More on Magnetic Moments

- Recall when a magnetic moment is placed in an external magnetic moment it will experience a torque (like a compass)

$$\vec{\tau} = \vec{\mu}_l \times \vec{B}$$

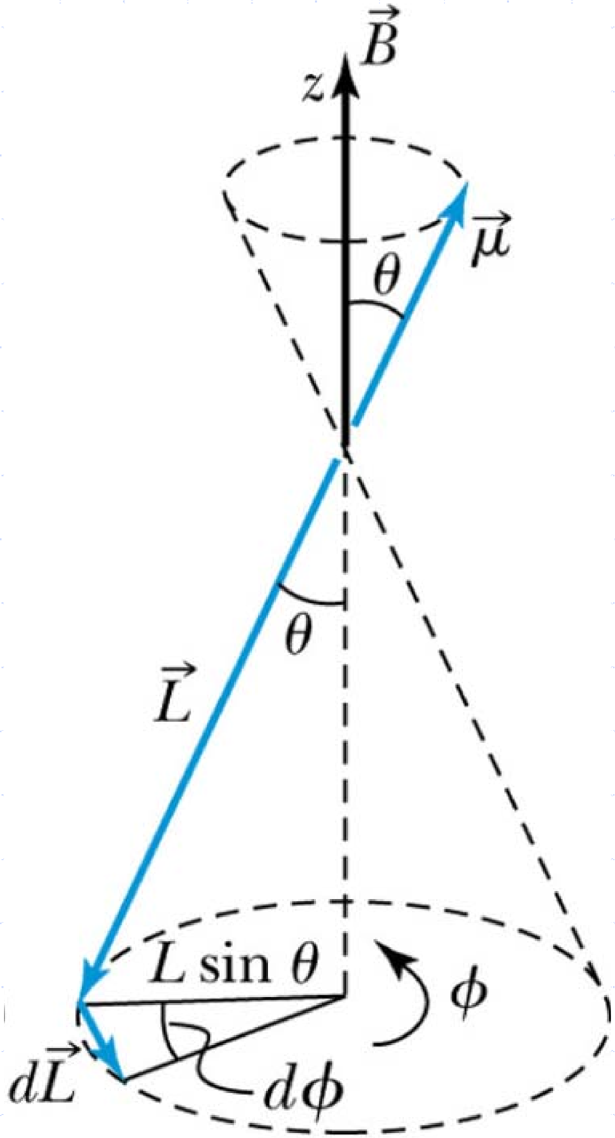
- If the orientational potential energy cannot be dissipated, the magnetic moment will precess about the magnetic field (like a spinning top)
- This is because

The $\vec{\tau} = \vec{\mu}_l \times \vec{B}$ direction will be in the x - y plane

and perpendicular to $\vec{L}$. Since $\vec{\tau} = \dfrac{d\vec{L}}{dt}$ the change

$d\vec{L}$ will also be perpendicular to $\vec{L}$. Thus $\vec{L}$ will precess.

More on Magnetic Moments



© 2006 Brooks/Cole - Thomson

More on Magnetic Moments

- This is called Larmor precession and we can use the figure to calculate ω_L

$$\omega_L = \frac{d\varphi}{dt} = \frac{1}{L\sin\theta}\frac{dL}{dt} = -\frac{e}{2m\mu_l\sin\theta}\frac{dL}{dt}$$

$$\tau = \frac{dL}{dt} = \mu_l B\sin\theta$$

$$\omega_L = \frac{eB}{2m} \text{ is the Larmor precession frequency}$$

More on Magnetic Moments

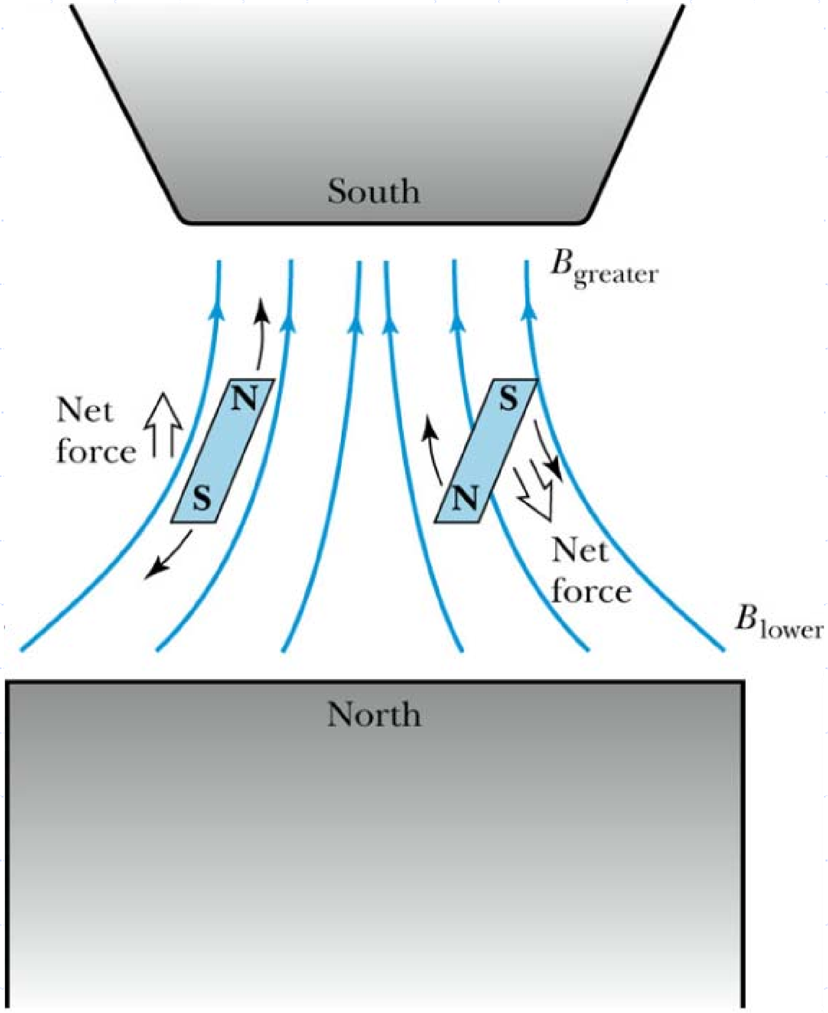
- If the magnetic field is uniform there will be no net force on the magnetic moment (but there will be a torque)
- If the magnetic field is nonuniform there will be a net force on the dipole

$$F = -\frac{\partial}{\partial z}\left(-\vec{\mu}_l \cdot \vec{B}\right) = \frac{\partial B_z}{\partial z}\mu_{l_z} = -\frac{\partial B_z}{\partial z} g_l \mu_B m_l$$

where we used $\vec{\mu} = -\frac{g_l \mu_B}{\hbar}\vec{L}$

More on Magnetic Moments

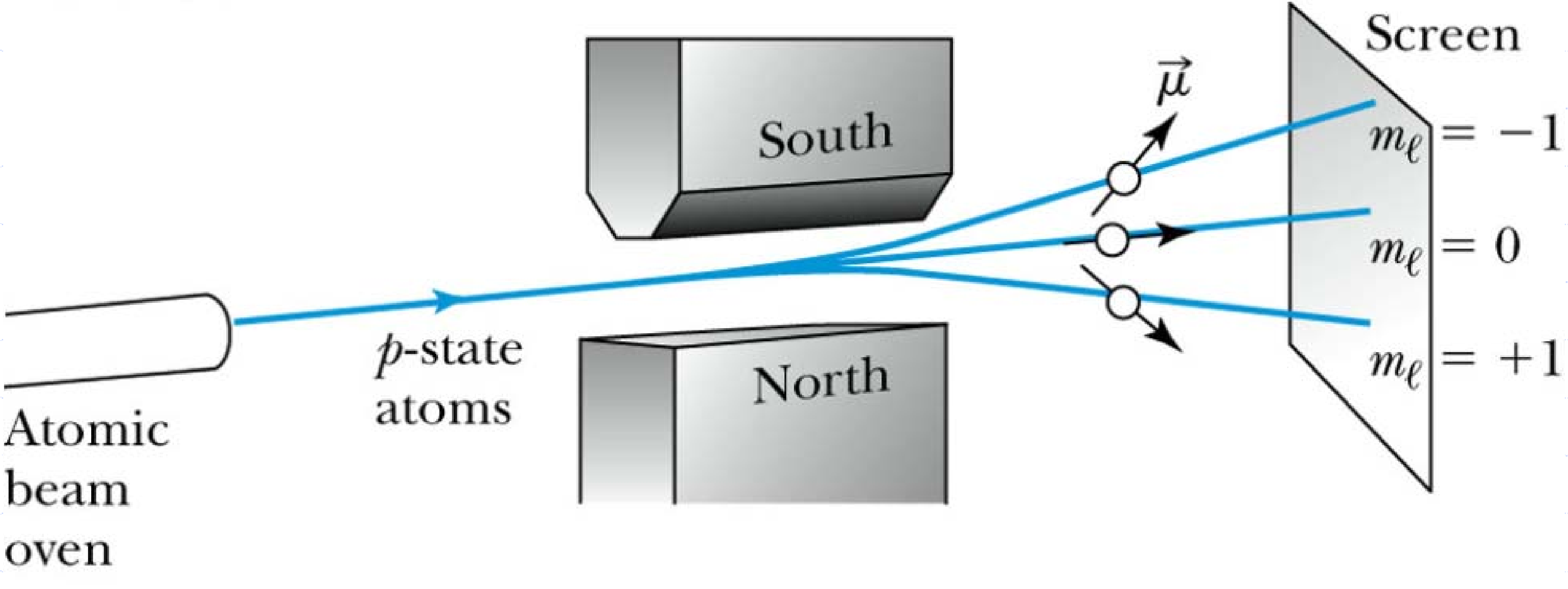
- Force on magnetic moments in a nonuniform B field



© 2006 Brooks/Cole - Thomson

Stern-Gerlach Experiment

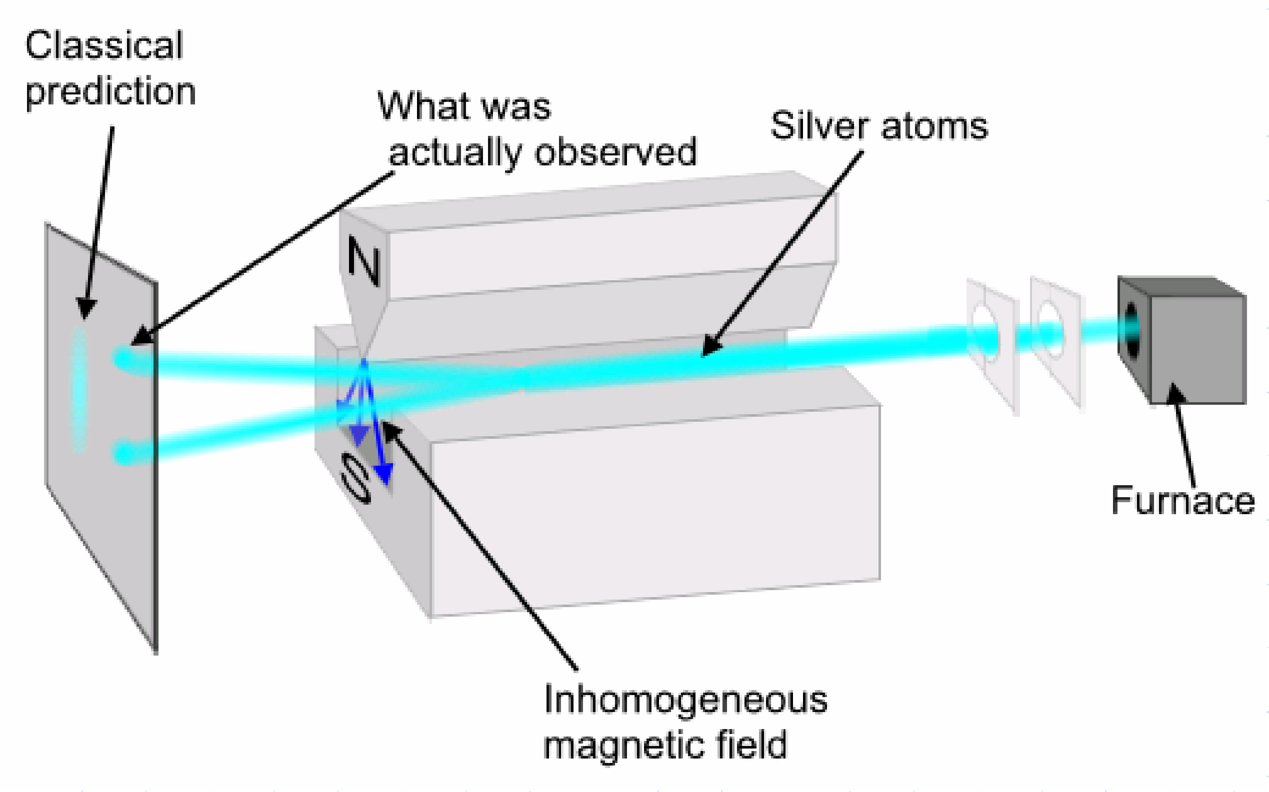
- What would be expected classically?



© 2006 Brooks/Cole - Thomson

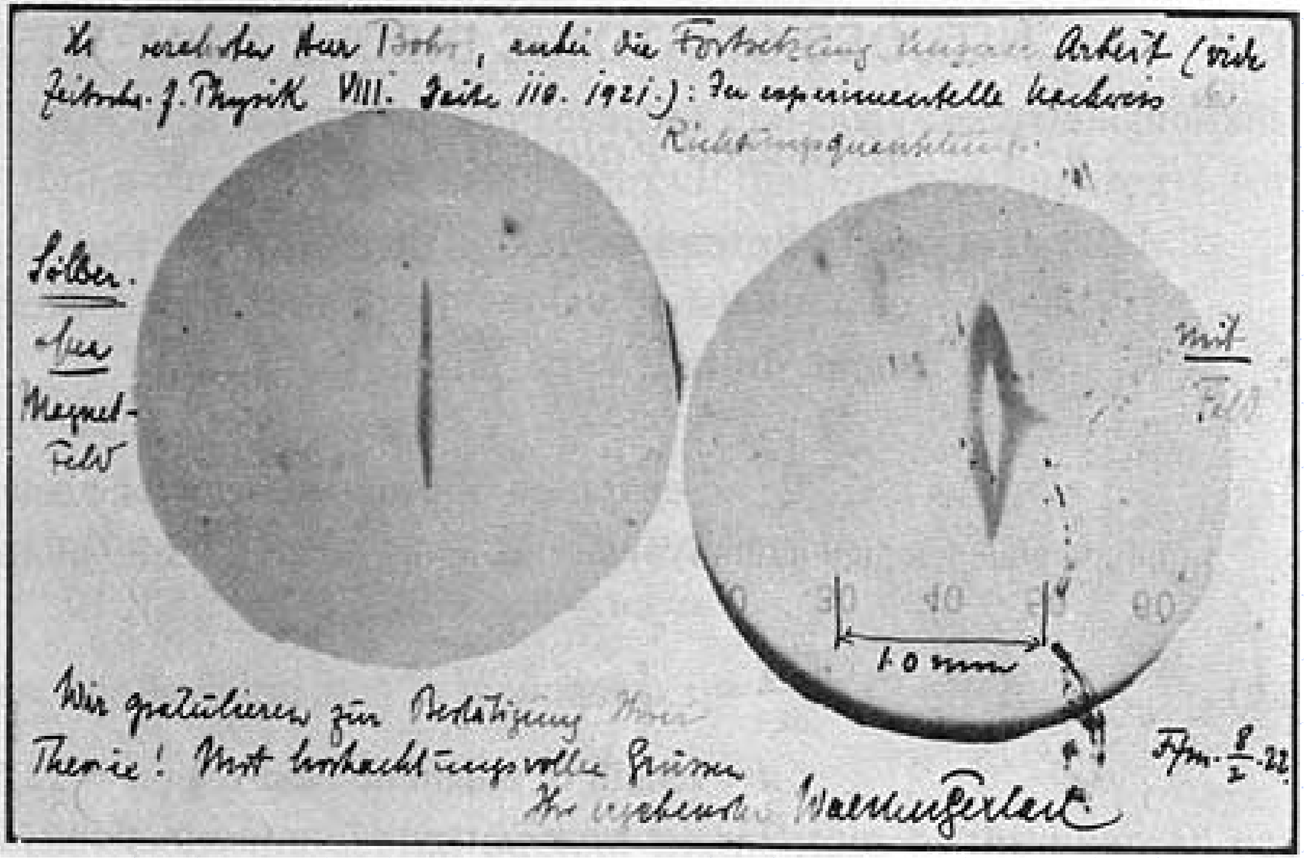
Stern-Gerlach Experiment

- Stern-Gerlach's experiment with silver atoms resulted in two distinct lines, not three



Stern-Gerlach Experiment

- Postcard sent to Bohr from Gerlach



Hochverehrter Herr Bohr, anbei die Fortsetzung unserer Arbeit (vide Zeitschr. f. Physik VIII. Seite 110. 1921): Der experimentelle Nachweis der Richtungsquantelung.

Silber.
Ohne Magnet-Feld

mit Feld

1.0 mm

Wir gratulieren zur Bestätigung Ihrer Theorie! Mit hochachtungsvollen Grüssen
Ihr ergebener Walther Gerlach

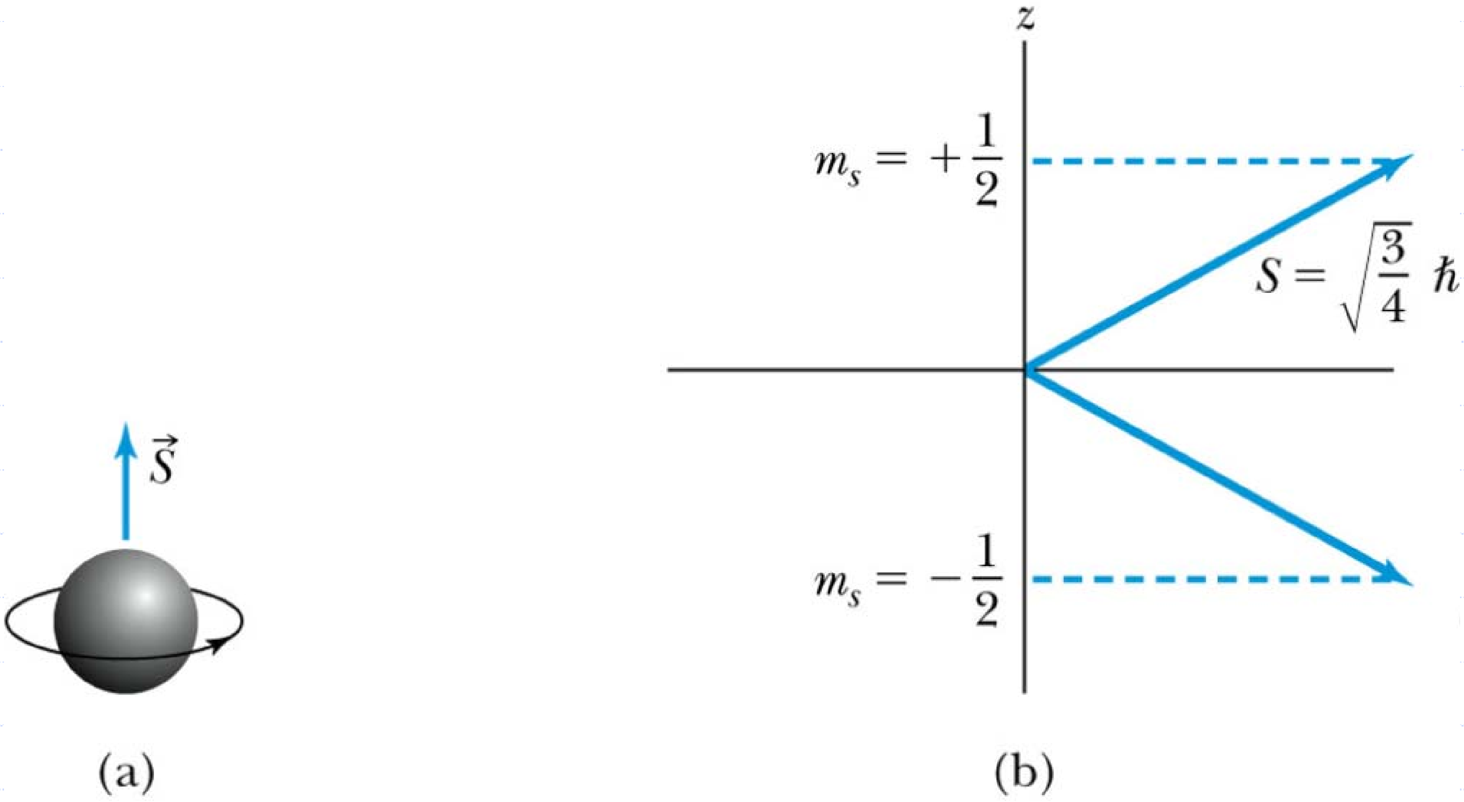
Ffm. 8/2. 22

Spin

- Phipps and Taylor repeated the Stern-Gerlach experiment using hydrogen and found the same result
- To explain these results Goudsmit and Uhlenbeck (grad students) proposed that in addition to orbital angular momentum, electrons had intrinsic angular momentum, or spin
  - In spite of its name, there is no classical analog for spin (even though we draw electrons like spinning tops)

Spin

- Spin of electrons = 1/2



© 2006 Brooks/Cole - Thomson

Spin

- The great thing about spin from our standpoint is that the spin angular momentum algebra is identical to that of orbital angular momentum

$$s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$$

$$m_s = -s, \ldots, s$$

$$S^2 \chi_{s,m_s} = s(s+1)\hbar^2 \chi_{s,m_s}$$

$$S_z \chi_{s,m_s} = m_s \hbar \chi_{s,m_s}$$

$$S^2 = S_x^2 + S_y^2 + S_z^2$$

$$\left[S_x, S_y\right] = i\hbar S_z, \left[S_y, S_z\right] = i\hbar S_x, \left[S_z, S_x\right] = i\hbar S_y$$

$$\left[S^2, S_x\right] = \left[S^2, S_y\right] = \left[S^2, S_z\right] = 0$$

Spin

- Spin is an intrinsic property of a particle or atom
- Different particles and atoms have different spins
  - Pions, Higgs bosons have spin 0
  - Electrons, positrons, muons, protons, neutrons, quarks have spin 1/2
  - Photons and W and Z-bosons have spin 1
  - Delta particles have spin 3/2
  - Gravitons (may have) spin 2

Spin

- This also means our complete hydrogen wave function must include a spin wave function

$$\Psi(x,t)=\psi(x)e^{-\frac{iEt}{\hbar}}\chi$$

- And we'll need to specify s, m_s as additional quantum numbers

Spin

- For the most part we will focus on electrons
  - Spin s = 1/2

$$S = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)\hbar^2} = \sqrt{\frac{3}{4}}\hbar$$

$$S_z = +\frac{\hbar}{2} \text{ or spin up or } \uparrow \text{ or } \left|\uparrow\right\rangle \text{ or } \chi_+$$

$$S_z = -\frac{\hbar}{2} \text{ or spin down or } \downarrow \text{ or } \left|\downarrow\right\rangle \text{ or } \chi_-$$

χ_+ and χ_- are the spin eigenfunctions (there are only 2)

Spin

- Just as there was an orbital magnetic dipole moment, so will there be a spin magnetic dipole moment

Orbital magnetic moment

$$\vec{\mu}_l = -\frac{g_l \mu_B}{\hbar}\vec{L} \text{ where } g_l = 1$$

$$\mu_{l_z} = -\frac{g_l \mu_B}{\hbar} L_z = -g_l \mu_B m_l$$

$$\Delta E = -\vec{\mu}_l \cdot \vec{B}$$

Spin magnetic moment

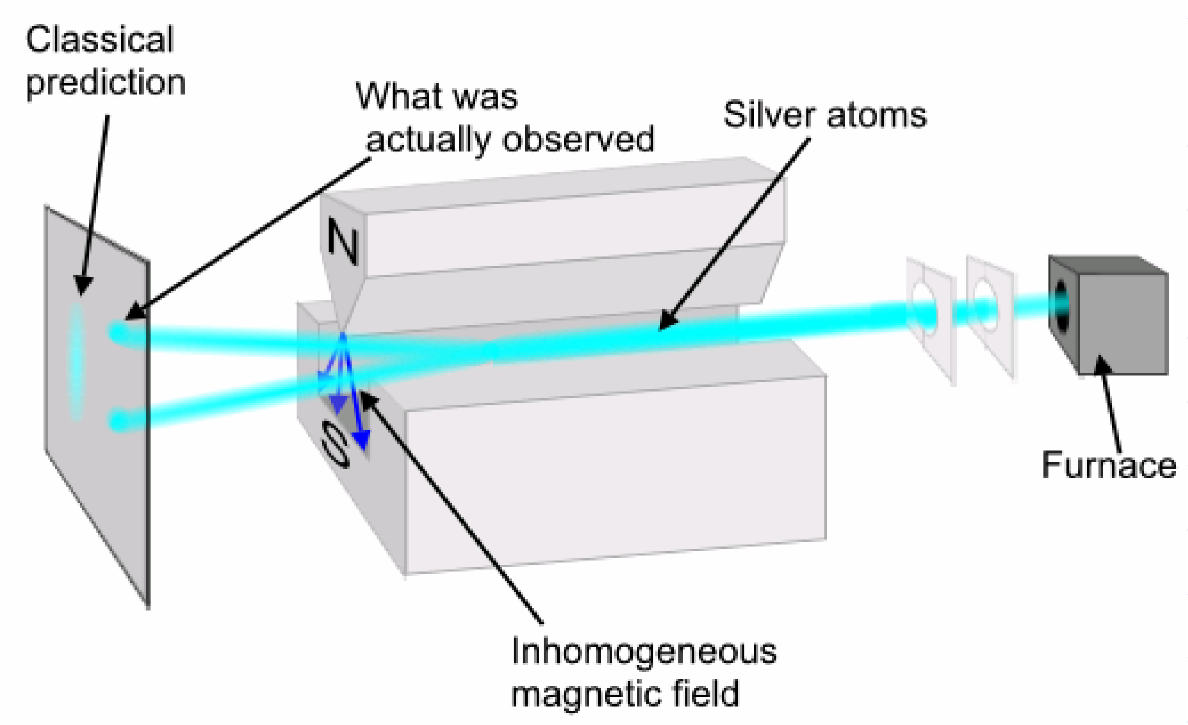
$$\vec{\mu}_s = -\frac{g_s \mu_B}{\hbar}\vec{S}$$

$$\mu_{s_z} = -\frac{g_s \mu_B}{\hbar} S_z = -g_l \mu_B m_s$$

$$\Delta E = -\vec{\mu}_s \cdot \vec{B}$$

Spin

- This explains the results from Stern-Gerlach and Phipps-Taylor

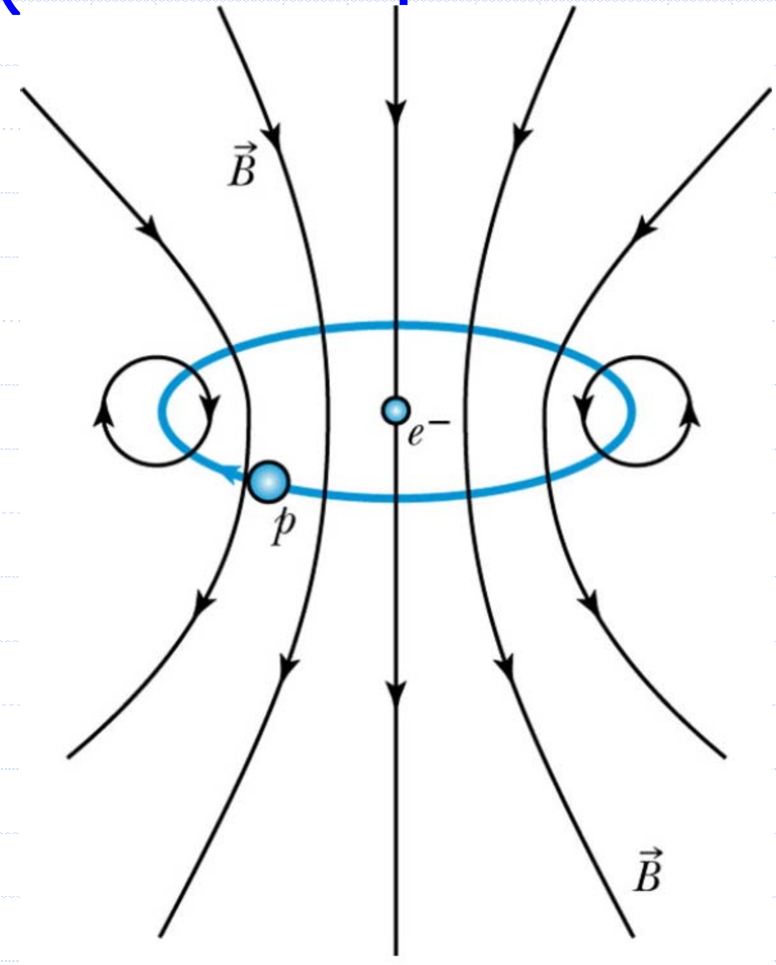


Spin

- Aside, even in the absence of an external magnetic field, the electron will still experience a magnetic field
- Thus we would expect a doublet splitting of the hydrogen atom spectral lines

Spin

- Magnetic field produced by orbiting proton (nuclear dipole moment)



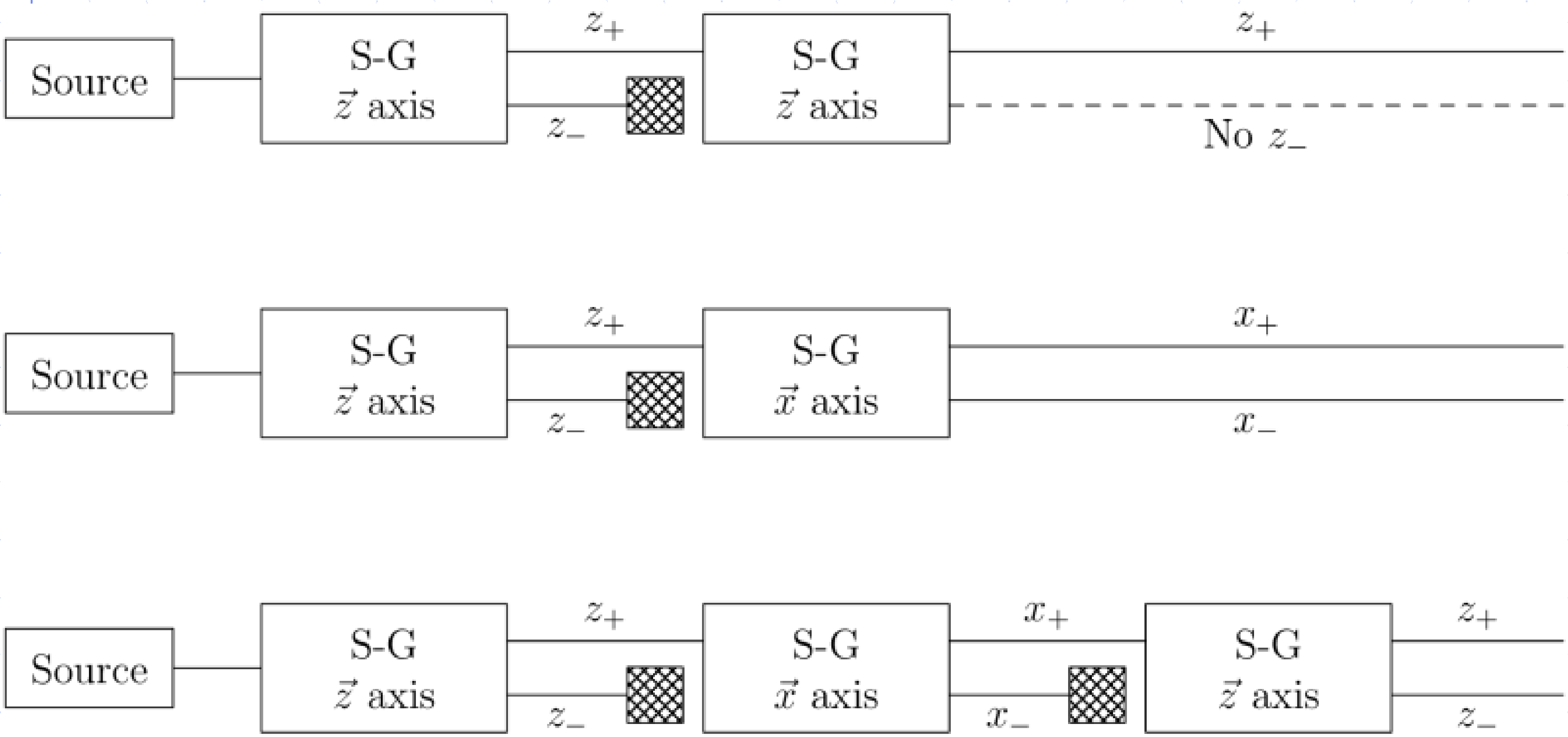
© 2006 Brooks/Cole - Thomson

Spin

- A polarized beam is one where all electrons have spin up or spin down
  - What is the spin wave function for a polarized beam with spin up?
- An unpolarized beam has equal amounts of spin up and spin down
  - What is the wave function for an unpolarized beam?

Spin

- Stern-Gerlach experiments



Spin

- The Stern-Gerlach apparatus can be used to measure the direction of the electron spin
- Because only one component of spin (S_x, S_y, S_z) can be measured at a time measurement of a spin component in an orthogonal direction will result in 50/50 probability of spin up/down along the new direction
  - Any previous knowledge of the original spin direction is destroyed