Measurement of the W Helicity in Top Quark Decays

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We present a search for right-handed helicity state W'-bosons in top quark events using t\bar{t} decays in the \( \mu + \text{jets} \) final states. A non-zero fraction of right-handed W'-bosons, \( f^+ \), would be evidence for a V+A current contribution to top quark decays. Using a Bayesian confidence interval, we find 0 < \( f^+ < 0.262 \) (90% CL). This is consistent with the standard model prediction of \( f^+ = 0.0 \).

This is version 2.4 of the conference note.

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References
I. INTRODUCTION

One test of the standard model (SM) is the measurement of the helicity of \( W \)-bosons in top quark decays. In the SM, the top quark decays via the V-A charged current interaction. At the Born level, this parity violating interaction limits decays of top quarks into \( W \)'s with longitudinal and left-handed helicity states with fractions \( f^0 \) and \( f^- \) respectively.

The branching ratio for \( f^0 \) is a function of the top quark mass \( (m_t) \) \( W \)-boson mass \( (M_W) \), and \( b \)-quark mass \[1\]. The effects of the \( b \)-quark mass are small and hence:

\[
f^0 = \frac{m_t^2}{m_t^2 + 2M_W^2}
\]

With the present measured values of the top quark and \( W \)-boson mass the SM prediction gives \( f^0 \approx 0.70 \) and \( f^- \approx 0.30 \). There are no decays into right-handed \( W \) helicity states \( (f^+ = 0.0) \) since the \( b \)-quark helicity in this case would necessarily be right-handed and hence greatly suppressed. The helicity states for antitop decays would be reversed from that given above. An early theoretical treatment of top quark decays is given in [2]. In this measurement we search for a non-zero value of \( f^+ \) that would be evidence for a possible V+A admixture to the \( t \rightarrow b \) current.

In Run I of the Tevatron, CDF measured \( f^+ = 0.11 \pm 0.15 \) and \( f^0 = 0.91 \pm 0.37 \pm 0.13 \) [3]. DØ obtained \( f^0 = 0.56 \pm 0.31 \) [4][5]. In addition to direct measurements, data on \( b \rightarrow s \gamma \) decays have been used to set a limit on \( W_R \) and \( W_L \) mixing [6],[7].

Our analysis consists of selecting events using nearly the same criteria used by the Winter 2004 lepton+jets \( t \bar{t} \) production cross section analyses. One exception is that we do not reject events that contain jets that are tagged with soft muons. In addition we also employ topological criteria to increase the expected number of signal versus background.

For events passing all selection criteria we perform a kinematic fit using the HITFIT package to select the \( b \)-jet associated with the leptonic \( W \). We use the term “leptonic \( W \)” as shorthand notation for the phrase “\( W \) that decays leptonically”. In the very small percentage of cases where the kinematic fit does not converge, we use a simpler \( \chi^2 \) method to select the \( b \)-jet associated with the leptonic \( W \). Both methods use the measured top mass and \( W \)-boson mass as contraints.

Once the \( b \)-jet associated with the leptonic \( W \) is identified we calculate \( \cos \theta^* \). We define \( \cos \theta^* \) as the cosine of the angle between the lepton momentum and the initial \( W \)-boson momentum when boosted to the rest frame of the leptonic \( W \). With this definition, the \( \cos \theta^* \) distribution for right-handed \( W \)'s is peaked towards \( \cos \theta^* = +1 \). We use the object momenta returned from the kinematic fit in calculating \( \cos \theta^* \).

We produce templates in \( \cos \theta^* \) for \( t \bar{t} \) signal assuming different V+A fractions \( f^+ \) and for \( Wjjjjj \) and “QCD” backgrounds. We rely on Monte Carlo to produce the different \( \cos \theta^* \) distributions except for the multijet background (called “QCD”), which is taken from data.

We use these templates in a binned likelihood fit to find the V+A fraction \( f^+ \) given by the data. The resulting log likelihood curves are interpreted using both Bayesian and frequentist approaches. We also use these templates in fits to ensembles of Monte Carlo events in order to test the veracity of our procedure and to estimate systematic uncertainties.
Our data samples consist of ROOT-tuples from the MUQCD and EMQCD skims processed with the Nefertiti version of top_analyze. Data from bad luminosity blocks are rejected using the top_dq package. The integrated luminosity of the surviving $\mu$+jets data is $158.4 \text{ pb}^{-1}$.

The Monte Carlo samples consist of events generated by ALPGEN or PYTHIA. All Monte Carlo events were processed through GEANT, DØRECO, and top_analyze. The standard $t\bar{t}$ sample corresponds to the Top group’s Tune A version. The standard $W^{+}jjjj$ sample corresponds to the Higgs group’s Tune A version. In order to test the hypothesis of a non-zero $f^+$ fraction, we also used ALPGEN samples which varied the $tWb$ coupling in increments from purely $V-A$ to purely $V+A$, resulting in $f^+$ varying from 0.0 to 0.3. (The fraction of decays into longitudinal $W$’s is $f^0 = 0.7$ for both $V-A$ and $V+A$ couplings, so $f^0$ does not change in any of our Monte Carlo samples.) We call these samples the $f^+$ samples. The $f^+$ samples corresponded to the Top group’s Tune A version.

We require events to have fired the correct lepton+jets L1, L2, and L3 triggers.

Our preselection criteria for the $\mu + jets$ channel are given in Table I. These criteria are identical to those used in the $t\bar{t}$ production cross section analysis for Winter 2004 conferences, except we do not reject events in which a soft muon is associated with a jet (i.e. we have no SLV veto).

Our final selection criteria are the preselection criteria plus a topological criterion used to further increase the expected ratio of $S/\sqrt{S + B}$. Presently we use a cut on a six-variable topological likelihood $L_t$ given in Table II. The topological likelihood variables and cut value were optimized by performing ensemble tests and choosing a definition and cut that minimized the average width of the 68% Bayesian confidence interval for the measurement of $f^+$.

The topological likelihood $L_t$ is based on six kinematic variables, defined as follows:

- Aplanarity $A$, defined as $3/2$ times the smallest eigenvalue of the normalized momentum tensor of the
jets and lepton. $\mathcal{A}$ is a measure of the deviation from flatness of the event, and $t\bar{t}$ events tend to have larger values than background events.

- $H_{T2}$, defined as the sum of the $E_T$'s of all the jets in the event except the highest-$E_T$ one, divided by the sum of the magnitudes of the longitudinal momenta of the jets, lepton, and neutrino ($p_z$ of the neutrino is calculated using a $W$ mass constraint). Top quark events will tend to be more central and thus have higher values of $H_{T2}$.

- $K'_{T_{\text{min}}}$, defined as the distance in $\eta - \phi$ space between the closest pair of jets multiplied by the $E_T$ of the lowest-$E_T$ jet in the pair, and divided by the $E_T$ of the $W$. Only the four leading-$E_T$ jets are considered in computing this variable. Jets arising from gluon radiation (as is the case for background) will tend to result in low values of $K'_{T_{\text{min}}}$.

- Sphericity $\mathcal{S}$ is defined as 3/2 times the sum of the two smallest eigenvalues of the normalized momentum tensor of the jets in the event. This variable is similar to $\mathcal{A}$, and $t\bar{t}$ events will tend to have larger values than background.

- $H_T$, defined as the as the scalar sum of all jet $P_T$ values $> 15$ GeV. Jets arising from gluon radiation in general have lower $P_T$ than jets in $t\bar{t}$ events so background events will tend to have smaller values of $H_T$ compared to signal events.

- HITFIT $\chi^2$, defined as the $\chi^2$ associated with a kinematic fit to the hypothesis of $t\bar{t}$ decays in the $e+$jets or $\mu+$jets final states. Signal events will naturally have smaller $\chi^2$ values than background events. HITFIT is the name of the software package used to perform the kinematic fit.

The efficiency of topological likelihood cut as a function of $f^+$ is shown in Figure 1. There is a slight variation in efficiency that we later account for as an error in the second matrix method. The nominal efficiency we use for $t\bar{t}$ events is taken from the $f^+=0.15$ sample.

The fact that we are using Monte Carlo distributions as input to the topological likelihood $L_t$ raises the question of how well data distributions agree with the Monte Carlo. We compare data and Monte Carlo distributions for preselected events for the transformed variables aplanarity, sphericity, $K'_{T_{\text{min}}}$, $H_T$, and HITFIT $\chi^2$. We also compare data and Monte Carlo distributions for preselected events for $E_T$, $P_T^{\mu}(\text{highest})$, and $P_T^{\mu}(\text{highest})$. For the contributing fractions of $t\bar{t}$, $Wjjjj$, and QCD we use the fractions determined by the second matrix method. The $t\bar{t}$ and $Wjjjj$ distributions are given by Monte Carlo samples described at the beginning of this section. The QCD distribution is taken from data by requiring preselection cuts except that lepton is required to be not isolated. The data/Monte Carlo comparisons are shown in the analysis note. There is relatively good agreement between data and Monte Carlo in all distributions.

### III. SIGNAL AND BACKGROUND DETERMINATION

We use the first and second matrix methods in order to determine the number of signal and background events in the selected data. Below we use the terms “loose”, “preselected”, and “selected” events. Preselected events are those events that pass the selection criteria in Table I. Loose events are those events that pass the preselection criteria except the isolation criterion. Specifically, $N_{\text{loose}}$ in the $\mu+$jets channel is the number of events passing all preselection criteria except for the Rat11 and Rattrk isolation criterion. Selected events are those events that pass the preselection cuts and the topological likelihood cut.
The first matrix method is used to determine the number of $t\bar{t} + W jjjj$ ($N_{\text{pr}}(N_{\text{pr}}^e + y)$ and $QCD(N_{\text{pr}}^e)$) events after preselection but before applying the topological likelihood cut [10].

The input to the first matrix method is given in Table III. The output from the first matrix method is given in Table IV. The efficiencies and their errors are taken from the production cross section note for the lepton plus jets decay channel [9]. $N_{\text{loose}}$ and $N_{\text{pr}}$ were defined in the first paragraph of this section. Here $\varepsilon_{\text{signal}}$ refers to both $t\bar{t}$ and $W jjjj$ events.

We use the second matrix method in order to determine the number of signal and background events in our final sample after all selection criteria [10].

The quantities for the second matrix method and their error for the $\mu$+jets and $e$+jets channels are given in Table V.

For $t\bar{t}$ events we calculate $\varepsilon_{\text{pr}}^{t\bar{t}}$ using the $f^+ = 0.15$ Monte Carlo sample. This minimizes the error due to
the variation of the $L_\ell$ efficiency as a function of $f^+$. The variation in the efficiency as a function of $f^+$ (Figure 1) is included in the error for $\varepsilon_{\text{sel}}^\ell$ listed in Table V.

The resulting number of $t\bar{t}$, $Wjjjj$ and QCD events resulting from the second matrix are given in Table VI. We use these numbers of signal and background events to perform ensemble tests and as input into our likelihood used to determine $f^+$. The uncertainties in these numbers include the uncertainty in the topological likelihood selection efficiency, which varies as a function of $f^+$.

IV. TEMPLATES

The input to the maximum likelihood fit requires templates of signal and backgrounds. The $t\bar{t}$ and $Wjjjj$ templates are generated using the Monte Carlo samples described in Section II. The $t\bar{t}$ templates are produced for $\Pi$ values from 0.0 to 0.3. The events are required to pass all selection cuts.

The QCD templates are found using data. For QCD, the events are required to pass all selection cuts with one difference. In the $\mu^+$-jets channel we define a QCD sample by requiring the high $P_T$ muon not to be isolated (i.e. to fail the Rat11 or Rattrk cuts).

To ease reading, the templates for signal and backgrounds for the $\mu^+$-jets and $e^+$-jets decay channels are shown in Section XI. In order to study certain systematic errors we also make templates varying the top quark mass and the jet energy scale corrections.

When used in the maximum likelihood fit, the templates are rebinned to have five bins. We chose five bins based on a study that varied the number of bins as 2, 5, 10, and 50 bins.

Our templates were produced using HITFIT to determine the $b-$ jet associated with the leptonic $W$ [11]. The object momenta returned from HITFIT’s constrained kinematic fir are used in the calculation of $\cos \theta^*$. In the case where there was no solution in HITFIT we employed a simpler $\chi^2$ method that compares the calculated and known hadronic and leptonic decay top quark masses and hadronic $W$ mass to select the $b-$jet, and used the object momenta from RECO in calculating $\cos \theta^*$. However these cases are a rare circumstance, occurring in about 0.5% of events.

The nominal JES corrections were applied to both data and Monte Carlo. The nominal parton and eta
dependent correction were used in selecting the $b$-jet associated with the leptonic $W$ (either with HITFIT or the simpler $\chi^2$ method).

V. MAXIMUM LIKELIHOOD FIT

We perform a binned maximum likelihood fit to extract the value of $f^+$, the fraction of $V+A$ in the top decay, most consistent with the data. As input to the fit we have the distributions of $\cos\theta^*$ in: the selected data events, ALPGEN $t\bar{t}$ Monte Carlo with $f^+ = 0.00, 0.05, 0.10, 0.15, 0.20, 0.25$, and $0.30$, ALPGEN $Wjjjj$ Monte Carlo, and QCD background from data. We also note in the ALPGEN $V+A$ samples, the fraction $f^0$ is constant (70%) for all samples.

For each $f^+$ value for which a template exists, we compute the likelihood of the data to be consistent with the sum of signal and background templates. The likelihood is computed by multiplying the Poisson probabilities of each template bin being consistent with the data, using a method in which the finite template statistics are explicitly accounted for [12]. We also have a prior expectation for the normalization of the background, which is expressed with a Gaussian term in the likelihood. We define the likelihood as

$$L(f^+) = \prod_{i=1}^{N_{\text{bins}}} e^{(n_{b,i} - \bar{n}_{b,i})^2 / 2\sigma_{b,i}^2} \times \prod_{j=1}^{N_{\text{bins}}} P(d_j; n_j) \times \prod_{k=0}^{N_{\text{sources}}} B(a_{jk}; A_{jk}, p_k).$$  

where $P(x; y)$ is the Poisson probability for $x$ events given an average value $y$ and $B(m; n, p)$ is the binomial probability for observing $m$ events out of $n$ possible given probability $p$.

In the Gaussian term, $N_{\text{bkg}}$ is the number of background sources ($Wjjjj$ and QCD in this analysis), $\bar{n}_{b,i}$ is the observed number of events for the $i$th background (i.e., from the second matrix method), $\sigma_{b,i}$ is the systematic uncertainty on the observed number, and $n_{b,i}$ is the expected number of events for the $i$th background. In the Poisson term, $d_j$ is the number of data events in the $j$th bin of the $\cos\theta^*$ distribution and $n_j$ is the predicted number of signal and background events in the $j$th bin of the distribution. In the binomial term, $a_{jk}$ is the actual number of Monte Carlo signal and background events in the $j$th template bin, $A_{jk}$ is the (unknown) expected number of Monte Carlo signal and background events in the $j$th template bin and $p_k$ is the probability (strength factor) for observing the $k$th source ($t\bar{t}, Wjjjj$ and QCD).

We minimize the $-\ln L$ for each trio of templates (two background templates and one $f^+$ template) (the $a_{jk}$) and data distribution (the $d_j$). The minimization is with respect to the strengths $p_k$ and the expected number of Monte Carlo (template) signal and background events $A_{jk}$. The procedure is simplified by performing the minimization of the $A_{jk}$ analytically. The result of the fit then gives the predicted number of events in each bin $n_j$.

To summarize, we minimize $-\ln L$ using the likelihood given above. However when comparing values of the likelihood at each value of $f^+$ we take the value of $-\ln L$ to be that using only the Gaussian and Poisson terms. This prescription still accounts for finite template statistics but removes the correlation between $L$ and the template statistics. The result is a distribution of $-\ln L$ points versus $f^+$. We fit these points to a parabola to estimate the likelihood as a function of $f^+$. 

A. Bayesian interpretation of results

Since in our assumed model \( f^+ \) must lie between 0 and 0.30, we use a Bayesian technique to determine a 68% CL range for the true value of \( f^+ \). We choose to use a flat prior probability in \( f^+ \). With this choice, finding a Bayesian confidence interval is equivalent to integrating the likelihood curve. If the parabola fit to the \(- \ln L\) points has its minimum in the allowed range, we take the value of that minimum (i.e. the maximum of \( L \)) as the most likely value \( x_{\text{ML}} \). We then find the points \( x_{\text{min}} \) and \( x_{\text{max}} \) such that:

\[
\frac{\int_{x_{\text{min}}}^{x_{\text{ML}}} L(x) \, dx}{\int_{0}^{0.30} L(x) \, dx} = \frac{\int_{x_{\text{ML}}}^{x_{\text{max}}} L(x) \, dx}{\int_{0.30}^{0} L(x) \, dx} = 0.34
\]

If \( x_{\text{ML}} \) lies outside the allowed range (or close enough to the boundary that the \( x_{\text{max}} \) or \( x_{\text{min}} \) cannot be found by both equations above), a single-sided range is reported:

\[
\frac{\int_{0}^{x_{\text{max}}} L(x) \, dx}{\int_{0}^{0.30} L(x) \, dx} = 0.68
\]

or:

\[
\frac{\int_{0.30}^{x_{\text{min}}} L(x) \, dx}{\int_{0.30}^{0} L(x) \, dx} = 0.68
\]

If \( x_{\text{ML}} \) is less than (or close to) 0.0, then \( x_{\text{min}} = 0 \) and \( x_{\text{max}} \) is calculated. If \( x_{\text{ML}} \) is greater than (or close to) 0.30, then \( x_{\text{max}} = 0.30 \) and \( x_{\text{min}} \) is calculated. This in all cases there is both an \( x_{\text{min}} \) and \( x_{\text{max}} \).

In the case where the \(- \ln L\) points form an “upside-down” parabola, \( x_{\text{ML}} \) is taken to be at the physical boundardy (\( f^+ = 0.0 \) or 0.30) with the smallest value of \(- \ln L\).

VI. RESULTS FROM ENSEMBLE TESTS

We test the performance of the maximum likelihood fit by means of Monte Carlo ensemble tests. For these tests, we assume a true value of \( f^+ \) and form a mock data set by drawing events from the appropriate Monte Carlo samples. Each data set so formed has the same number of \( \mu^+\text{jets} \) as we observe in the real data sample (Table VI), but the number of signal and background is varied according to the binomial distribution. (Also once the number of background events in a particular mock data set is determined, the number of \( Wjjjj \) and QCD events is allowed to fluctuate binomially as well).

The mock data set is then fit according to the same procedure used for fitting the real data. By repeating the process a large number of times (1000 unless otherwise stated) we can investigate the statistical properties of the maximum likelihood fit. The results are given in Table VII. Note that while the general trend
TABLE VII: Results of Monte Carlo ensemble tests on mock data samples that model the current real data sample. The results in this table are for $\cos\theta^*$ templates having five bins.

<table>
<thead>
<tr>
<th>True $f^+$</th>
<th>Ave. Bayesian result</th>
<th>Ave. size of 68% CL range</th>
<th>Fraction in 68% CL range</th>
<th>Fraction with good parabolas</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.07</td>
<td>0.16</td>
<td>0.697</td>
<td>0.704</td>
</tr>
<tr>
<td>0.05</td>
<td>0.12</td>
<td>0.17</td>
<td>0.590</td>
<td>0.822</td>
</tr>
<tr>
<td>0.10</td>
<td>0.12</td>
<td>0.17</td>
<td>0.692</td>
<td>0.794</td>
</tr>
<tr>
<td>0.15</td>
<td>0.15</td>
<td>0.17</td>
<td>0.817</td>
<td>0.867</td>
</tr>
<tr>
<td>0.20</td>
<td>0.18</td>
<td>0.17</td>
<td>0.711</td>
<td>0.868</td>
</tr>
<tr>
<td>0.25</td>
<td>0.20</td>
<td>0.17</td>
<td>0.697</td>
<td>0.863</td>
</tr>
<tr>
<td>0.30</td>
<td>0.21</td>
<td>0.16</td>
<td>0.633</td>
<td>0.819</td>
</tr>
</tbody>
</table>

TABLE VIII: Summary of the systematic errors on $f^+$.

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top mass</td>
<td>0.11</td>
</tr>
<tr>
<td>Jet energy scale</td>
<td>0.04</td>
</tr>
<tr>
<td>W + jets model</td>
<td>0.08</td>
</tr>
<tr>
<td>$t\bar{t}$ model</td>
<td>0.05</td>
</tr>
<tr>
<td>Total</td>
<td>0.15</td>
</tr>
</tbody>
</table>

is reasonable (the average of the most likely value of $f^+$ increases as the true $f^+$ increases), the change in the average result is much less than the change in true $f^+$. This is an unavoidable consequence of having the result defined over a finite range of values.

Also shown in the table is the average size of the 68% confidence interval (CL) and the fraction of times in which the 68% CL range contains the true value of $f^+$. Note the average size of the 68% CL is slightly over half the allowed range of $f^+$. The fact that the fraction of times in which the 68% CL contains the true $f^+$ value is 82% for $f^+ = 0.15$ is a reflection of this average size. Both the fraction of times in which the 68% CL contains the true value of $f^+$ and the fraction of times there is a good parabola are symmetric about $f^+ = 0.15$ which makes sense. The main point here is that these ensemble tests show that the Bayesian confidence interval behaves properly. When one averages over all possible values of $f^+$, the probability for the true $f^+$ to be in the 68% Bayesian confidence interval is 69.1%, which is reasonable agreement.

VII. STATISTICAL AND SYSTEMATIC ERRORS

Statistical uncertainties in both the data and templates are handled by the likelihood fit using Poisson statistics for the data and binomial statistics for the templates.

Sources of systematic errors arise from the uncertainties in the top mass, jet energy scale, and Monte Carlo models of signal and background. Variations in these parameters can change the measurement in two ways: by altering the estimate of the background in the final sample (i.e., if the final selection efficiency changes) and by modifying the shape of the $\cos\theta^*$ templates.

We estimate the magnitude of these uncertainties by running ensemble tests using the standard templates, but with the mock data drawn from samples with the appropriate parameter varied. The signal and background content of the ensembles is fixed to the values we expect using our nominal final selection efficiencies, but the background constraint input to the maximum likelihood fit is varied to reflect the shifted final selection efficiency. The results are summarized in Table VIII, and details of the calculations are given below.
TABLE IX: Bayesian result for $f^+$ for various confidence levels. This result includes statistical errors only.

<table>
<thead>
<tr>
<th>Result for $f^+$</th>
<th>CL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; f^+ &lt; 0.188$</td>
<td>68%</td>
</tr>
<tr>
<td>$0 &lt; f^+ &lt; 0.262$</td>
<td>90%</td>
</tr>
<tr>
<td>$0 &lt; f^+ &lt; 0.281$</td>
<td>95%</td>
</tr>
</tbody>
</table>

To estimate the systematic error due to the uncertainty of the top quark mass we use $f^+ = 0.0$ samples with the top quark mass set to 170, 175, and 180 GeV.

The jet energies against which the selection criteria are applied contain the nominal jet energy scale corrections. Additionally, we apply parton and eta dependent corrections to these jet energies when calculating $\cos\theta^*$.

To estimate the systematic error due to the uncertainty of the jet energy scale we vary the jet energy scale by $\pm \sigma$ about the nominal value. The nominal value is used to correct jets back to the particle level energy and to equate the energy scales of jets in data and Monte Carlo. To estimate the systematic error due to the uncertainty of the jet energy scale we first modify the jet energies to have the values given by varying the jet energy scale by $\pm \sigma$. The missing $E_T$ is then adjusted to account for changes in the relevant components of the jet scale shift (e.g. missing $E_T$ is changed to reflect uncertainty in hadronic response, but not uncertainty in out-of-cone showering). We use these new jet energies and missing $E_T$ when applying the selection criteria. Next we apply parton and eta dependent corrections to the uncorrected jet energies, but now vary these parton and eta dependent corrections by $\pm \sigma$. The parton and eta dependent corrections are used to correct the energy of the jet to that of the original parton. We then calculate $\cos\theta^*$ using these jet energies within HitFit.

Another source of systematic uncertainty is the model of the $Wjjjj$ background. The nominal background sample used is the Higgs’ group Tune A ALPGEN sample. An alternate sample is the Top group’s iqopt10 ALPGEN sample, which was generated with a factorization scale of $< p_T (\text{jet}) >^2$.

Finally we consider the model of $t\bar{t}$ decays. As an alternative to the nominal Alpgen $t\bar{t}$ sample, we generate Pythia samples of $t\bar{t}$ in the $\mu +$jets decay channel, passed through a parton-level selection such that the surviving events have a $\cos\theta^*$ distribution identical to that expected for purely right-handed, longitudinal, or left-handed $W$’s. We then combine these three separate sets of events in the correct proportions to mimic any given $f^+$ value.

VIII. RESULTS FROM DATA

The results of applying our maximum likelihood fit to the $\cos\theta^*$ distribution observed in the data are shown in Figure 2 for $\mu +$jets events. The parabola that best fits the combined $- \ln L$ points has a minimum outside the physically-allowed region (at $f^+ = -0.128 \pm 0.403$). The Bayesian confidence intervals for different confidence levels are given in Table IX. This result includes statistical errors only.

We also show plots comparing the data distribution to the best fit model ($f^+ = 0.0$). In Figure 3 the data is shown as the points with error bars, the best fit signal template as the dashed histogram, the best fit background template as the dotted histogram, and the sum as the solid histogram. The best fit templates are normalized according to the fitted signal and background levels at the best fit $f^+$ point (0.0 in our case).

We also give the number of signal and background events resulting from the best fit using each $f^+$ signal
FIG. 2: Result of the maximum likelihood fit for $f^+$ on the $\mu+$jets data.

FIG. 3: Comparison of $\mu+$jets data (points with errors bars) to the sum of the best-fit templates of signal and background (solid histogram). The signal and background contributions are shown separately as the dashed and dotted histogram.

The systematic errors in the last section are included in the fit by convoluting a Gaussian function with model (Table X).
The results including systematic errors for different confidence levels are given in Table XI. The maximum likelihood distribution including statistical errors and including both statistical and systematic errors is shown in Figure 4. These results are interpreted as, for any one experiment, there is a $x\%$ chance that the true value of $f^+$ lies within the $x\%$ confidence interval.

The standard model (SM) value of $f^+ = 0$ is consistent with our results, but increased data and Monte Carlo statistics will be required to rule out any non-SM value.

<table>
<thead>
<tr>
<th>$f^+$</th>
<th>$t\bar{t}$</th>
<th>Wjjjj + QCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>10.0</td>
<td>19.8</td>
</tr>
<tr>
<td>0.05</td>
<td>10.0</td>
<td>19.8</td>
</tr>
<tr>
<td>0.10</td>
<td>9.3</td>
<td>19.9</td>
</tr>
<tr>
<td>0.15</td>
<td>9.3</td>
<td>19.9</td>
</tr>
<tr>
<td>0.20</td>
<td>9.1</td>
<td>19.9</td>
</tr>
<tr>
<td>0.25</td>
<td>8.3</td>
<td>19.9</td>
</tr>
<tr>
<td>0.30</td>
<td>8.0</td>
<td>19.9</td>
</tr>
</tbody>
</table>

**TABLE X:** Number of signal and background events resulting from the best fit using each $f^+$ signal model.

**TABLE XI:** Bayesian result for $f^+$ for various confidence levels. This result includes both statistical and systematic errors.

A width given by the total systematic error with the Gaussian resulting from the maximum likelihood fit. The standard model (SM) value of $f^+ = 0$ is consistent with our results, but increased data and Monte Carlo statistics will be required to rule out any non-SM value.

**FIG. 4:** Results of the combined $\tau$+jets and $\mu$+jets maximum likelihood fits including statistical errors only (solid line) and including both statistical and systematic errors (dashed line).
### IX. FREQUENTIST INTERPRETATION

As an alternative to the Bayesian approach we also construct confidence intervals for our result using a frequentist approach based on a paper by Feldman and Cousins [13]. This is only background information for the present.

In this analysis, we do not have a single measured number of events \( n \) but rather several measured numbers that make up bins in a histogram. Again, closely following the pedagogy of Feldman and Cousins [13], consider that we have a set of measured data \( N \equiv \{ n_i \} \), together with an assumed expected (known) background \( B \equiv \{ b_i \} \) and a signal contribution \( T \equiv \{ \mu_i | f^+ \} \). In this case each bin \( i \) corresponds to a Poisson process [14].

\[
P(n_i | \mu_i) = (\mu + b)^n \cdot \frac{\exp[-(\mu + b)]}{n!}.
\]

Ordering in this case is carried out using \( \Delta \ln L = \ln L - \ln L_{\text{best}} \) where

\[
L(f^+) = \prod_{i=1}^{N_{\text{bins}}} e^{(n_i - n_i^*)^2/2n_i^*} \times \prod_{j=1}^{N_{\text{bins}}} P(d_j; n_j) \times \prod_{k=0}^{N_{\text{interes}}} B(a_{jk}; A_{jk}, p_k),
\]

as used earlier in this analysis.

We proceed as follows.

- Perform ensemble tests for each \( f^+ \) Monte Carlo sample using the standard \( \cos \theta^* \) templates. For each mock data sample, calculate \( \Delta \ln L = \ln L - \ln L_{\text{best}} \). The value of \( \ln L_{\text{best}} \) is that evaluated at the minimum of the \( \ln L \) fit. In the case where minimum is outside the physical boundard (0.0 or 0.3) \( \ln L_{\text{best}} \) is evaluated at the boundary. The value of \( \ln L \) is that evaluated at the true \( f^+ \) value of the mock data sample.

- The values of \( \Delta \ln L = \ln L - \ln L_{\text{best}} \) are histogrammed for each set of ensemble tests performed at the different values of \( f^+ \). The histograms at each \( f^+ \) are integrated to find that value of \( \Delta \ln L = \Delta \ln L_{\text{critical}} \) corresponding to the desired confidence interval (68\%, 90\%, etc.) This is the acceptance region for each value of \( f^+ \).

- A plot is made of \( \Delta \ln L_{\text{critical}} \) for each \( f^+ \) ensemble test.

- The \( - \ln L \) curve from the data (or some mock data sample) is plotted on the same graph. The points or point where the two curves intersect give the desired confidence interval.

Following the steps itemized above, the frequentist approach gives the limits for various confidence levels in Table XII. This result includes statistical errors only.
TABLE XIII: Frequentist result for $f^+$ for various confidence levels. This result includes both statistical and systematic errors.

<table>
<thead>
<tr>
<th>Confidence Interval</th>
<th>0.0</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
<th>0.25</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>68%</td>
<td>0.679</td>
<td>0.681</td>
<td>0.676</td>
<td>0.677</td>
<td>0.687</td>
<td>0.668</td>
<td>0.694</td>
</tr>
<tr>
<td>90%</td>
<td>0.900</td>
<td>0.892</td>
<td>0.916</td>
<td>0.890</td>
<td>0.887</td>
<td>0.903</td>
<td>0.898</td>
</tr>
<tr>
<td>95%</td>
<td>0.952</td>
<td>0.942</td>
<td>0.961</td>
<td>0.939</td>
<td>0.945</td>
<td>0.954</td>
<td>0.949</td>
</tr>
</tbody>
</table>

TABLE XIV: Given a true $f^+$, the fraction of times that the true value of $f^+$ falls within the Feldman-Cousins confidence interval.

Systematic errors can be included in a similar manner to that used in the Bayesian approach. That is, one convolutes a Gaussian function with a width given by the total systematic error with the Gaussian resulting from the maximum likelihood fit. The resulting $-\ln L$ curve is plotted on the same graph as the $\Delta \ln L_{critical}$ points as was done above. The results are given in Table XIII. The acceptance curves and the $-\ln L$ curves for statistical errors only and combined systematic and statistical errors are shown in Figure 5. This means that 95% frequentist confidence interval runs from 0 to 0.257. The frequentist confidence interval is constructed in such a way that in 95% of mock Monte Carlo experiments, the true value of $f^+$ lies within the given 95% confidence interval.

![FIG. 5: Acceptance curves for 68%, 90%, and 95% confidence intervals. Also shown are the results of the combined $e^+$-jets and $\mu^+$-jets maximum likelihood fits including statistical errors only (solid line) and including both statistical and systematic errors (dashed line).](image)

Just as in the case of the Bayesian interpretation, we can use ensemble tests to check whether the frequentist approach behaves sensibly. Table XIV gives the fraction of times that the true value of $f^+$ falls within the Feldman-Cousins confidence interval. Thus the Feldman-Cousins coverage behaves sensibly.
X. CONCLUSIONS

We have measured the fraction of right-handed W’s ($f^+$) in top decays using the muon plus jets decay channels. Using a Bayesian interpretation for confidence intervals and including both statistical and systematic uncertainties we find

$$0 < f^+ < 0.262 \ (90\% \ CL)$$

This means, for any one experiment, there is a 90% chance that the true value of $f^+$ lies within the 90% confidence interval.

We have also performed a frequentist analysis based on Feldman-Cousins [13] that gives a similar result. This measurement is in agreement with the Standard Model prediction of $f^+ = 0.0$.

XI. TEMPLATE PLOTS

The $\cos\theta^*$ templates used in the maximum likelihood fit are given in this section for the $\mu$+jets channel. The events pass all selection criteria.
These are the templates for the $\mu$+jets channel for $f^+ = 0.0, 0.10, 0.20, \text{ and } 0.30$.

![Graph showing $\cos\theta^*$ distribution for $\mu$+jets $t\bar{t}$ signal for $f^+ = 0.0, 0.10, 0.20, 0.30$](image_url)

**FIG. 6:** $\cos\theta^*$ distribution for $\mu$+jets $t\bar{t}$ signal for $f^+ = 0.0, 0.10, 0.20, 0.30$

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[8] Internal/EB from the DØ Top Group Home page DØ Top Analysis and Data Sample for the Winter Conferences 2004 v0.3.
[9] Internal/EB from the DØ Top Group Home page Measurement of the t\bar{t} Production Cross Section at $\sqrt{s} = 1.96$ TeV in the Lepton Plus Jets Final States Using a Topological Method v0.5.
[10] Internal/EB from the DØ Top Group Home page Measurement of the W Helicity in Top Quark Decays v2.3
[11] Internal/EB from the DØ Top Group Home page Direct Measurement of the Top Quark Mass in the Lepton Plus Jets Channel Using Run II Data v0.1
[14] Internal/EB from the DØ Top Group Home page Measurement of the W Helicity in t\bar{t} Decays at $\sqrt{s} = 1.96$ TeV in the Lepton + Jets Final States Using a Lifetime Tag v1.4