## Statistical Combination of ATLAS and CMS Higgs Searches



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## The Standard Model of Particle Physics




## The Standard Model of Particle Physics



$$
\mathcal{L}_{S M}=
$$

$$
\underbrace{\frac{1}{4} \mathbf{W}_{\mu \nu} \cdot \mathbf{W}^{\mu \nu}-\frac{1}{4} B_{\mu \nu} B^{\mu \nu}-\frac{1}{4} G_{\mu \nu}^{a} G_{a}^{\mu \nu}}_{\text {kinetic energies and self-interactions of the gauge bosons }}
$$

$+\underbrace{\bar{L} \gamma^{\mu}\left(i \partial_{\mu}-\frac{1}{2} g \tau \cdot \mathbf{W}_{\mu}-\frac{1}{2} g^{\prime} Y B_{\mu}\right) L+\bar{R} \gamma^{\mu}\left(i \partial_{\mu}-\frac{1}{2} g^{\prime} Y B_{\mu}\right) R}$

Sutratow Tyedom

kinetic energies and electroweak interactions of fermions

$$
\begin{aligned}
& +\quad \underbrace{\frac{1}{2}\left|\left(i \partial_{\mu}-\frac{1}{2} g \tau \cdot \mathbf{W}_{\mu}-\frac{1}{2} g^{\prime} Y B_{\mu}\right) \phi\right|^{2}-V(\phi)}_{W^{ \pm}, Z, \gamma, \text { and Higgs masses and couplings }} \\
& +\quad \underbrace{g^{\prime \prime}\left(\bar{q} \gamma^{\mu} T_{a} q\right) G_{\mu}^{a}}_{\text {interactions between quarks and gluons }}+\underbrace{\left(G_{1} \bar{L} \phi R+G_{2} \bar{R} \phi_{c} L+\text { h.c. }\right)}_{\text {fermion masses and couplings to Higgs }}
\end{aligned}
$$



The standard model makes many predictions that are testable in very different experimental environments.

- Non-trivial aspects of the theory have been tested to < 1 ppm


$$
a_{\mu}(\exp )=11659208(6) \times 10^{-10}(0.5 \mathrm{ppm})
$$



$$
\left\{\begin{array}{l}
\left(D_{\mu} \phi\right)^{+} D^{\mu} \phi-V(\phi)-\frac{1}{4} F_{\mu \nu} F \mu \\
D_{\mu} \phi=\partial_{\mu} \phi-i . e A_{\mu} \phi \\
=\partial_{\mu} A_{V}-\partial_{\nu} A_{\mu} \\
\phi)=\phi \phi^{t} \phi+\beta\left(\phi^{x} \phi\right)^{2} \\
\alpha<0, \beta>_{0}
\end{array}\right.
$$






## Standard Model Higgs Properties







The Higgs boson can be produced via different interactions.
Production cross section $\sigma$ depends on the unknown Higgs mass

## Standard Model Higgs Properties



The Higgs boson then decays in one of several possible final states
The fraction of each decay mode also depends on the unknown Higgs mass

## Triggering



## 3D view/




From the many, many collision events, we impose some criteria to select $n$ candidate signal events. We hypothesize that it is composed of some number of signal and background events.

$$
\operatorname{Pois}(n \mid s+b)
$$

The number of events that we expect from a given interaction process is given as a product of

- $L$ : a time-integrated beam intensity (units $1 / \mathrm{cm}^{2}$ ) that serves as a measure of the amount of data that we have collected or the number of trials we have had to produce signal events
- $\sigma$ : "cross-section" (units $\mathrm{cm}^{2}$ ) a quantity that can be calculated from theory
- $\varepsilon$ : fraction of signal events selected by selection criteria

The selection efficiency and the theoretical cross-section have experimental and theoretical systematic uncertainties and we parametrize them with nuisance parameters $\alpha$

$$
s=L \epsilon(\alpha) \sigma(\alpha)
$$

In addition to the rate of interactions, our theories predict the distributions of angles, energies, masses, etc. of particles produced

- we form functions of these called discriminating variables $m$,
- and use Monte Carlo techniques to estimate f(m) In addition to the hypothesized Higgs signal process, there are known background processes.
- thus, the distribution of $f(m)$ is a mixture model
- the full model is a marked Poisson process

background process

$$
P(\mathbf{m} \mid s)=\operatorname{Pois}(n \mid s+b) \prod_{j}^{n} \frac{s f_{s}\left(m_{j}\right)+b f_{b}\left(m_{j}\right)}{s+b}
$$

Here is an example prediction from search for $\mathrm{H} \rightarrow \mathrm{ZZ}$ and $\mathrm{H} \rightarrow \mathrm{WW}$

- sometimes multivariate techniques are used



$$
P(\mathbf{m} \mid s)=\operatorname{Pois}(n \mid s+b) \prod_{j}^{n} \frac{s f_{s}\left(m_{j}\right)+b f_{b}\left(m_{j}\right)}{s+b}
$$

## Data-driven background determination

Regions in the data with negligible signal expected are used as control samples

- simulated events are used to estimate extrapolation coefficients
- extrapolation coefficients have large theoretical and experimental uncertainties


Figure 10: Flow chart describing the four data samples used in the $H \rightarrow W W^{(*)} \rightarrow \ell \boldsymbol{\nu} \ell \nu$ analysis. S.R and C.R. stand for signal and control regions, respectively.

## Data-driven background determination

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| S.R. |
| :---: |
| $H \rightarrow W W$ |
| $W W$ |
| Top |
| $W+$ jets |



Figure 10: Flow chart describing the four data samples used in the $H \rightarrow W W^{(*)} \rightarrow \ell \boldsymbol{\nu} \ell \nu$ analysis. S.R and C.R. stand for signal and control regions, respectively.

## Constraints on Nuisance Parameters

Many uncertainties have no clear statistical description or it is impractical to provide Traditionally, we use Gaussians, but for large uncertainties it is clearly a bad choice

- quickly falling tail, bad behavior near physical boundary, optimistic p-values, ...

For systematics constrained from control samples and dominated by statistical uncertainty, a Gamma distribution is a more natural choice [PDF is Poisson for the control sample]

- longer tail, good behavior near boundary, natural choice if auxiliary is based on counting For "factor of 2" notions of uncertainty log-normal is a good choice
- can have a very long tail for large uncertainties

None of them are as good as an actual model for the auxiliary measurement, if available

To consistently switch between frequentist, Bayesian, and hybrid procedures, need to be clear about prior vs. likelihood function

| PDF | Prior | Posterior |
| :--- | :--- | :--- |
| Gaussian | uniform | Gaussian |
| Poisson | uniform | Gamma |
| Log-normal | reference | Log-Normal |



Each experiment combines multiple searches for the Higgs to improve power

The Standard Model Higgs imposes relations between the different searches

- There is only one Higgs boson with unknown mass $m_{H}$
- giving rise to a mild form of the look-elsewhere effect (LEE)
- There are well defined branching ratios for a given value of $m_{H}$
- There is a common production cross-section $\sigma_{S M}$ for a given value of $m_{H}$
- though we often choose to consider $\mu=\sigma / \sigma$ см as a parameter assuming the branching ratios are given by Standard Model

In other theories, these relations are violated, exacerbating the LEE

The different searches also suffer from common systematic uncertainties from detector performance, luminosity uncertainty, etc.

- the ability to incorporating these correlations imposes some constraints in the strategy employed by the individual searches


## Combinations across experiments

The theory imposes the same relationships among searches performed by different experiments (eg. ATLAS and CMS)

- uncertainties associated with detector performance are uncorrelated

However, we use the same theoretical tools for predicting the rates and distributions associated with the signal and several backgrounds

- Even for data-driven approaches, we often rely on simulation for extrapolation
- requires coordination between experiments


There is a long history of combining Higgs searches across experiments

- At LEP collider, combining four experiments (around 1999)
- At Tevatron, combining two experiments

Combinations at the LHC pose new challenges -- toy exercise in 2010

- RooStats: a new tools to address these challenges [see talk by G. Schott, Wed.]


## The 2010 ATLAS+CMS Higgs Combination Exercise

The exercise was based on "toy" data and models, though realistic in complexity

- An intense effort between in June 2010, toy results shown July 6
- Initial meetings were mainly focused on
- aligning language, philosophy, strategy, and priorities.
- discussion practical and technical issues

Early on we decided the initial combination would be based on $\mathrm{H} \rightarrow \mathrm{WW}+0 \mathrm{j}$ and that the analyses would be number counting in a few channels

- attempt to provide inputs in a technology neutral way as well as a RooStats workspace format
- early discussions on form of constraint terms (Gaussian, gamma, lognormal)
- later discussions on methods, test statistics, etc.

Took ~1 month to prepare and validate inputs

- Four days from the time the inputs were shared to final results!
- Very impressive and encouraging exercise... but still an exercise.


## Tables, Formulae, and Workspaces

The ATLAS input:

- Poisson terms 3 signal regions and 6 control regions
- Uncertainties in extrapolation coefficients treated with truncated Gaussians and individual systematics on extrapolation coefficients were summed in quadrature
- thus, unable to identify any correlated systematic (eg. theory uncertainty)
- after discussions, decided to use this approach for initial exercise, but the need to evolve parametrization for real combination was recognized.



## Tables, Formulae, and Workspaces

The CMS input:

- cleanly tabulated effect on each background due to each source of systematic
- systematics broken down into uncorrelated subsets
- used lognormal distributions for all systematics, Poissons for observations

Started with a txt input, defined a mathematical representation, and then prepared the RooStats workspace


## Visualization of the ATLAS+CMS Workspace

The full model has
12 observables and 50 parameters


At this point, no correlated systematics across experiments
parameter of interest

$$
\mu=\frac{\sigma B R}{\sigma_{S M} B R_{S M}}
$$

RooStats supports several statistical methods used in high energy physics

- Common test statistics
- simple likelihood ratio (LEP)

$$
Q_{L E P}=L_{s+b}(\mu=1) / L_{b}(\mu=0)
$$

- ratio of profiled likelihoods (Tevatron) $Q_{T E V}=L_{s+b}(\mu=1, \hat{\hat{\nu}}) / L_{b}\left(\mu=0, \hat{\nu}^{\prime}\right)$
- profile likelihood ratio (LHC)

$$
\lambda(\mu)=L_{s+b}(\mu, \hat{\hat{\nu}}) / L_{s+b}(\hat{\mu}, \hat{\nu})
$$

- Sampling strategies
- toy MC randomizing nuisance parameters according to $\pi(\nu)$
- a Bayes-frequentist hybrid (prior-predictive)
- toy MC with nuisance parameters fixed (Neyman Construction)
- assuming asymptotic distribution (Wilks and Wald)
- Bayesian (different priors for the parameter of interest)

During the next four days, we tried to obtain results with as many of these methods as possible

Despite the complexity, we were able to go from inputs to results in 4 days!

- not only did we get results for the combination, we did it with six techniques
- a testament to the power and flexibility of the workspace technology and the RooFit/RooStats tools

The results were based upon loosely representative toy models. The CMS results were more powerful, as they were using multivariate analyses and systematic uncertainties are not so extreme.

Hybrid test statistics distributions



|  | test statistics | significance <br> (no syst.) | significance <br> (with syst.) |
| :---: | :---: | :---: | :---: |
| ATLAS | $\mathrm{Q}_{\text {LEP }}$ | 3.78 | $3.07 \pm 0.01$ |
|  | $\mathrm{Q}_{\text {TEV }}$ | - | $2.8 \pm 0.1$ |
|  | $\lambda(\mu)$ |  | - |
| CMS | $\mathrm{Q}_{\text {LEP }}$ | $6.22 \pm 0.02$ | $4.77 \pm 0.02$ |
|  | $\mathrm{Q}_{\text {TEV }}$ | - | $>4.6$ |
|  | $\lambda(\mu)$ | - | $4.3 \pm 0.1$ |
| COMBI | $\mathrm{Q}_{\text {LEP }}$ | - | $>4.6$ |
|  | $\mathrm{Q}_{\text {TEV }}$ | - | $>3.5$ |
|  | $\lambda(\mu)$ | - | - |

$\mathbf{G b \%} \mathbf{C L}$ upper IImıts: results with systematics (except it indicated otherwise)

- computing the p -value for significance in this approach is challenging:
- speed improvements would be useful
- or use importance sampling techniques
- CMS distribution (and results previous slide) made with a RooFit-independent tool
Grégory Schott - ATLAS-CMS statistics meeting - 01.07.2010

| technique | test stat | rule | sampling | ULATLAS | UL CMS | UL COMBI |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Feldman-Cousins (no <br> syst.) | $\lambda(\mu)$ | $\mathrm{CL}_{\mathrm{S}+\mathrm{B}}$ | toys | $0.69 \pm 0.05$ | - | - |
| Profile LR (Wilks) | $\lambda(\mu)$ | $\mathrm{CL}_{\mathrm{S}+\mathrm{B}}$ | asymptotic | 0.79 | 0.28 | 0.25 |
| Feldman-Cousins++ | $\lambda(\mu)$ | $\mathrm{CL}_{\mathrm{S}+\mathrm{B}}$ | toys | $0.78 \pm 0.05$ | $0.26 \pm 0.02$ | $0.23 \pm 0.02$ |
| Hybrid | $\mathrm{Q}_{\mathrm{LEP}}$ | $\mathrm{CL}_{\mathrm{s}}$ | toys | $\sim 0.68$ | $0.29 \pm 0.03$ <br> $($ LandS $)$ | - |
| Hybrid | $\mathrm{Q}_{\text {LEP }}$ | $\mathrm{CL}_{\mathrm{S}+\mathrm{B}}$ | toys | -0.61 | - | - |
| Bayesian | $\mathrm{n} / \mathrm{a}$, flat prior on r | MCMC* | 0.72 | 0.31 | 0.28 |  |

Kyle Cranmer (NYU)
PhyStat 2011, January, 18, 2011

## Some lessons learned

In general, this combination has been a great success

- in our first meeting we were already discussing correlated systematics between ATLAS and CMS

We need to identify each of the backgrounds estimated from theory, because

- they are affected by luminosity uncertainty
- their theoretical uncertainties are correlated between experiments
- separate production modes: the qg, qQ, and gg parts uncertainties in the parton density functions affect different processes in a different way, lumping them all together may be missing some essential physics.

We need to separate and individually parametrize the effect of individual systematics

- the ability to correlate across experiments (and for different channels within the same experiment) requires the ability to relate parameters in the model in a consistent way
- consistent procedures are needed for assessing effect of common systematics

Attempt to directly incorporate model for control samples when feasible

- superior to approximating by Gaussian, Gamma, etc. (though often not feasible)

Since our toy exercise in July, ATLAS and CMS have formed an official LHC Higgs Combination Group

- kick-off meeting was in December
- first working meeting was last week
- focusing on validation of RooStats [link]

The goal for the group is to show a combined ATLAS+CMS Higgs combination this summer -- with real data!

Good luck to the LHC-HCG in 2011!

## Backup

We know that the $\mathrm{W}, \mathrm{Z}$ bosons are massive, but explicit mass terms for the W,Z break the electroweak gauge symmetry.

- massless W,Z only have transverse polarizations
Hogs mechanism:
- Add $\phi_{1}$ a new [complex doublet of] Scalar field [s] with specific potential and $V(\phi)$ interactions with $\mathrm{W}, \mathrm{Z}$
- generates masses for W,Z

$$
V(\phi)=\mu^{2}|\phi|^{2}+\lambda|\phi|^{4}
$$

Goldstone modes (become longitudinal polarizations of massive $\mathrm{W}, \mathrm{Z}$ )


Interactions with fermions:


## ATLAS CMS Open Statistics Forum

Tuesday 06 July 2010 from 16:30 to 18:30 (Europe/Zurich) at CERN ( 40-S2-B01 - Salle Bohr )

Description The first results from the Toy combination Exercise will be presented

Tuesday 06 July 2010

| 16:30-16:50 | Historical Introduction $20^{\prime}$ <br> Speaker: Robert Cousins (UCLA) <br> Material: Slides |
| :--- | :--- |
|  | Combination Strategy 20' <br> Speaker: Kyle Cranmer (NYU) |
|  | Material: Slides |


| 17:30-17:50 | Inputs for the Toy Combination Exercise $20^{\prime}$ |
| :--- | :--- |
|  | The $H \rightarrow W$ Winputs used for the combination. |
| Speaker: William Quayle (Wisconsin) |  |
|  | Material: Slides |

18:00-18:20 Toy Exercise Conclusions 20'
Speaker: Grégory Schott (Universităt Karlsruhe)
Material: Slides 틔 园 给
http://indico.cern.ch/conferenceDisplay.py?confld=100458

The ATLAS+CMS statistics committees are looking into a different way to avoid setting limits where we have no sensitivity (instead of $\mathrm{CL}_{\mathrm{s}}$ )

- idea: don't quote limit below some threshold defined by an N- $\sigma$ downward fluctuation of b-only pseudo-experiments


Let's consider a simplified problem that has been studied quite a bit to gain some insight into our more realistic and difficult problems

- number counting with background uncertainty
- in our main measurement we observe $n_{o n}$ with $s+b$ expected

$$
\operatorname{Pois}\left(n_{\mathrm{on}} \mid s+b\right)
$$

- and the background has some uncertainty
- but what is "background uncertainty"? Where did it come from?
- maybe we would say background is known to $10 \%$ or that it has some pdf $\pi(b)$
- then we often do a smearing of the background:

$$
P\left(n_{\mathrm{on}} \mid s\right)=\int d b \operatorname{Pois}\left(n_{\mathrm{on}} \mid s+b\right) \pi(b),
$$

-Where does $\pi(b)$ come from?

- did you realize that this is a Bayesian procedure that depends on some prior assumption about what $b$ is?


## The "on/off" problem

Now let's say that the background was estimated from some control region or sideband measurement.

- We can treat these two measurements simultaneously:
- main measurement: observe $n_{o n}$ with $s+b$ expected
- sideband measurement: observe $n_{\text {off }}$ with $\tau b$ expected

$$
\underbrace{P\left(n_{\text {on }}, n_{\text {off }} \mid s, b\right)}_{\text {joint model }}=\underbrace{\operatorname{Pois}\left(n_{\text {on }} \mid s+b\right)}_{\text {main measurement }} \underbrace{\operatorname{Pois}\left(n_{\text {off }} \mid \tau b\right)}_{\text {sideband }}
$$

- In this approach "background uncertainty" is a statistical error
- justification and accounting of background uncertainty is much more clear How does this relate to the smearing approach?

$$
P\left(n_{\mathrm{on}} \mid s\right)=\int d b \operatorname{Pois}\left(n_{\mathrm{on}} \mid s+b\right) \pi(b)
$$

- while $\pi(b)$ is based on data, it still depends on a prior $\eta(b)$

$$
\pi(b)=P\left(b \mid n_{\text {off }}\right)=\frac{P\left(n_{\text {off }} \mid b\right) \eta(b)}{\int d b P\left(n_{\text {off }} \mid b\right) \eta(b)} .
$$

Recommendation: where possible, one should express uncertainty on a parameter as a statistical (random) process

- explicitly include terms that represent auxiliary measurements in the likelihood

Recommendation: when using a Bayesian technique, one should explicitly express and separate the prior from the objective part of the probability density function

## Example:

- By writing $P\left(n_{\text {on }}, n_{\text {off }} \mid s, b\right)=\operatorname{Pois}\left(n_{\text {on }} \mid s+b\right) \operatorname{Pois}\left(n_{\text {off }} \mid \tau b\right)$.
- the objective statistical model is for the background uncertainty is clear
- One can then explicitly express a prior $\eta(b)$ and obtain:

$$
\pi(b)=P\left(b \mid n_{\mathrm{off}}\right)=\frac{P\left(n_{\mathrm{off}} \mid b\right) \eta(b)}{\int d b P\left(n_{\mathrm{off}} \mid b\right) \eta(b)} .
$$

Goal of Bayesian-frequentist hybrid solutions is to provide a frequentist treatment of the main measurement, while eliminating nuisance parameters (deal with systematics) with an intuitive Bayesian technique.

$$
P\left(n_{\text {on }} \mid s\right)=\int d b \operatorname{Pois}\left(n_{\text {on }} \mid s+b\right) \pi(b), \quad \quad p=\sum_{n=n_{\text {obs }}}^{\infty} P(n \mid s)
$$

Principled version (eg. $\mathrm{Z}_{\ulcorner }$):

- clearly state prior $\eta(b)$; identify control samples (sidebands) and use:

$$
\pi(b)=P\left(b \mid n_{\text {off }}\right)=\frac{P\left(n_{\text {off }} \mid b\right) \eta(b)}{\int d b P\left(n_{\text {off }} \mid b\right) \eta(b)} .
$$

Ad-hoc version (eg. $\mathbf{Z}_{\mathrm{N}}$ ):

- unable or unwilling to justify $\pi(b)$, so go straight to some distribution
- eg. a Gaussian, truncated Gaussian, log normal, Gamma, etc...
- often the case for real systematic uncertainty (eg. MC generators, different background estimation techniques, etc.)

Recommendation: Avoid ad hoc priors if possible.

Now on a real PROOF cluster with 30 machines

- real world example throws millions of toys experiments, does full fit on 50 parameters for each toy.
- also supports producing simple shells scripts for use with GRID or batch queues

Now importance sampling is also implemented,

- following presentation at Banff with particle physics \& statistics experts
- allows for 1000x speed increase!
- Still being tested in detail



