Spin

- Many quantum experiments are done with photon polarization instead of electron spin
- Here is the correspondence between the two

Electron	Photon
$\vert 1\rangle_z$	H
$\vert 0\rangle_z$	V
$\vert 1\rangle_x$	$+$ D
$\vert 0\rangle_x$	- D
SG	Polarizer

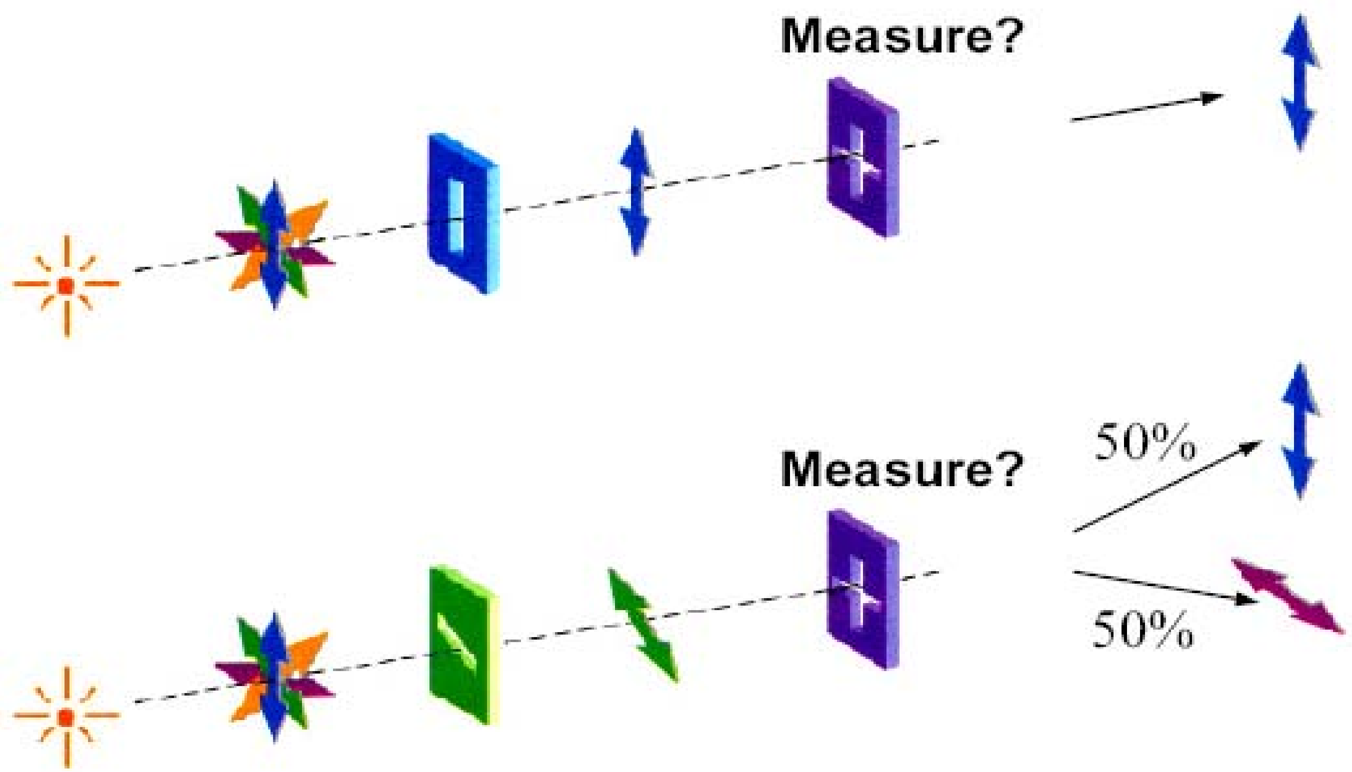
- And the measurement probabilities are the same

Spin

- From our study of spin and Stern-Gerlach experiments we know we must choose a direction (basis) to measure S_z for electrons or linear polarization for photons

Electron	SG	Photon	Filter	Probability of detection
$\|1\rangle_z$	$+z$	H	H	1
$\|1\rangle_z$	$-z$	H	V	0
$\|1\rangle_z$	$+x$	H	$+D$	50%
$\|1\rangle_z$	$-x$	H	$-D$	50%

Spin or Polarization



Quantum Cryptography

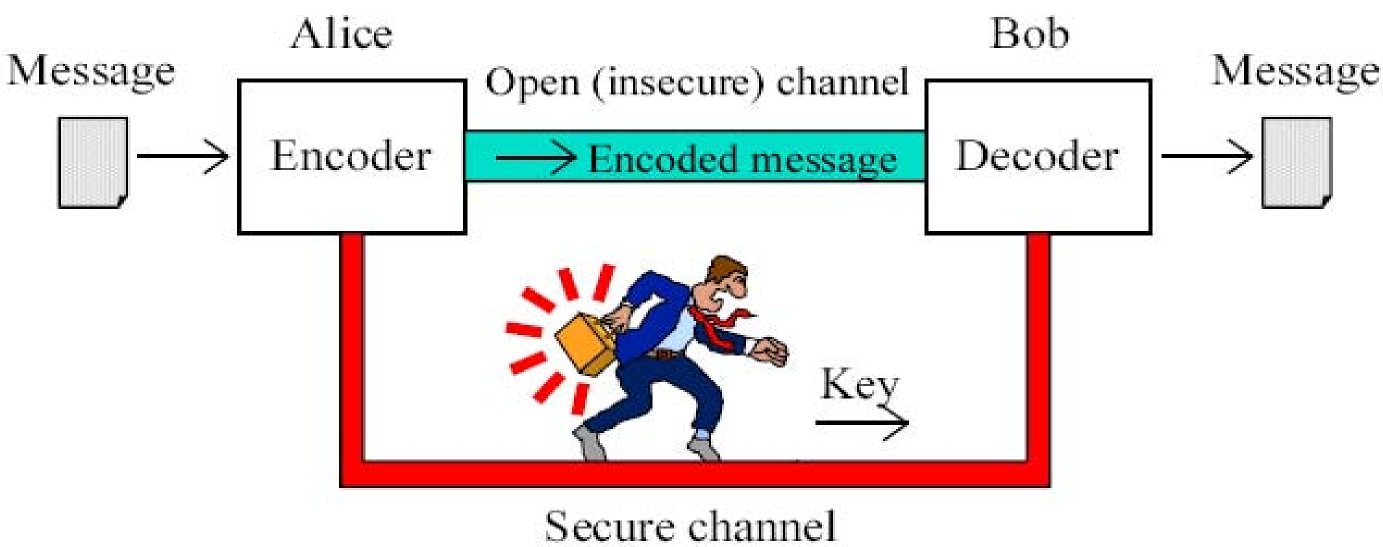
- How does the left side of the room pass a note to the right side of the room without me learning the contents of the message?
- The only mathematically proven way to transmit a message is to use a one-time pad
  - This requires a key the same length as the message and can be used only once

Quantum Cryptography

- Note, the figures on the next few slides were take from a talk on quantum cryptography by Vadim Makarov from NTNU in Norway

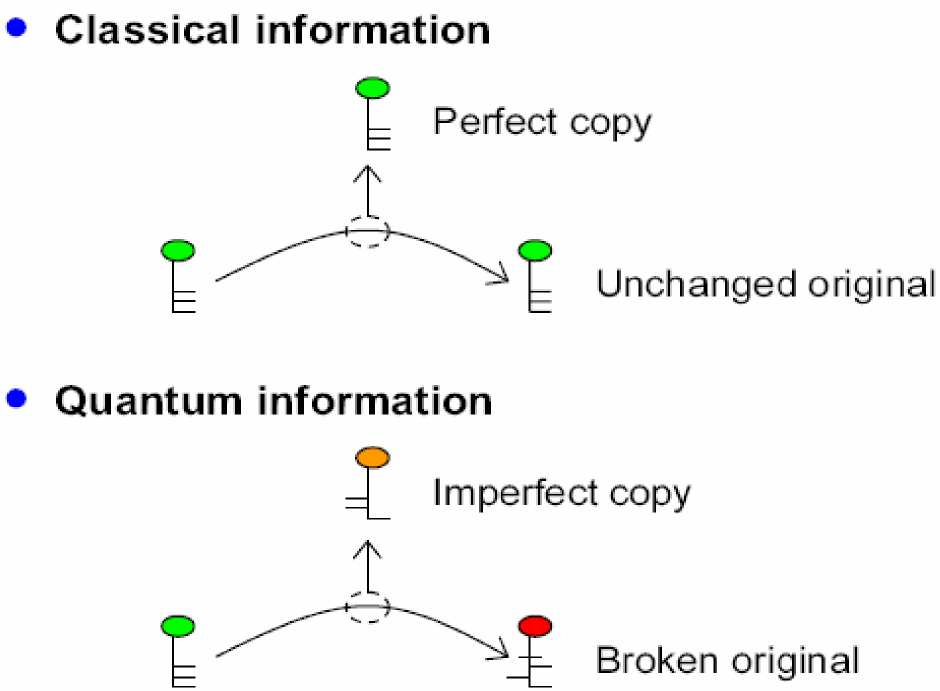
Quantum Cryptography

- Classical key cryptography requires a secure channel for key distribution
  - Vulnerable, authentication not assured, evesdropping
- Quantum cryptography can distribute the key using an open channel

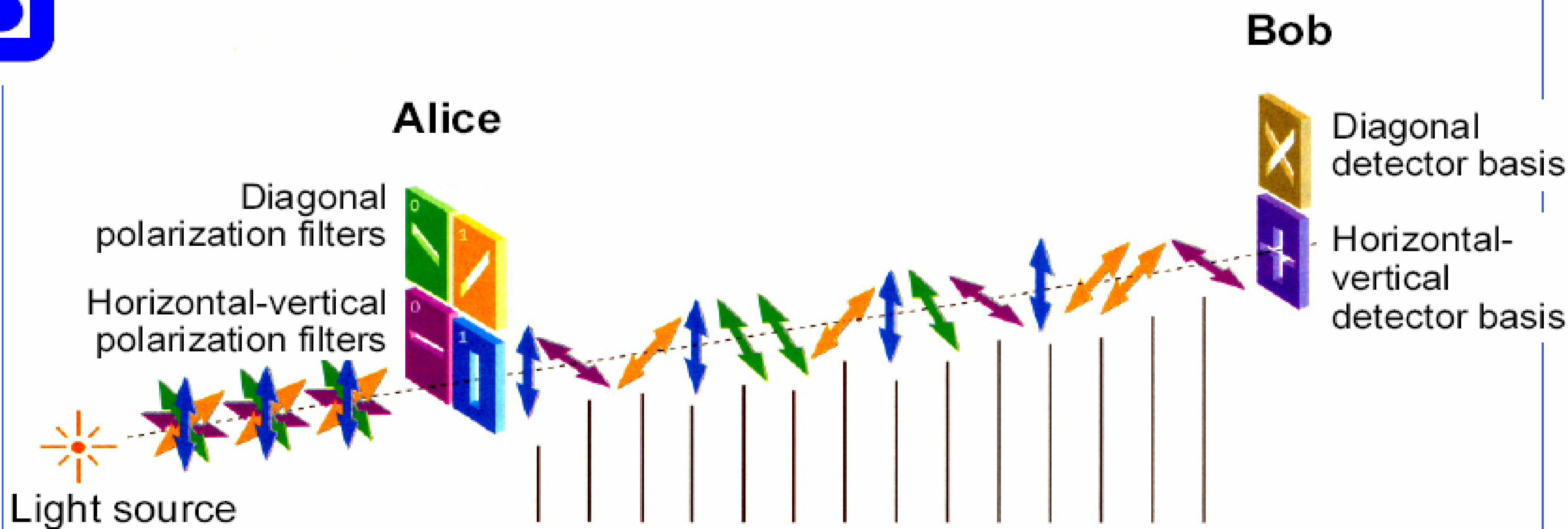


Quantum Cyptography

- In addition, a message transmitted classically can be passively monitored
- A message transmitted quantum mechanically



Quantum Cryptography



Alice's bit sequence	1	0	1	1	0	0	1	1	0	0	1	1	1	0
Bob's detection basis	+	×	+	+	×	×	+	+	×	+	×	×	+	+
Bob's measurement	1	0	0	1	0	0	1	1	0	0	0	1	0	0
Retained bit sequence	1	–	–	1	0	0	–	1	0	0	–	1	–	0

Quantum Cryptography

- Alice generates a random key
- Alice generates a random set of analyzers
- Alice sends the results to Bob
- Bob generates a random set of analyzers
- Alice and Bob publicly exchange what analyzers were used (sifting)
- Alice and Bob check whether randomly selected entries agree
- The remaining results are kept as a key

Quantum Cryptography

Vienna, 21. April 2004

World Premiere: Bank Transfer via Quantum Cryptography Based on Entangled Photons

Press conference and demonstration of the ground-breaking experiment:
21 April 2004, 11:30, Vienna City Hall – Steinsaal

A collaboration of:
group of Professor Anton Zeilinger, *Vienna University*; *ARC Seibersdorf research GmbH*; *City of Vienna*; *Wien Kanal Abwassertechnologien GmbH* and *Bank Austria – Creditanstalt*

Today, the *Bank Austria Creditanstalt* has, on behalf of the City of Vienna, performed the World's first bank transfer encoded via quantum cryptography.

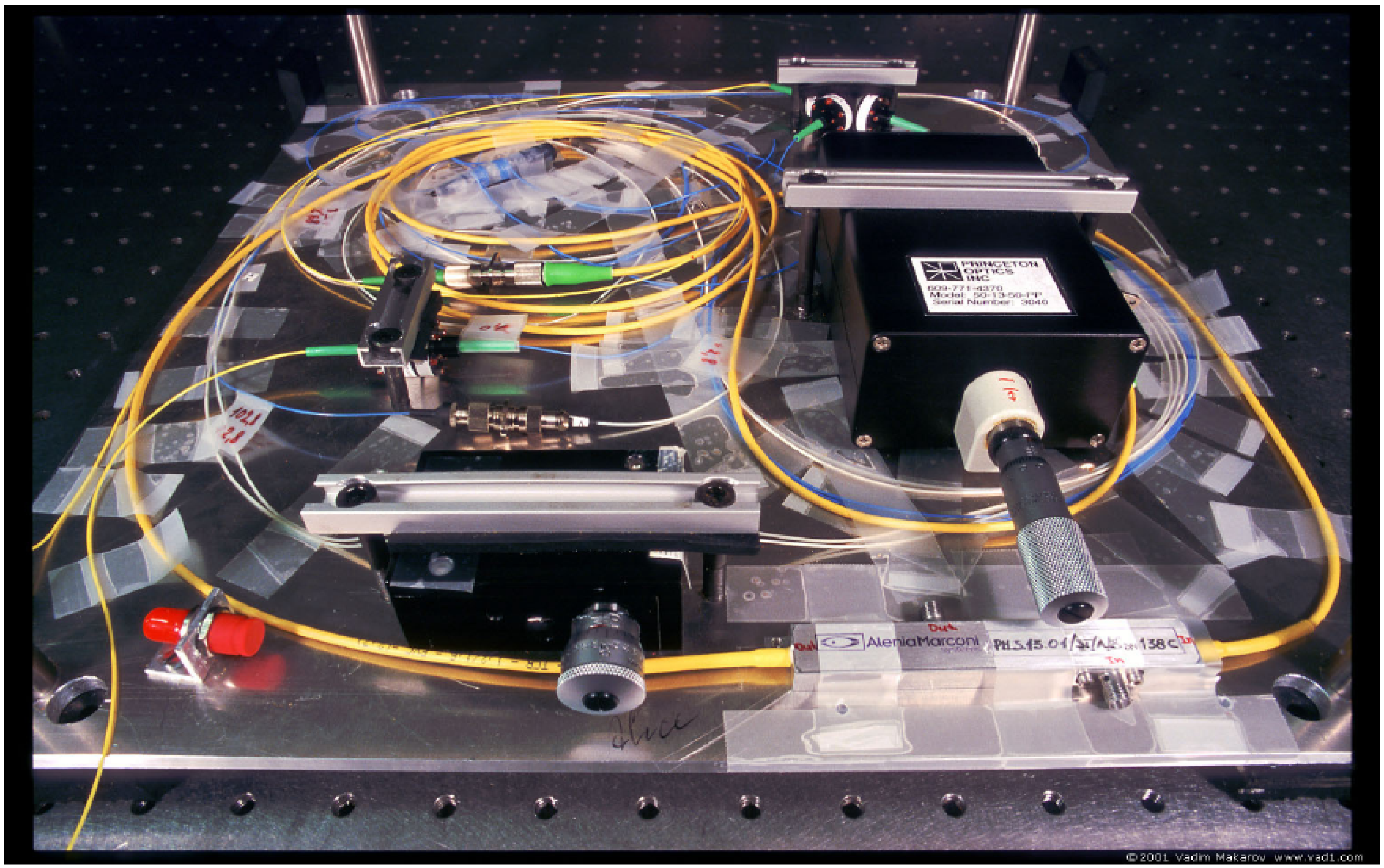
This novel technology was demonstrated by the group of Professor Anton Zeilinger, Vienna University in collaboration with the group *Quantum Technologies* (*Information Technologies Division*) of *Seibersdorf research*. The bank transfer was initiated by Vienna's Mayor Dr. Michael Häupl, and executed by the Director of the *Bank Austria Creditanstalt*, Dr. Erich Hampel. The information was sent via a glass fiber cable, laid by the company *Wien Kanal Abwassertechnologien* from the Vienna City Hall to the *Bank Austria Creditanstalt* branch office "Schottengasse".

Entangled photon pairs enable absolutely secure transfer of information

In quantum cryptography, a data key for encoding messages is created using quantum technologies. It provides solutions for two problems yet unsolved by today's commonly used classical cryptography systems: The **creation** and the **transfer** of absolutely random keys. On the one hand, the security of the produced keys is based on the laws of Nature – and not on the complicated mathematical procedures used by today's systems. On the other hand, quantum cryptography simplifies the distribution of the keys. Trustworthy human messengers who personally deliver a key, still the common carriers of information in cases of highly confidential transfer of information, are finally a thing of the past. The keys can now be produced simultaneously by transmitter and receiver – the transfer is made redundant.

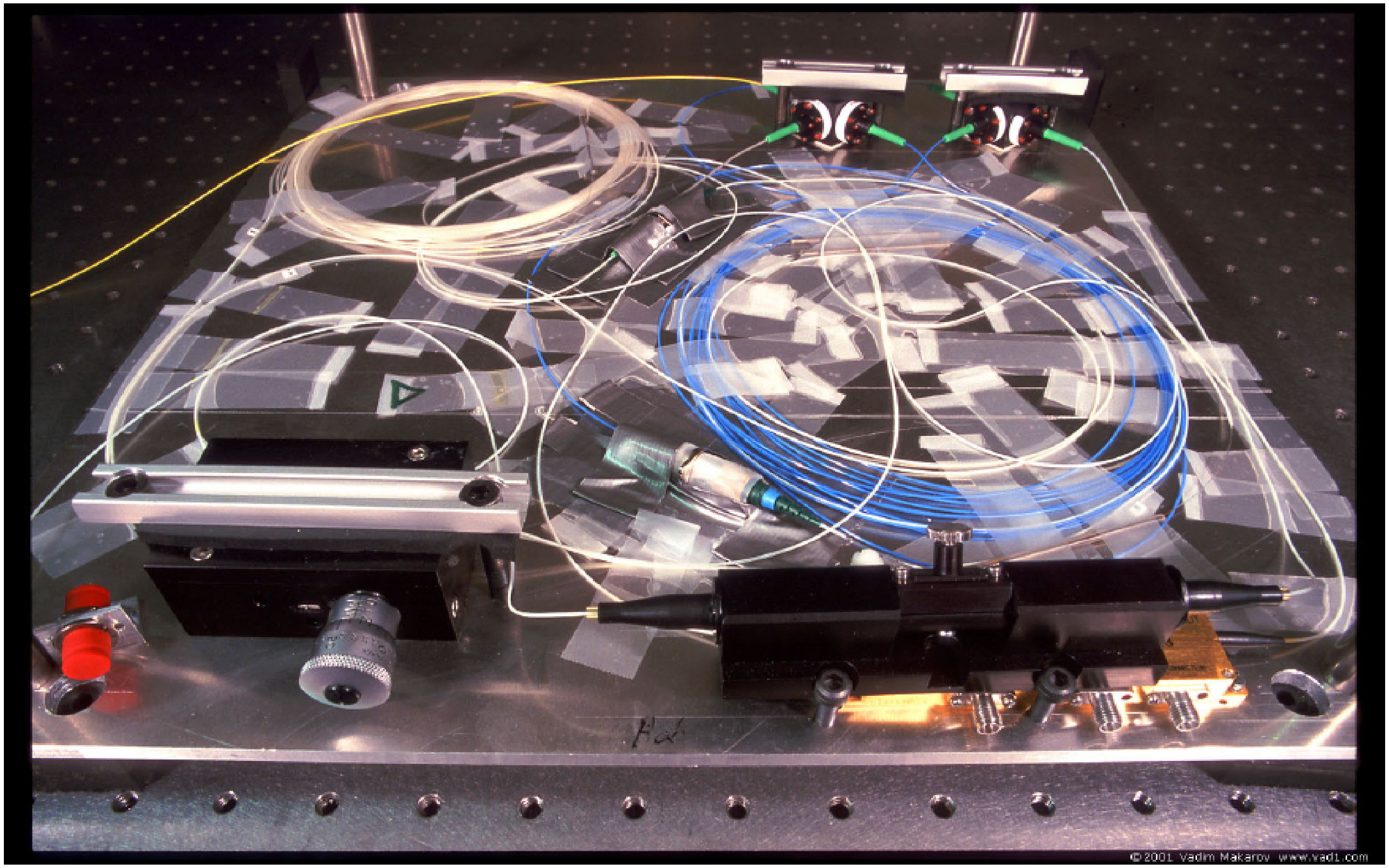
Quantum Cryptography

- Here is Alice



Quantum Cryptography

- Here is Bob



Qubit

- A Qubit (or Qbit) is a quantum system with exactly two degrees of freedom

a bit can be 1 or 0 $\left(|1\rangle \text{ or } |0\rangle\right)$

a qubit is superposition of states $|\psi\rangle = \alpha|1\rangle + \beta|0\rangle$

where $\alpha^2 + \beta^2 = 1$

and as we learned previously

$\mathrm{P}\left(|1\rangle\right) = \alpha^2$ and $\mathrm{P}\left(|0\rangle\right) = \beta^2$

- Examples
  - Electron with spin up and spin down
  - Photon with horizontal and vertical polarization

Entanglement

- Recall the EPR paradox

EINSTEIN-PODOLSKY-ROSEN PARADOX, (2)

If you measure the momentum **p**, then the momentum of the red is **-p**. Since the momentum of the red was measured without disturbing it, that quantity must be regarded as **real**.

If you measure the position q_1, then the position of the red is q_2. Since the position of the red was measured without disturbing it, that quantity must be regarded as **real**.

Since both the momentum and the position of the red can be known without disturbing the red itself, both quantities must be regarded as real. BUT THE QUANTUM MECHANICS IMPLIES THAT THE TWO CANNOT BE REAL AT THE SAME TIME. This shows that something is wrong with the quantum mechanics.

Entanglement

- Describes quantum states that have to be referenced to each other even though they are spatially separated
- Say I can generate electron (or photon) pairs such that they always have opposite spins

$$\psi = \frac{1}{\sqrt{2}}\left(\left|\uparrow,\downarrow\right\rangle - \left|\downarrow,\uparrow\right\rangle\right)$$

  - What is the probability of A measuring up?
  - What is the probability of A measuring down?
  - What is the probability of B measuring up when A measures down?
- We say the two spins are entangled

Quantum Teleportation

- Quantum teleportation transfers a quantum state to an arbitrarily far location using a distributed entangled state and the transmission of classical information
- The state at A is physically not transferred but an identical copy appears at B

Quantum Teleportation

- Recall that spin can be measured with different SG analyzers (bases)
- We'll define

$$|\uparrow\rangle_z = |\uparrow\rangle \text{ and } |\downarrow\rangle_z = |\downarrow\rangle$$

$$|\uparrow\rangle_x = |\rightarrow\rangle \text{ and } |\downarrow\rangle_x = |\leftarrow\rangle$$

$$\text{and } |\uparrow\rangle_z = \frac{1}{\sqrt{2}}\left(|\rightarrow\rangle + |\leftarrow\rangle\right)$$

$$\text{and } |\downarrow\rangle_z = \frac{1}{\sqrt{2}}\left(|\rightarrow\rangle - |\leftarrow\rangle\right)$$

Quantum Teleportation

- Let's define some operations
- These don't destroy the quantum state

X is a rotation about the x-axis

$$X\left(a|\uparrow\rangle + b|\downarrow\rangle\right) = a|\downarrow\rangle + b|\uparrow\rangle$$

Z is a rotation about the z-axis

$$Z\left(a|\uparrow\rangle + b|\downarrow\rangle\right) = a|\uparrow\rangle - b|\downarrow\rangle$$

$C-X$ flips the second spin if the first is down

$C-X$ keeps the second spin if the first is up

$$C-X\left(a|\uparrow,\uparrow\rangle + b|\uparrow,\downarrow\rangle + c|\downarrow,\uparrow\rangle + d|\downarrow,\downarrow\rangle\right) =$$
$$a|\uparrow,\uparrow\rangle + b|\uparrow,\downarrow\rangle + c|\downarrow,\downarrow\rangle + d|\downarrow,\uparrow\rangle$$

Quantum Teleportation

- We want to transfer an arbitrary state from Lab A to Lab B

$$a\left|\uparrow\right\rangle + b\left|\downarrow\right\rangle$$

- We also start with an entangled state

$$\frac{1}{\sqrt{2}}\left(\left|\uparrow,\downarrow\right\rangle - \left|\downarrow,\uparrow\right\rangle\right)$$

- Then the overall state is

$$\frac{a}{\sqrt{2}}\left|\uparrow,\uparrow,\downarrow\right\rangle - \frac{b}{\sqrt{2}}\left|\downarrow,\uparrow,\downarrow\right\rangle - \frac{a}{\sqrt{2}}\left|\uparrow,\downarrow,\uparrow\right\rangle + \frac{b}{\sqrt{2}}\left|\downarrow,\downarrow,\uparrow\right\rangle$$

Quantum Teleportation

- Spin 1 is the unknown spin to be transported
- Spin 2 is the first spin of the entangled pair and is located Lab A
- Spin 3 is the second spin of the entangled pair and is located in Lab B

Quantum Teleportation

- Apply the C-X operation to spins 1 and 2 in Lab A

$$\frac{a}{\sqrt{2}}\left|\uparrow,\uparrow,\downarrow\right\rangle - \frac{b}{\sqrt{2}}\left|\downarrow,\uparrow,\downarrow\right\rangle - \frac{a}{\sqrt{2}}\left|\uparrow,\downarrow,\uparrow\right\rangle + \frac{b}{\sqrt{2}}\left|\downarrow,\downarrow,\uparrow\right\rangle$$

becomes

$$\frac{a}{\sqrt{2}}\left|\uparrow,\uparrow,\downarrow\right\rangle - \frac{b}{\sqrt{2}}\left|\downarrow,\downarrow,\downarrow\right\rangle - \frac{a}{\sqrt{2}}\left|\uparrow,\downarrow,\uparrow\right\rangle + \frac{b}{\sqrt{2}}\left|\downarrow,\uparrow,\uparrow\right\rangle$$

Quantum Teleportation

- Measure spin 2 in the z basis

$$\frac{a}{\sqrt{2}}\left|\uparrow,\uparrow,\downarrow\right\rangle - \frac{b}{\sqrt{2}}\left|\downarrow,\downarrow,\downarrow\right\rangle - \frac{a}{\sqrt{2}}\left|\uparrow,\downarrow,\uparrow\right\rangle + \frac{b}{\sqrt{2}}\left|\downarrow,\uparrow,\uparrow\right\rangle$$

- The probability of getting spin up is

$$\left|\frac{a}{\sqrt{2}}\right|^2 + \left|\frac{b}{\sqrt{2}}\right|^2 = \frac{|a|^2 + |b|^2}{2} = \frac{1}{2}$$

  - And also 1/2 for getting spin down

- Let's say we measure up, then the new state will be

$$a\left|\uparrow,\uparrow,\downarrow\right\rangle + b\left|\downarrow,\uparrow,\uparrow\right\rangle$$

Quantum Teleportation

- Measure spin 1 in the x basis

first write $a|\uparrow,\uparrow,\downarrow\rangle + b|\downarrow,\uparrow,\uparrow\rangle$ in the x basis

$$\frac{a}{\sqrt{2}}|\rightarrow,\uparrow,\downarrow\rangle + \frac{a}{\sqrt{2}}|\leftarrow,\uparrow,\downarrow\rangle + \frac{b}{\sqrt{2}}|\rightarrow,\uparrow,\uparrow\rangle + \frac{b}{\sqrt{2}}|\leftarrow,\uparrow,\uparrow\rangle$$

- Then the probability of measuring spin right is

$$\left|\frac{a}{\sqrt{2}}\right|^2 + \left|\frac{b}{\sqrt{2}}\right|^2 = \frac{|a|^2 + |b|^2}{2} = \frac{1}{2}$$

  - And 1/2 for getting spin left

- Let's say we measure right, then the new state will be

$$a|\rightarrow,\uparrow,\downarrow\rangle + b|\rightarrow,\uparrow,\uparrow\rangle$$

Quantum Teleportation

- Now call Lab B and tell them the results
- Lab B responds by

$$\text{If } \downarrow \text{ for spin 2 and} \rightarrow \text{ for spin 1: do nothing}$$

$$\text{If } \downarrow \text{ for spin 2 and} \leftarrow \text{ for spin 1: apply Z to spin 3}$$

$$\text{If } \uparrow \text{ for spin 2 and} \rightarrow \text{ for spin 1: apply X to spin 3}$$

$$\text{If } \uparrow \text{ for spin 2 and} \leftarrow \text{ for spin 1: apply Z then X to spin 3}$$

- Our example is case 3

$$a\left|\rightarrow,\uparrow,\downarrow\right\rangle + b\left|\rightarrow,\uparrow,\uparrow\right\rangle \text{ becomes}$$

$$a\left|\rightarrow,\uparrow,\uparrow\right\rangle + b\left|\rightarrow,\uparrow,\downarrow\right\rangle$$

Quantum Teleportation

- Thus the state of spin 3 in Lab B is

$$a\left|\rightarrow,\uparrow,\uparrow\right\rangle + b\left|\rightarrow,\uparrow,\downarrow\right\rangle$$

$$a\left|\uparrow\right\rangle + b\left|\downarrow\right\rangle$$

- Exactly the state we were trying to transmit

Quantum Teleportation

brief communications

Quantum teleportation across the Danube

A real-world experiment marks a step towards worldwide quantum communication.

Efficient long-distance quantum teleportation[1] is crucial for quantum communication and quantum networking schemes[2]. Here we describe the high-fidelity teleportation of photons over a distance of 600 metres across the River Danube in Vienna, with the optimal efficiency that can be achieved using linear optics. Our result is a step towards the implementation of a quantum repeater[3], which will enable pure entanglement to be shared between distant parties in a public environment and eventually on a worldwide scale.

Quantum teleportation is based on a quantum channel, here established through a pair of polarization-entangled photons shared between Alice and Bob (Fig. 1). We have implemented this by using an 800-metre-long optical fibre installed in a public sewer system located in a tunnel underneath the River Danube, where it is exposed to temperature fluctuations and other environmental factors.

For Alice to be able to transfer the unknown polarization state of an input photon $|\chi\rangle_b$, she has to perform a joint Bell-state measurement on the input photon *b* and her member, *c*, of the shared entangled photon pair (*c* and *d*). Our scheme allows her to identify two of the four Bell states, the optimum achievable with only linear optics[4,5].

As a result of this Bell-state measurement, Bob's 'receiver' photon *d* will be projected into a well defined state that already contains full information on the original input photon *b*, except for a rotation that depends on the specific Bell state that Alice observed. Our teleportation scheme therefore also includes active feed-forward of Alice's measurement results, which is achieved by means of a classical microwave channel together with a fast electro-optical modulator (EOM). It enables Bob to perform the unitary transformation on photon *d* to obtain an exact replica of Alice's input photon *b*.

Specifically, if Alice observes the $|\psi^-\rangle_{bc}$ Bell state, which is the same as the initial entangled state of photons *c* and *d*, then Bob already possesses the original input state. But if Alice observes the $|\psi^+\rangle_{bc}$ state, he introduces a π-phase shift between the horizontal and vertical polarization components of photon *d* by applying a voltage pulse of 3.7 kV on the EOM. For successful operation, Bob has to set the EOM correctly before photon *d* arrives. Because of the reduced velocity of light within the fibre-based quantum channel (two-thirds of that *in vacuo*), the classical signal arrives about 1.5 microseconds before the photon.

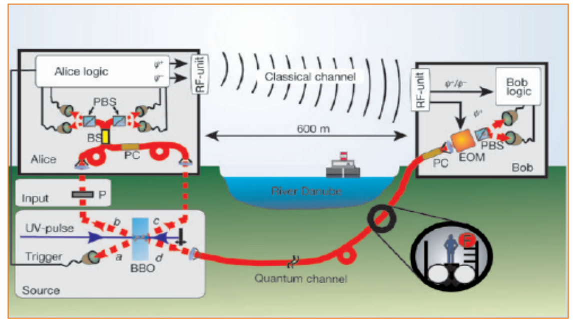


Figure 1 Long-distance quantum teleportation across the River Danube. The quantum channel (fibre F) rests in a sewage-pipe tunnel below the river in Vienna, while the classical microwave channel passes above it. A pulsed laser (wavelength, 394 nm; rate, 76 MHz) is used to pump a β-barium borate (BBO) crystal that generates the entangled photon pair *c* and *d* and photons *a* and *b* (wavelength, 788 nm) by spontaneous parametric down-conversion. The state of photon *b* after passage through polarizer P is the teleportation input; *a* serves as the trigger. Photons *b* and *c* are guided into a single-mode optical-fibre beam splitter (BS) connected to polarizing beam splitters (PBS) for Bell-state measurement. Polarization rotation in the fibres is corrected by polarization controllers (PC) before each run of measurements. The logic electronics identify the Bell state as either $|\psi^-\rangle_{bc}$ or $|\psi^+\rangle_{bc}$ and convey the result through the microwave channel (RF unit) to Bob's electro-optic modulator (EOM) to transform photon *d* into the input state of photon *b*.

We demonstrated the teleportation of three distinct polarization states: linear at 45°, left-handed circular or horizontal. The teleportation fidelity achieved was 0.84, 0.86 or 0.90 for the 45°, for each of these input states, respectively. These fidelities comfortably surpass the classical limit of 0.66 (ref. 6) and prove that our teleportation system is operating correctly. Without operation of the EOM, however, Bob observes a completely mixed polarization for the 45° and circular polarization input states, causing the observed fidelity for these states to drop to 0.54 and 0.59, respectively, in the absence of active unitary transformation. The deviation from the random fidelity of 0.5 is due to statistical fluctuations in the observed counts.

Each measurement run lasted for 28 h and the rate of successful teleportation events was 0.04 per second. Polarization stability proved to be better than 10° on the fibre between Alice's and Bob's labs, corresponding to an ideal teleportation fidelity of 0.97 over a full measurement run. Hence, despite the exposure of our system to the environment, high-fidelity teleportation was still achievable without permanent readjustments.

We have demonstrated quantum teleportation over a long distance and with high fidelity under real-world conditions outside a laboratory. Our system combines for the first time, to our knowledge, an improved Bell-state analyser with active unitary transformation, enabling a doubling of the efficiency of teleportation compared with earlier experiments based on independent photons[7,8]. Our experiment demonstrates feed-forward of measurement results, which will be essential for linear-optics quantum computing[9–11], and constitutes a step towards the full-scale implementation of a quantum repeater.

Rupert Ursin*, Thomas Jennewein*†, Markus Aspelmeyer*, Rainer Kaltenbaek*, Michael Lindenthal*, Philip Walther*, Anton Zeilinger*†

**Institute for Experimental Physics, University of Vienna, 1090 Vienna, Austria*
e-mail: rupert.ursin@univie.ac.at
†Austrian Academy of Science, 1090 Vienna, Austria

1. Bennett, C. H. *et al. Phys. Rev. Lett.* 70, 1895–1899 (1993).
2. Bose, S., Vedral, V. & Knight, P. L. *Phys. Rev. A* 57, 822–829 (1998).
3. Briegel, H. J., Dür, W., Cirac, J. I. & Zoller, P. *Phys Rev. Lett.* 81, 5932–5935 (1998).
4. Lütkenhaus, N., Calsamiglia, J. & Suominen, K.-A. *Phys. Rev. A* 59, 3295–3300 (1999).
5. Kim, Y.-H. *et al. Phys. Rev. Lett.* 86, 1370–1373 (2001).
6. Popescu, S. *Phys. Rev. Lett.* 72, 797–799 (1994).
7. Bouwmeester, D. *et al. Nature* 390, 575–579 (1997).
8. Marcikic, I., de Riedmatten, H., Tittel, W., Zbinden, H. & Gisin, N. *Nature* 421, 509–513 (2003).
9. Riebe, M. *et al. Nature* 429, 734–737 (2004).
10. Barrett, M. D. *et al. Nature* 429, 737–739 (2004).
11. Gottesmann, D. & Chuang, I. L. *Nature* 402, 390–393 (1999).

Competing financial interests: declared none.