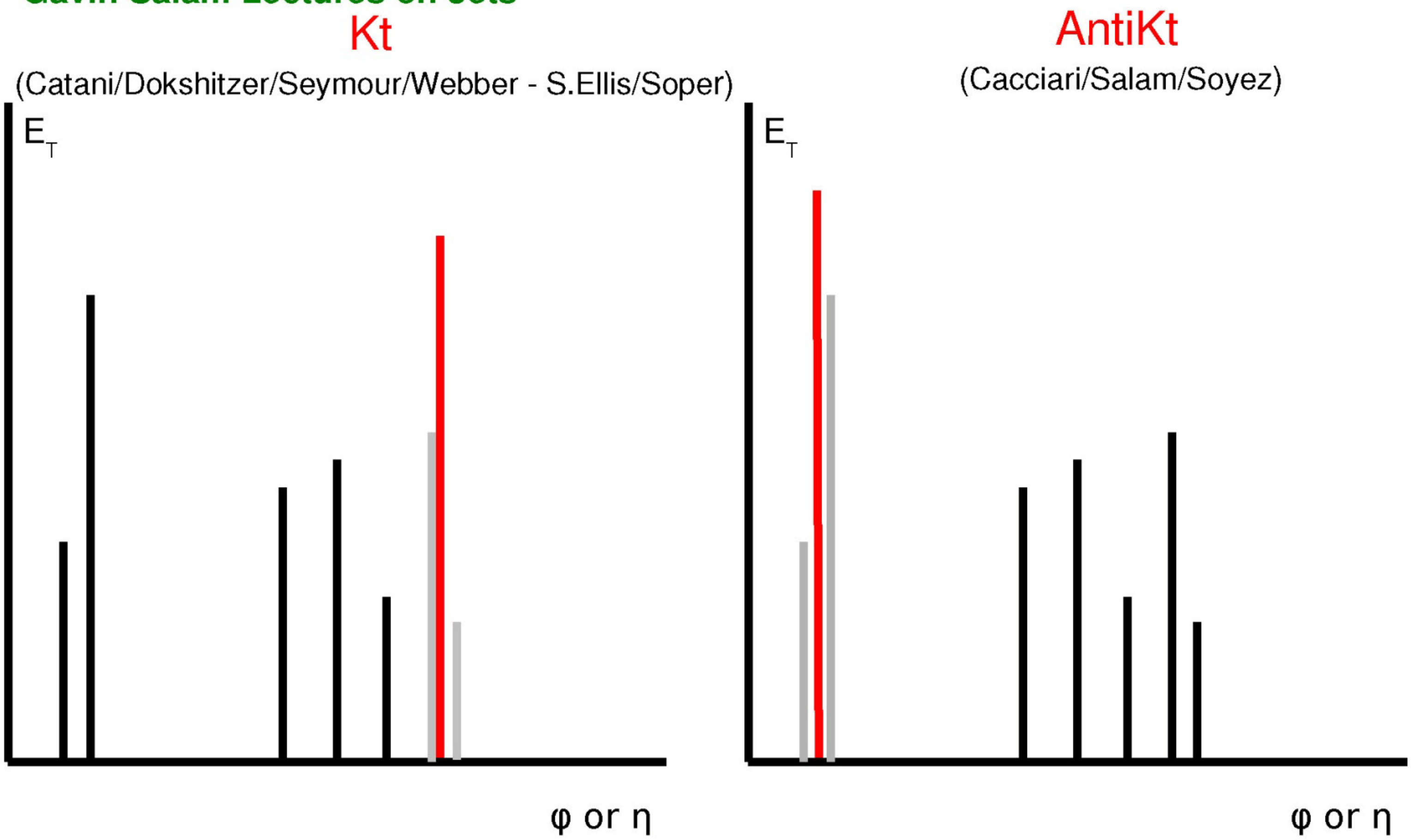

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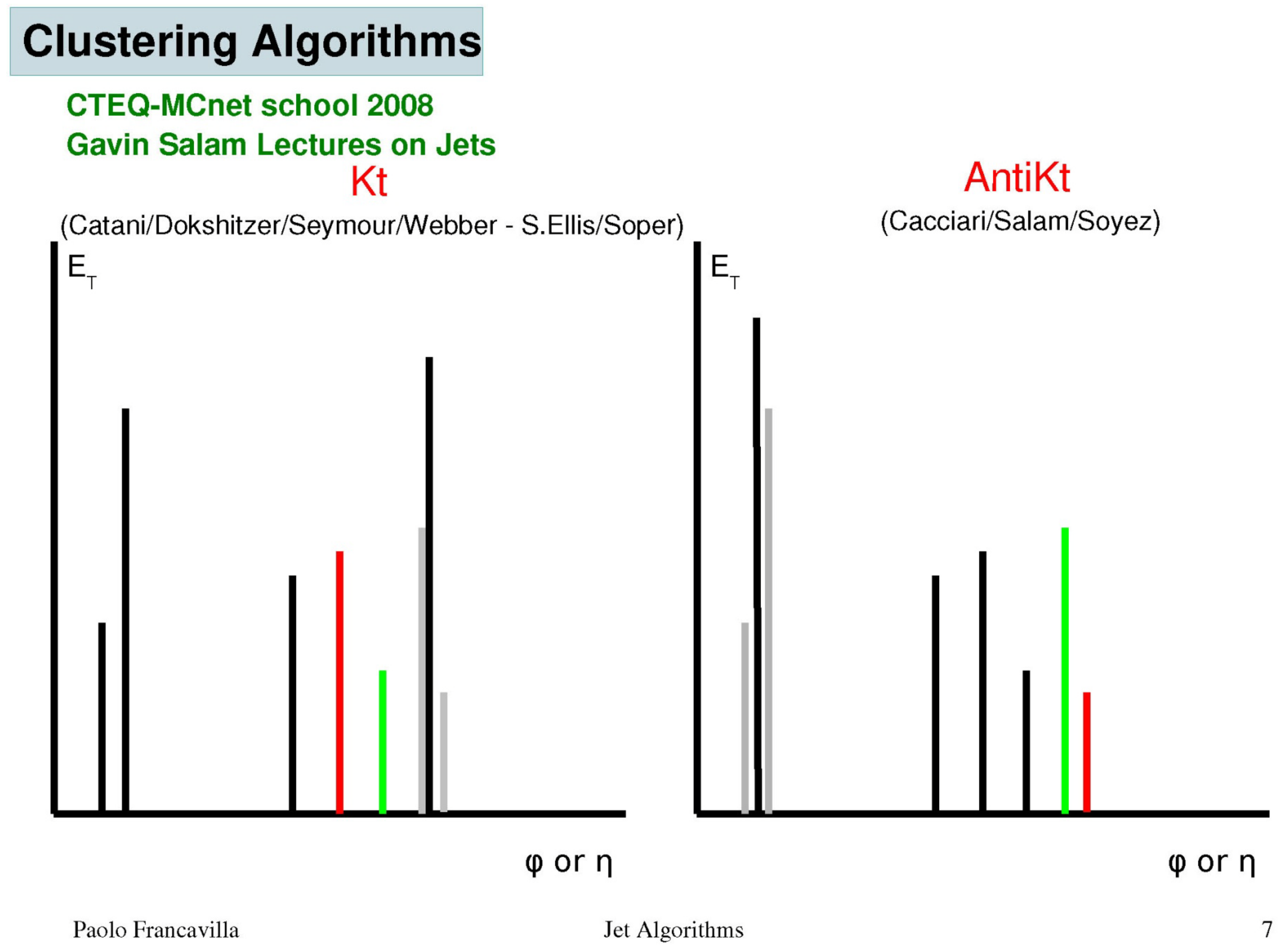

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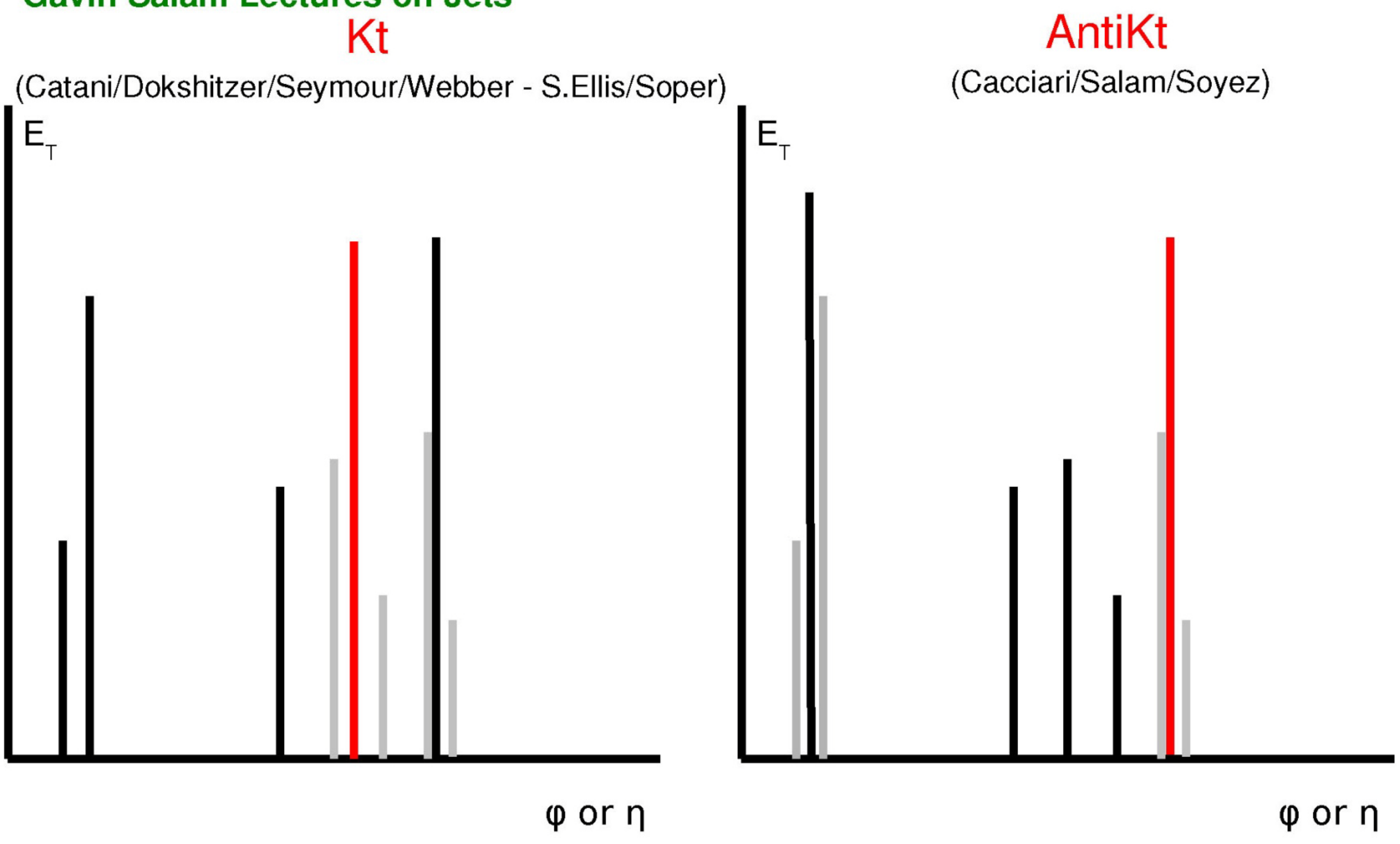

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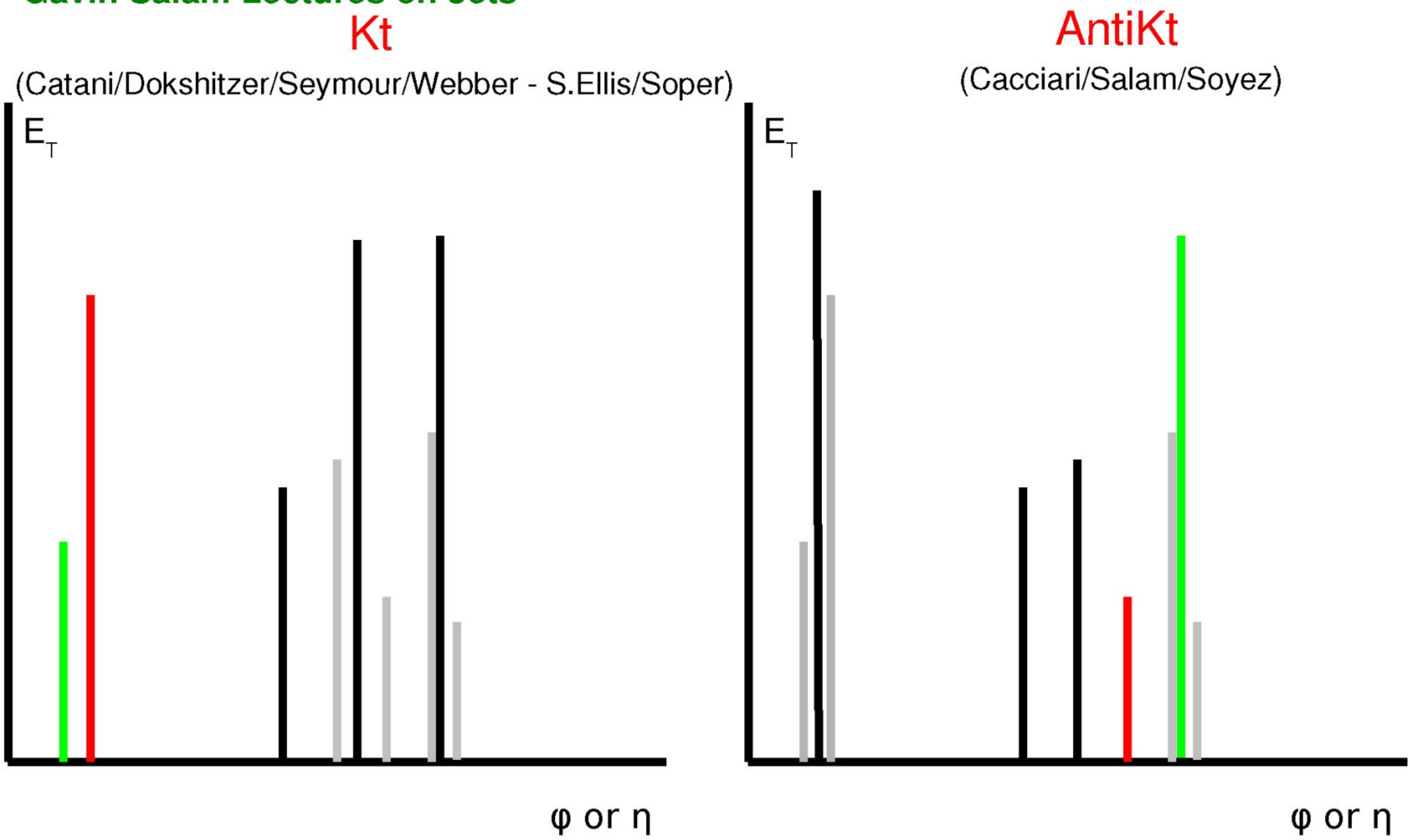

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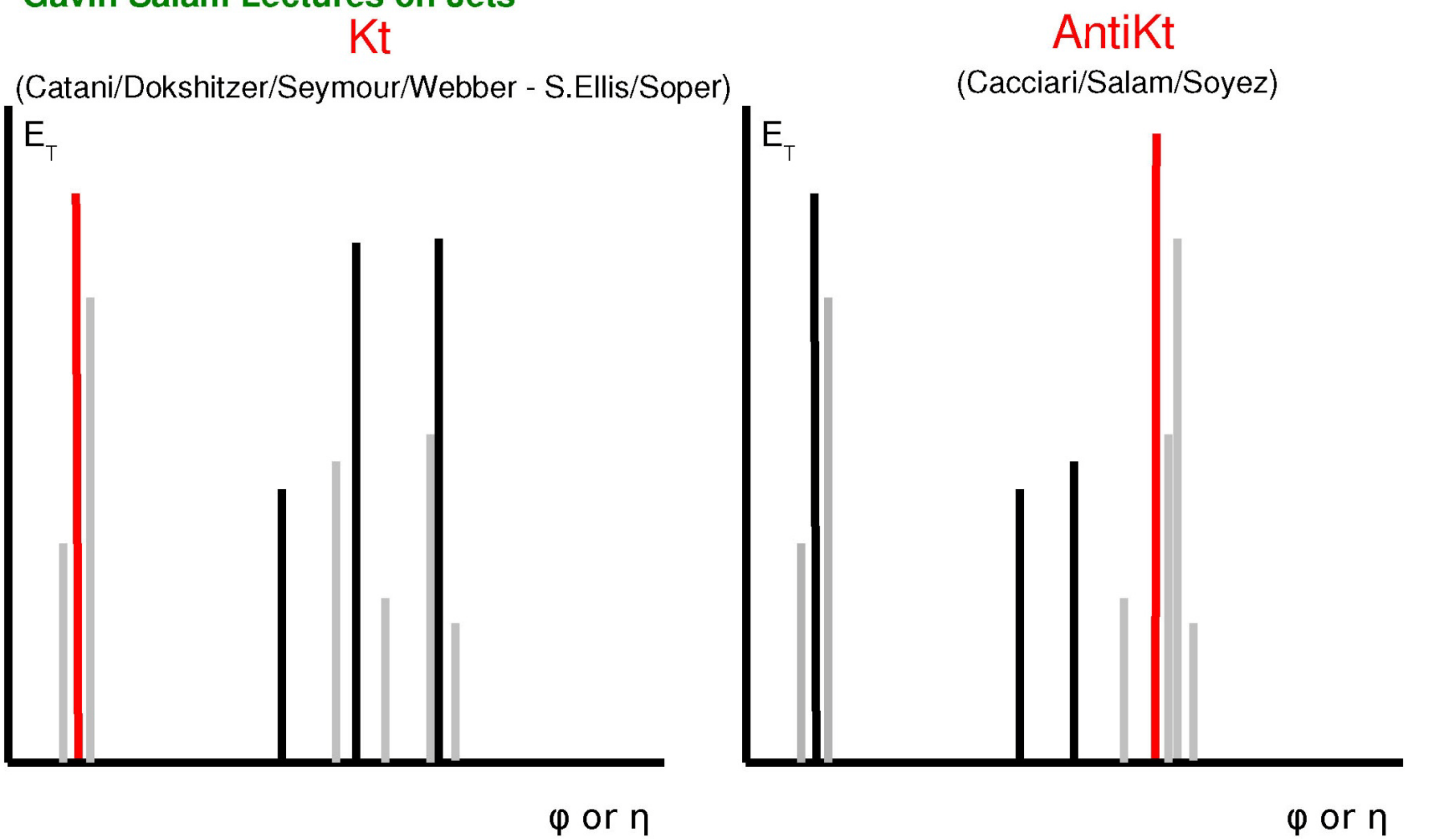

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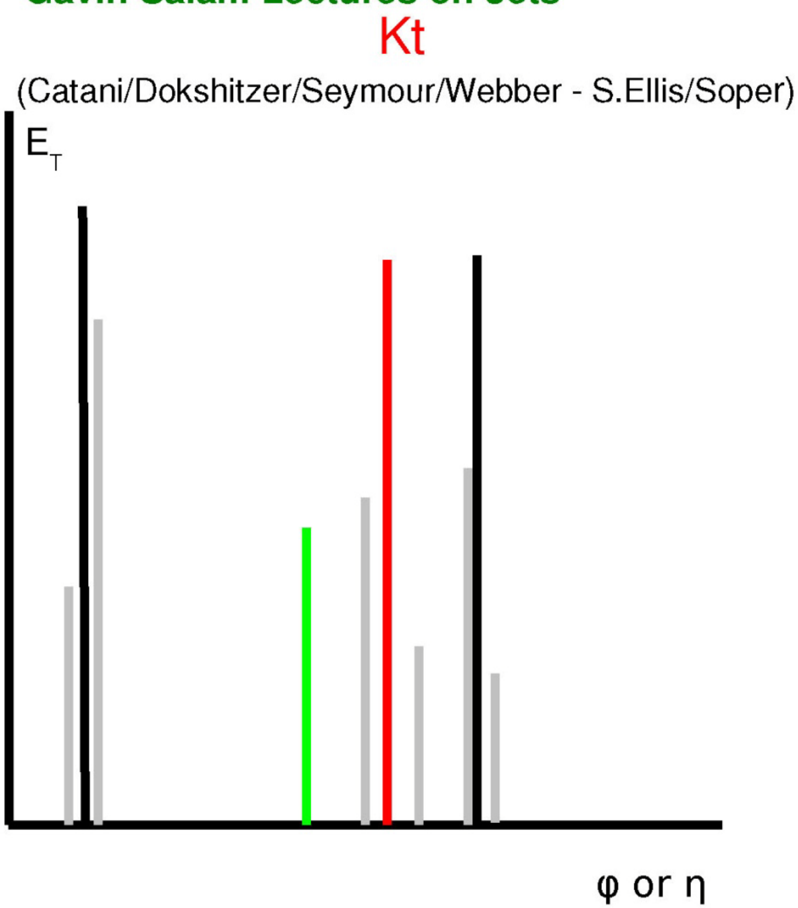

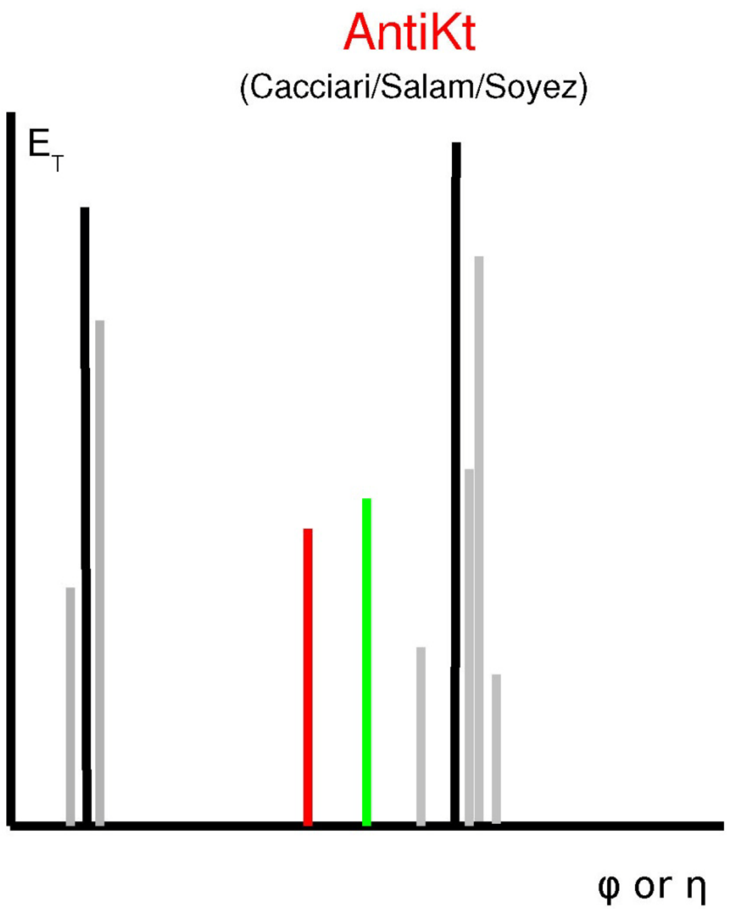

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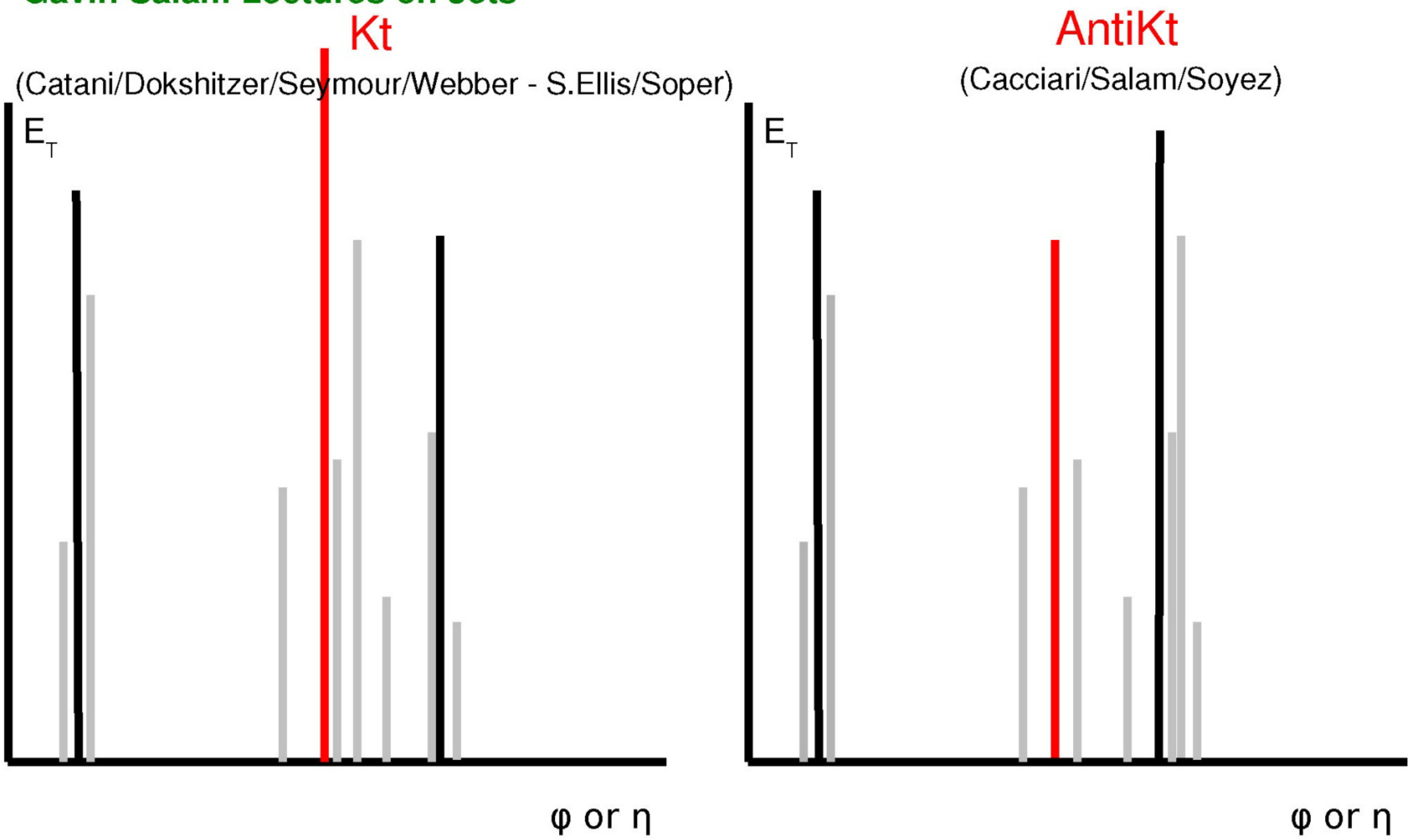

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