Introduction to Hadronic Final State Reconstruction in Collider Experiments
(Part IV)

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What is response?

Reconstructed calorimeter signal
- Based on the direct measurement – the raw signal
- May include noise suppression

Has the concept of signal (or energy) scale
- Mostly understood as the basic signal before final calibrations
- Does not explicitly include particle or jet hypothesis

Uses only calorimeter signal amplitudes, spatial distributions, etc.

\[ E_{raw} = A_{peak} \times \left[ \text{ADC} \rightarrow \text{nA} \right] \times \left[ \text{HV} \times \text{cross-talk} \times \text{purity} \right] \times \left[ \text{nA} \rightarrow \text{MeV} \right] \]
Slow signal collection in liquid argon calorimeters

~450 ns @ 1 kV/mm drift time versus 40 MHz/25 ns bunch crossing time

Measure only \( I_0 = I(t_0) \) (integrate <25 ns)

Applying a fast bi-polar signal shaping

Shaping time ~15 ns

With well known shape

Shaped pulse integral = 0

Net average signal contribution from pile-up = 0

Need to measure the pulse shape (time sampled readout)

Total integration ~25 bunch crossings

23 before signal, 1 signal, 1 after signal
What is digital filtering

Unfolds the expected (theoretical) pulse shape from a measured pulse shape

Determines signal amplitude and timing

Minimizes noise contributions
Noise reduced by ~1.4 compared to single reading
Note: noise depends on the luminosity

Requires explicit knowledge of pulse shape

Folds triangular pulse with transmission line characteristics and active electronic signal shaping
Characterized by signal transfer functions depending on \( R, L, C \) of readout electronics, transmission lines

Filter coefficients from calibration system

Pulse “ramps” for response
Inject known currents into electronic chain
Use output signal to constrain coefficients

Noise for auto-correlation
Signal history couples fluctuations in time sampled readings

Signal amplitude \( A_{\text{peak}} \) (energy):

\[
A_{\text{peak}} = \sum_{i=1}^{N_{\text{i}}} a_i (s_i - p), \quad \text{with}
\]

\( a_i \) digital filter coefficient
\( s_i \) reading in time sample
\( p \) pedestal reading

Signal peak time \( t_{\text{peak}} \):

\[
A_{\text{peak}} t_{\text{peak}} = \sum_{i=1}^{N_{\text{i}}} b_i (s_i - p)
\]

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**Determines signal amplitude and timing**

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**Filter coefficients from calibration system**

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  - Inject known currents into electronic chain
  - Use output signal to constrain coefficients
- **Noise for auto-correlation**
  - Signal history couples fluctuations in time sampled readings

Signal amplitude $A_{\text{peak}}$ (energy):

$$A_{\text{peak}} = \sum_{i=1}^{N_s} a_i (s_i - p), \text{ with } \begin{cases} a_i & \text{digital filter coefficient} \\ s_i & \text{reading in time sample} \\ p & \text{pedestal reading} \end{cases}$$

Signal peak time $t_{\text{peak}}$:

$$A_{\text{peak}} t_{\text{peak}} = \sum_{i=1}^{N_s} b_i (s_i - p)$$


Constraints for digital filter coefficients $a_i$:

$$\sum_{i=1}^{N_s} a_i g_i = 1, \text{ with } g_i \text{ being the normalized physics pulse shape}$$

$$\sum_{i=1}^{N_s} a_i \frac{\partial g_i}{\partial t} = 0$$
ATLAS Digital Filter Coefficients

- $S_i$: signal amplitude in sample $i$
- $P$: analog-to-digital converter pedestal
- $N_s$: number of digital samples (def. 5)
- $a_i$: digital filter coefficient
- $R_{ij}$: noise auto-correlation
- $g_i$: normalized physics pulse shape

Filter Coefficients

- Determined by:
  - Known pulse shape
  - Minimizing noise

To 1st order independent of time jitter!

\[
\sum_{i=1}^{N_s} a_i g_i = 1 \\
\sum_{i=1}^{N_s} a_i \frac{\partial g_i}{\partial t} = \sum_{i=1}^{N_s} a_i g_i' = 0
\]

\[
a_i = \sum_{j=1}^{N_s} R_{ij}^{-1} \left( \lambda g_j - \mu g_j' \right)
\]

\[
\lambda = \frac{1}{\Delta} \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} R_{ij}^{-1} g_i' g_j'
\]

\[
\mu = \frac{1}{\Delta} \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} R_{ij}^{-1} g_i g_j'
\]

\[
\Delta = \left( \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} R_{ij}^{-1} g_i' g_j' \right) \cdot \left( \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} R_{ij}^{-1} g_i g_j \right) - \left( \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} R_{ij}^{-1} g_i g_j' \right)^2
\]
What does signal or energy scale mean?

Indicates a certain level of signal reconstruction

Standard reconstruction often stops with a basic signal scale

Electromagnetic energy scale is a good reference

Uses direct signal proportionality to electron/photon energy

Accessible in test beam experiments

Can be validated with isolated particles in collision environment

Provides good platform for data and simulation comparisons

Does not necessarily convert the electron signal to the true photon/electron energy!

Hadronic signals can also be calculated on this scale

Good platform for comparisons to simulations

But does not return a good estimate for the deposited energy in non-compensating calorimeters – see later discussion!

Is not a fundamental concept of physics!

Is a calorimeter feature

Definition varies from experiment to experiment

Recall electrons/photons in sampling calorimeters:

\[
E_{\text{vis}} = N_{\times} \int_{0}^{d_{\text{active}}} dE/dx = N_{\times} \Delta E \propto E_0
\]

Electron sampling fraction \( S_e \) relates signal and deposited energy:

\[
S_e = \frac{E_{\text{vis}}}{E_{\text{dep}}} \approx \frac{E_{\text{vis}}}{E_0} \rightarrow E_{\text{rec}}^{\text{em}} = \frac{1}{S_e} E_{\text{vis}} = c_e A \equiv E_{\text{dep}} \approx E_0
\]

with \( c_e \) being the electron calibration constant.

\( (S_e \) is a unitless fraction, \( c_e \) converts a signal unit into an energy unit, e.g. nA → MeV)

Response often denoted \( e = e(E_{\text{dep}}) = E_{\text{rec}}^{\text{em}} (c_e, A) \)
Single hadron response:
\[
\pi(E_0) = f_{\text{em}}(E_0) \cdot e + (1 - f_{\text{em}}(E_0)) \cdot h
\]
\[
\begin{cases}
f_{\text{em}}(E_0) & \text{intrinsic em fraction} \\
h & \text{response of pure hadronic shower branch}
\end{cases}
\]

Non-compensation measure:
\[
e = \frac{1}{\pi \cdot f_{\text{em}}(E_0) + (1 - f_{\text{em}}(E_0)) \cdot h/e}
\]

Popular parametrization by Groom et al.:
\[
f_{\text{em}}(E_0) = 1 - \left( \frac{E_0}{E_{\text{base}}} \right)^{m-1}
\]

\[
m = 0.80 - 0.85, \quad E_{\text{base}} = \begin{cases} 
1.0 \text{ GeV} & \text{for } \pi^\pm \\
2.6 \text{ GeV} & \text{for } p
\end{cases}
\]

Observable

\[
e = \frac{e}{\pi} = \frac{1}{1 - (1 - h/e)(E_0/E_{base})^{m-1}}
\]

provides experimental access to characteristic calorimeter variables in pion test beams by fitting \( h/e \), \( E_{base} \) and \( m \) from the energy dependence of the pion signal on electromagnetic energy scale:

\[
e = \frac{E_0}{\pi E_{rec}(\pi)} \approx \frac{E_{dep}}{E_{rec}(\pi)}
\]

Note that \( e/h \) is often constant, for example: in both H1 and ATLAS about 50% of the energy in the hadronic branch generates a signal independent of the energy itself.
Complex mixture of hadrons and photons

Not a single particle response
Carries initial electromagnetic energy

Mainly photons

Very simple response model
Assume the hadronic jet content is represented by 1 particle only
Not realistic, but helpful to understand basic response features

More evolved model
Use fragmentation function in jet response
This has some practical considerations
E.g. jet calibration in CDF

Gets non-compensation effect
Does not address acceptance effect due to shower overlaps
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\[
\frac{j(E_{\text{jet}})}{e} = f_{\gamma}^{\text{jet}} + \left(1 - f_{\gamma}^{\text{jet}}\right) \cdot \left(f_{\text{em}} + \left(1 - f_{\text{em}}\right) \frac{h}{e}\right)
\]

\[
f_{\text{em}} = f_{\text{em}}(E_{\text{jet}}^{\text{had}}), \quad E_{\text{jet}}^{\text{had}} = \left(1 - f_{\gamma}^{\text{jet}}\right) E_{\text{jet}}
\]

[single particle approximation]

\[
f_{\text{em}} = 1 - \left(\frac{E_{\text{jet}}^{\text{had}}}{E_{\text{base}}}\right)^{1-m}
\]

[Groom's parameterization]
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\[
\begin{align*}
\frac{j(E_{\text{jet}})}{e} & = f_{\gamma}^\text{jet} \\
& + \left(1 - f_{\gamma}^\text{jet}\right) \int \left[f_{\text{em}}(E_{\text{had}}) + \left(1 - f_{\text{em}}(E_{\text{had}})\right) \frac{h}{e}\right] dE_{\text{had}} \\
& = f_{\gamma}^\text{jet} \\
& + \left(1 - f_{\gamma}^\text{jet}\right) \sum_{\text{hadrons}} \left[1 - \left(\frac{E_{\text{had}}}{E_{\text{base}}}\right)^{m-1} + \left(\frac{E_{\text{had}}}{E_{\text{base}}}\right)^{m-1} \frac{h}{e}\right] \\
& = f_{\gamma}^\text{jet} + \left(1 - f_{\gamma}^\text{jet}\right) \sum_{\text{hadrons}} \left(1 + \left(\frac{E_{\text{had}}}{E_{\text{base}}}\right)^{m-1} \left(h/e - 1\right)\right)
\end{align*}
\]
### Noise

- **Fluctuations of the “zero” or “empty” signal reading**
  - Pedestal fluctuations
- **Independent of the signal from particles**
  - At least to first order
- **Mostly incoherent**
  - No noise correlations between readout channels
    - Noise in each channel is independent oscillator
- **Gaussian in nature**
  - Pedestal fluctuations ideally follow normal distribution around 0
  - Width of distribution (1 $\sigma$) is noise value

### Signal significance

- **Noise can fake particle signals**
  - Only signals exceeding noise can be reliably measured
- **Signals larger than $3 \times$ noise are very likely from particles**
  - Gaussian interpretation of pedestal fluctuations
- **Calorimeter signal reconstruction aims to suppress noise**
  - Average contribution = 0, but adds to fluctuations!

---

**Reading ($\sigma_{\text{noise}}$)**

- **Small signal:**
  - Noise only
  - Signal on top of noise
  - Sum of noise and signal
  - Signal after noise suppression
Noise

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Spatial Coordinate/Calorimeter Cell
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Small signal, first and second particle:
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  Signal after noise suppression
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**Reading ($\sigma_{\text{noise}}$)**

**Small signal, two particle, sum:**

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**Spatial Coordinate/Calorimeter Cell**
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< true signal!