Classical Statistics

- What is the speed distribution of the molecules of an ideal gas at temperature T?
  - Maxwell speed distribution

$$F(v)dv = C' e^{-\frac{\beta m v^2}{2}} v^2 dv$$

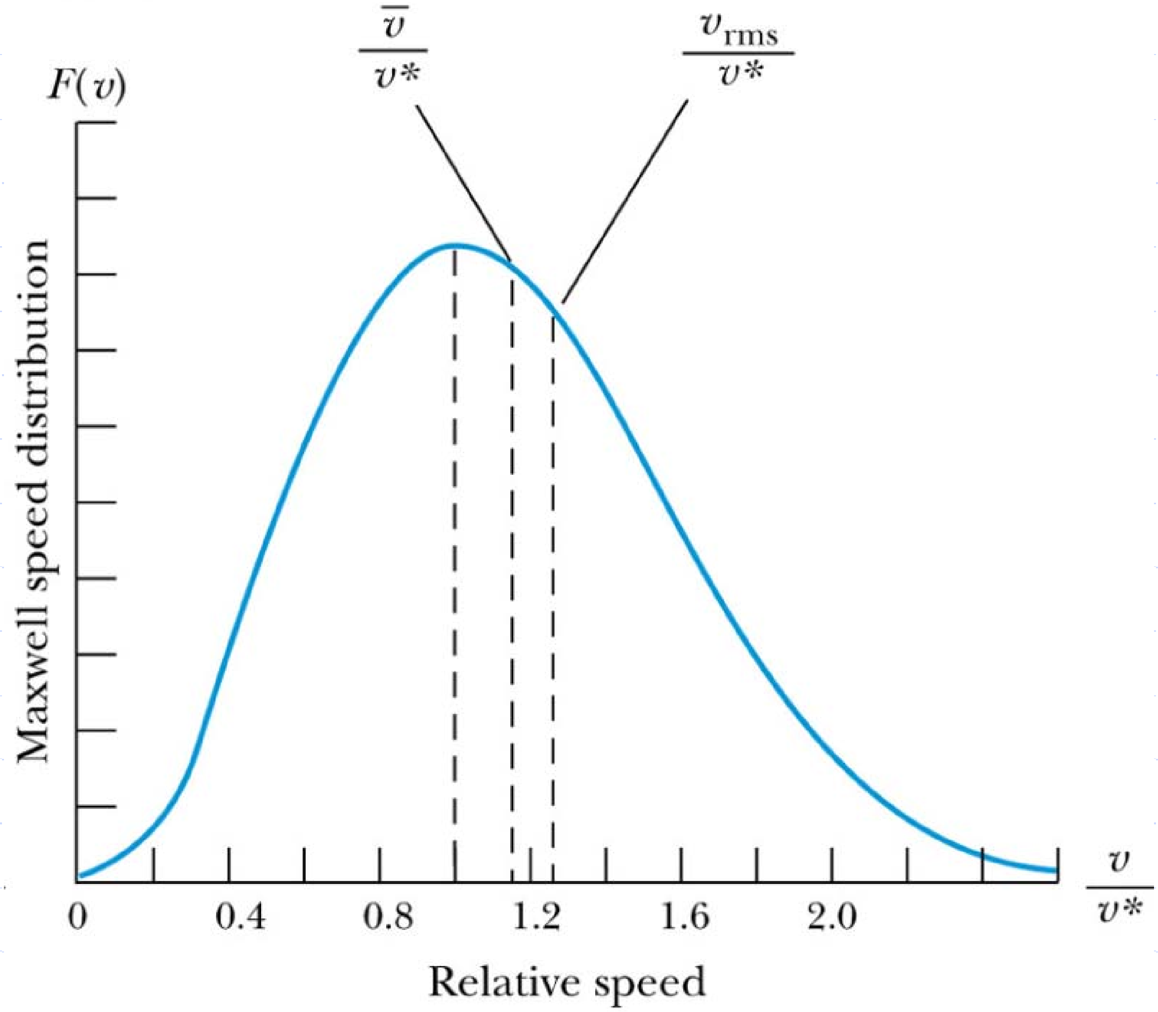
$$\text{where } \beta = \frac{1}{kT} \text{ and } C' = 4\pi \left(\frac{\beta m}{2\pi}\right)^{3/2}$$

- This is the probabilitity of finding a particle with speed between v and v +dv
  - And if we cared to calculate it we would find

$$\overline{v^2} = \int_0^\infty v^2 F(v)dv = \frac{3kT}{m}$$

Classical Statistics

- Maxwell speed distribution



© 2006 Brooks/Cole - Thomson

Classical Statistics

- In quantum mechanics we are generally interested in the energy
- Maxwell's velocity distribution can be converted into a statement about energy

$$\text{Using } E = \frac{mv^2}{2}$$

$$dE = mv\,dv$$

$$dv = \frac{dE}{mv} = \frac{dE}{\sqrt{2mE}}$$

$$\text{then } F(v)\,dv = F(E)\,dE$$

$$\text{where } F(E) = C'' e^{-\beta E} E^{1/2}$$

Classical Statistics

- The factor $F_{MB} = Ae^{-\beta E}$ is called the Maxwell-Boltzmann factor
  - F_{MB} gives the probability that a state of energy E is occupied at temperature T
    - The $E^{1/2}$ factor is not universal but rather specific to the molecular speed problem
- We need one more element called the density of states g(E)
  - g(E) is the number of states available per unit energy interval

$g(E)dE$ is the number of energy states in the range from E to $E + dE$

Classical Statistics

- We have then

  $n(E) = g(E)F_{MB}$ is the number of particles with energy E per unit energy

- This is useful because we can then calculate thermodynamic properties of interest

$$n = \int_0^\infty F_{MB}\, g(E)\, dE$$

$$U = \int_0^\infty E F_{MB}\, g(E)\, dE$$

$$C_V = \frac{\partial U}{\partial T}$$

$$P = -\frac{\partial U}{\partial V} \text{ at } T = 0$$

Quantum Statistics

- We would like to apply these same ideas using quantum mechanics
- To give some idea of how quantum mechanics will change things consider the following example
  - Let a gas be made of only two particles A and B
  - Let there be three quantum states s=1,2,3

Quantum Statistics

- Maxwell-Boltzmann case
  - The two particles are distinguishable and two particles can occupy the same state
  - There are 9 distinct states shown on the right

1	2	3
AB		
	AB	
		AB
A	B	
B	A	
	A	B
	B	A
A		B
B		A

Quantum Statistics

- Bose-Einstein case
  - The particles are identical bosons
  - Any number of particles can occupy the same state
  - Let B=A
  - There are 6 distinct states shown on the right

1	2	3
AA		
	AA	
		AA
A	A	
	A	A
A		A

Quantum Statistics

- Fermi-Dirac case
  - The particles are identical fermions
  - No two particles can occupy the same quantum state (Pauli exclusion principle)
  - There are 3 distinct states

1	2	3
A	A	
	A	A
A		A

Quantum Statistics

- The number of particles with energy E for the three cases is given by

$$n(E) = g(E)F \text{ where}$$

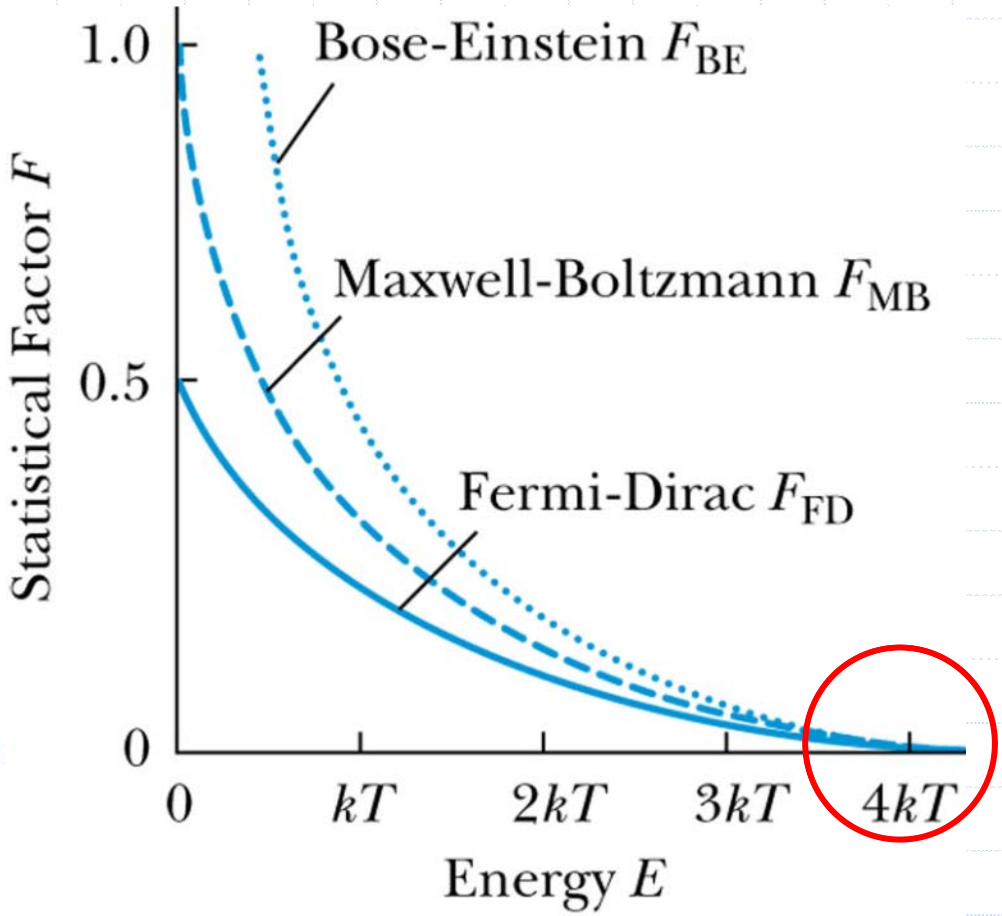
$$F_{MB} = Ae^{-\beta E} \text{ for distinguishable particles (classical)}$$

$$F_{BE} = \frac{1}{B_2 e^{\beta E} - 1} \text{ for identical bosons}$$

$$F_{FD} = \frac{1}{B_1 e^{\beta E} + 1} \text{ for identical fermions}$$

Quantum Statistics

- The three probability distributions with $A = B_1 = B_2 = 1$



© 2006 Brooks/Cole - Thomson

Fermi-Dirac Statistics

- This is the case for electrons so let's study it a bit more
- The constant B_1 can be written

$$B_1 = e^{-\beta E_F}$$

$$\text{then } F_{FD} = \frac{1}{e^{\beta(E-E_F)}+1}$$

E_F is called the Fermi energy

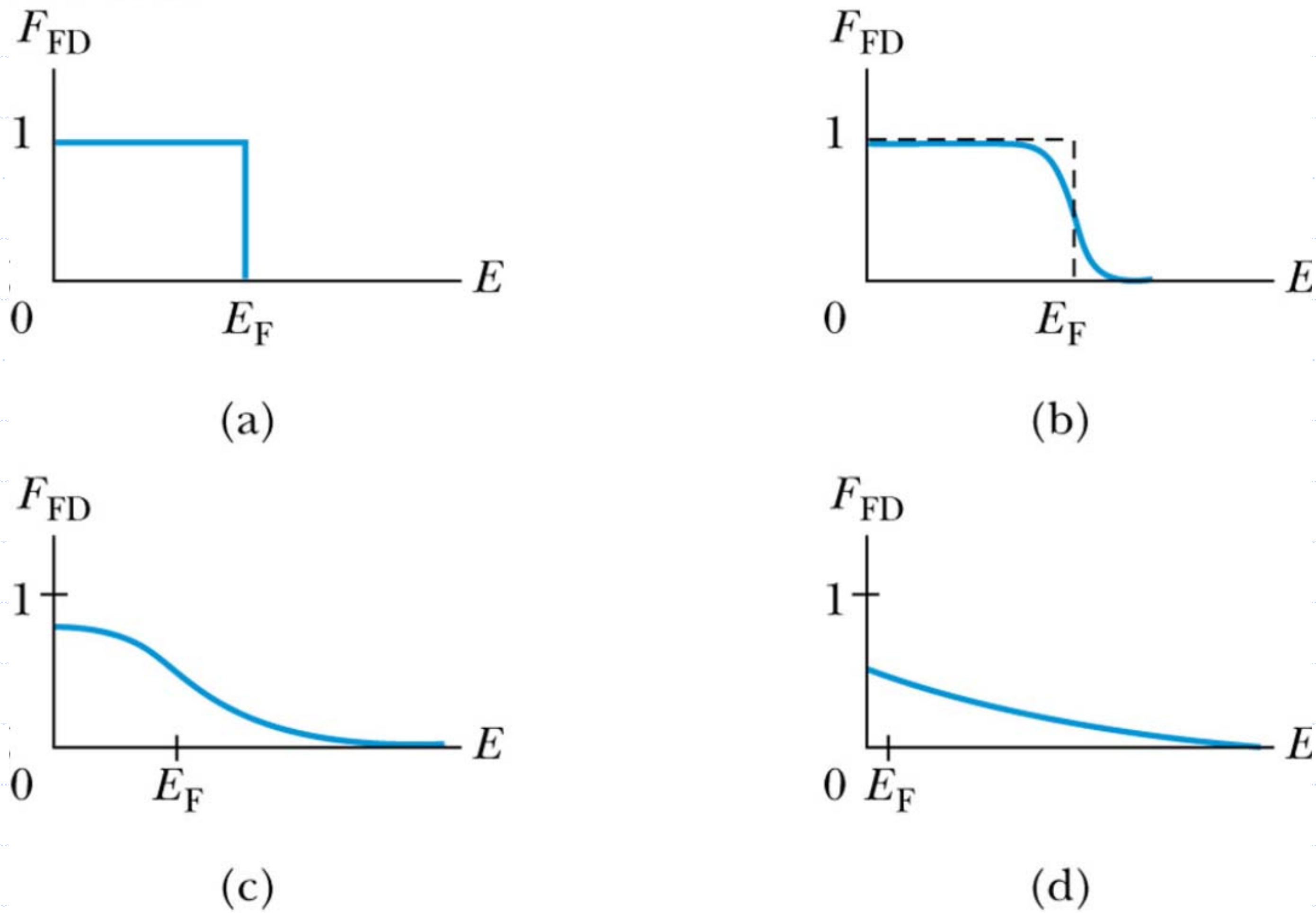
- You will hear a lot about the Fermi energy in solid state physics

Fermi-Dirac Statistics

- Comments
  - When $E=E_F$, $F_{FD} = ½$
    - The Fermi energy E_F is the energy of the highest occupied energy level at T=0
  - As $T \rightarrow 0$, $F_{FD}=1$ for $E<E_F$ and $F_{FD}=0$ for $E>E_F$
    - At/near T=0, the fermions will occupy the lowest energy states available (consistent with the Pauli exclusion principle)
    - As T increases, the can occupy higher energy states

Fermi-Dirac Statistics

- Define $k_B T_F = E_F$



© 2006 Brooks/Cole - Thomson