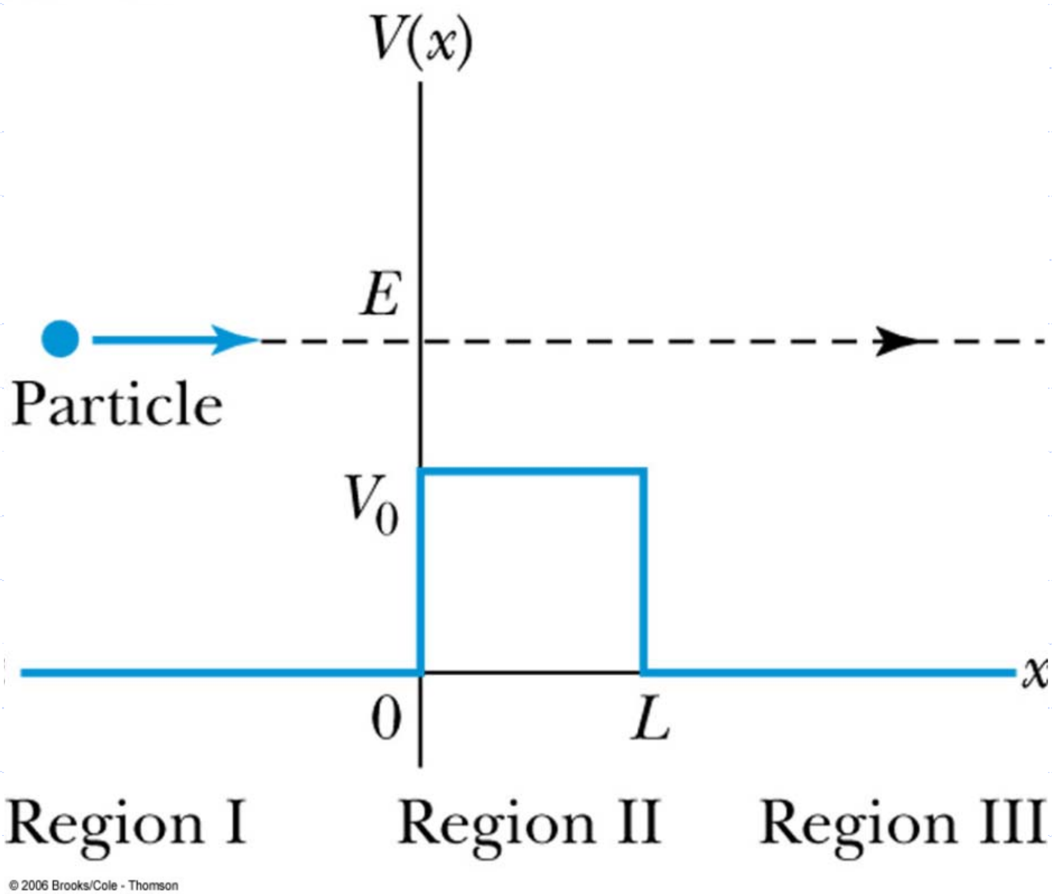
Square Barrier

- Barrier with $E>V_0$
  - What is the classical motion of the particle?



Square Barrier

- In regions I and III we need to solve

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_I}{dx^2} = E\psi_I$$

$$\frac{d^2\psi_I}{dx^2} + k^2\psi_I = 0 \text{ where } k_I = k_{III} = k = \frac{\sqrt{2mE}}{\hbar}$$

- In region II we need to solve

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2} + V_0\psi_{II} = E\psi_{II}$$

$$\frac{d^2\psi_{II}}{dx^2} + k_{II}^2\psi_{II} = 0 \text{ where } k_{II} = \frac{\sqrt{2m(E - V_0)}}{\hbar}$$

Square Barrier

- The solution in Region I contains the incident and reflected wave

$$\psi_{\text{I}} = Ae^{ikx} + Be^{-ikx}$$

- The solution in Region III contains the transmitted wave

$$\psi_{\text{III}} = Ee^{ikx} + Fe^{-ikx} \rightarrow Ee^{ikx}$$

- The solution in Region II is

$$\psi_{\text{II}} = Ce^{ik_{II}x} + De^{-ik_{II}x}$$

Square Barrier

- As usual we require continuity of ψ and dψ/dx at the boundaries
  - At x=0 this gives A and B in terms of C and D
  - At x=L this gives C and D in terms of E
- The results are

$$A = \left[\cos k_{II} L - i \frac{k^2 + k_{II}^2}{2kk_{II}} \sin k_{II} L \right] e^{ikL} E$$

$$B = i \frac{k_{II}^2 - k^2}{2kk_{II}} \sin\left(k_{II} L\right) e^{ikL} E$$

Square Barrier

- We define reflection R and transmission T coefficients

$$R = \left|\frac{B}{A}\right|^2 = \frac{\left(k^2 - k_{II}^2\right)^2 \sin^2 k_{II} L}{4k^2 k_{II}^2 + \left(k^2 - k_{II}^2\right)^2 \sin^2 k_{II} L}$$

$$T = \left|\frac{E}{A}\right|^2 = \frac{4k^2 k_{II}^2}{4k^2 k_{II}^2 + \left(k^2 - k_{II}^2\right)^2 \sin^2 k_{II} L}$$

- And I'll leave it to you to show that R+T=1

Square Barrier

- Using relations for k and k_{II}, we can rewrite the transmission coefficient T as

$$\text{with } k = \frac{\sqrt{2mE}}{\hbar} \text{ and } k_{II} = \frac{\sqrt{2m(E - V_0)}}{\hbar}$$

$$T = \frac{4E(E - V_0)}{4E(E - V_0) + V_0^2 \sin^2\left[\sqrt{2m(E - V_0)}\,\frac{L}{\hbar}\right]}$$

Square Barrier

- There is one interesting feature
  - With E and V_0 fixed, the transmission coefficient T oscillates between 1 and a minimum value as the barrier width is varied

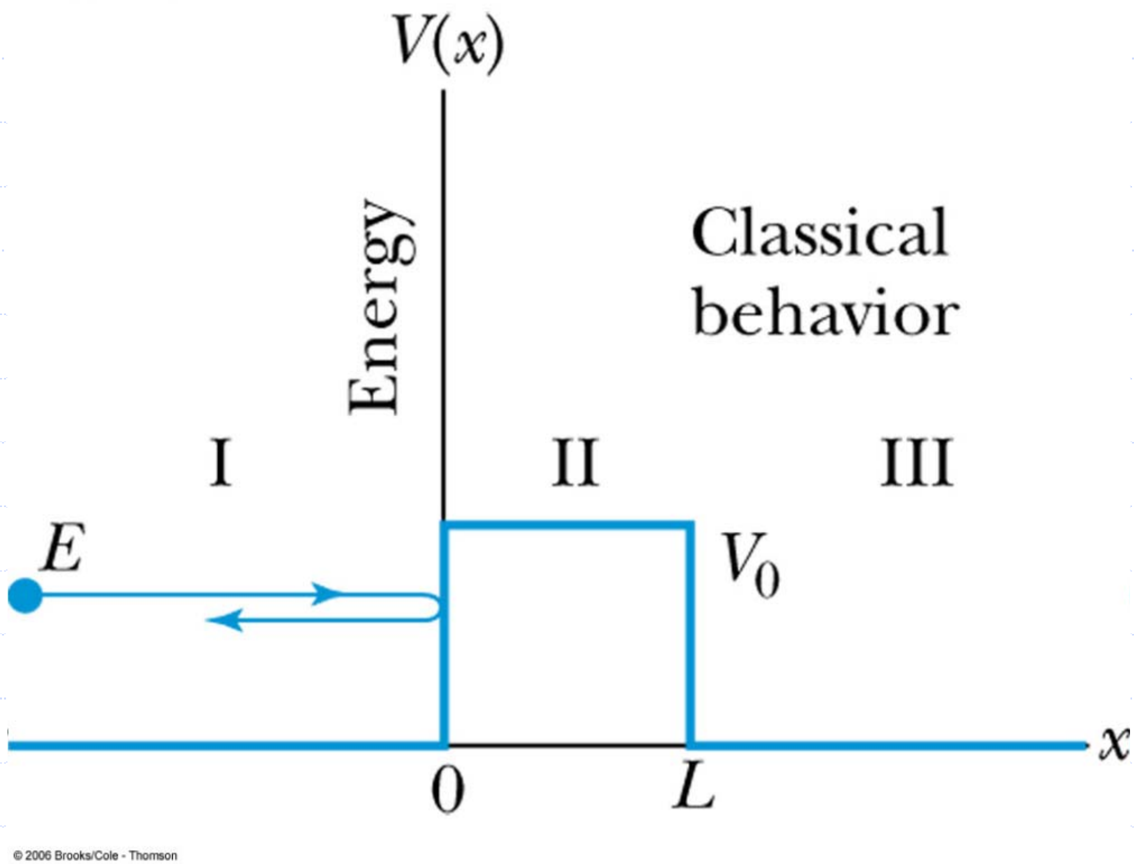
$$T_{\max} = 1$$

$$T_{\min} = \frac{4E\left(E - V_0\right)}{4E\left(E - V_0\right) + V_0^2}$$

  - We call the wave in the case of T=1 a resonance
  - A resonance is obtained when $k_{II}L = n\pi$
    - This means T=1 at values of L=λ/2 in region II
    - That is, a standing wave will exist in region II

Square Barrier

- Barrier with $E<V_0$
  - What is the classical motion of the particle?



Square Barrier

- In regions I and III we need to solve

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_I}{dx^2} = E\psi_I$$

$$\frac{d^2\psi_I}{dx^2} + k^2\psi_I = 0 \text{ where } k_I = k_{III} = k = \frac{\sqrt{2mE}}{\hbar}$$

- In region II we need to solve

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2} + V_0\psi_{II} = E\psi_{II}$$

$$\frac{d^2\psi_{II}}{dx^2} - \kappa_{II}^2\psi_{II} = 0 \text{ where } \kappa_{II} = \kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

Square Barrier

- The solution in Region I contains the incident and reflected wave

$$\psi_{\mathrm{I}} = Ae^{ikx} + Be^{-ikx}$$

- The solution in Region III contains the transmitted wave

$$\psi_{\mathrm{III}} = Ee^{ikx} + Fe^{-ikx} \rightarrow Ee^{ikx}$$

- The solution in Region II is

$$\psi_{\mathrm{II}} = Ce^{\kappa x} + De^{-\kappa x}$$

Square Barrier

- We could again apply boundary conditions on ψ and dψ/dx
- But it's easier to note the difference between this case and the one previous is

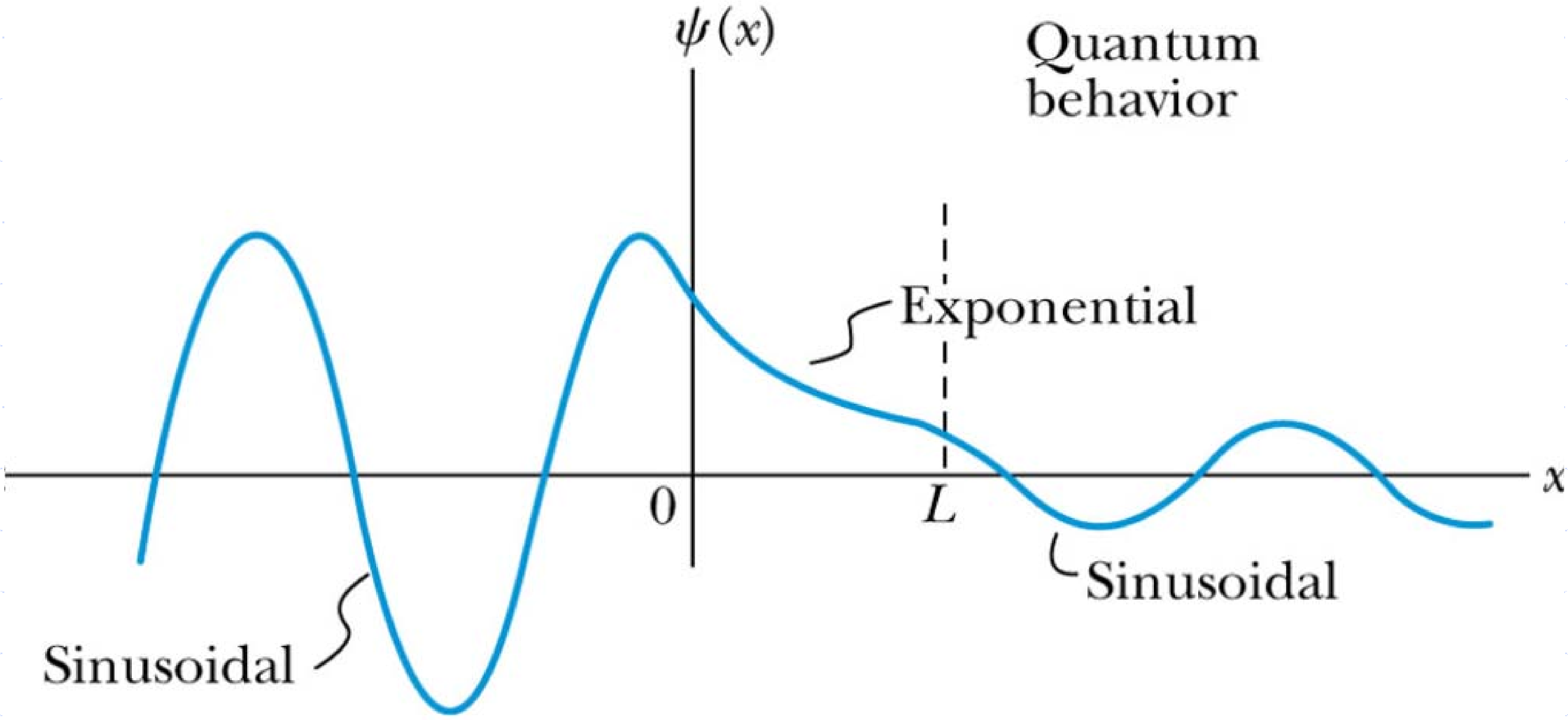
$$k_{II} \to -i\kappa$$

$$\text{and thus}\ \sin k_{II}L \to \sin(-i\kappa L) = -\sinh(\kappa L)$$

- Thus for E < V_0, T becomes

$$T = \frac{4E(V_0 - E)}{4E(V_0 - E) + V_0^2 \sinh^2\left[\sqrt{2m(V_0 - E)}\,\frac{L}{\hbar}\right]}$$

Square Barrier



Square Barrier

- Thus we get a finite transmission probability T even though $E < V_0$
  - This is called tunneling
  - You can think of tunneling in terms of the uncertainty principle
    - As shown in Thornton and Rex, when the particle is in region II, the uncertainty in kinetic energy is $V_0 - E$
    - The uncertainty in energy is comparable to the barrier height and there is a probability that particles could have sufficient energy to cross the barrier

Square Barrier

- For κL >> 1, the tunneling probability T becomes

$$T = 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)e^{-2\kappa L}$$

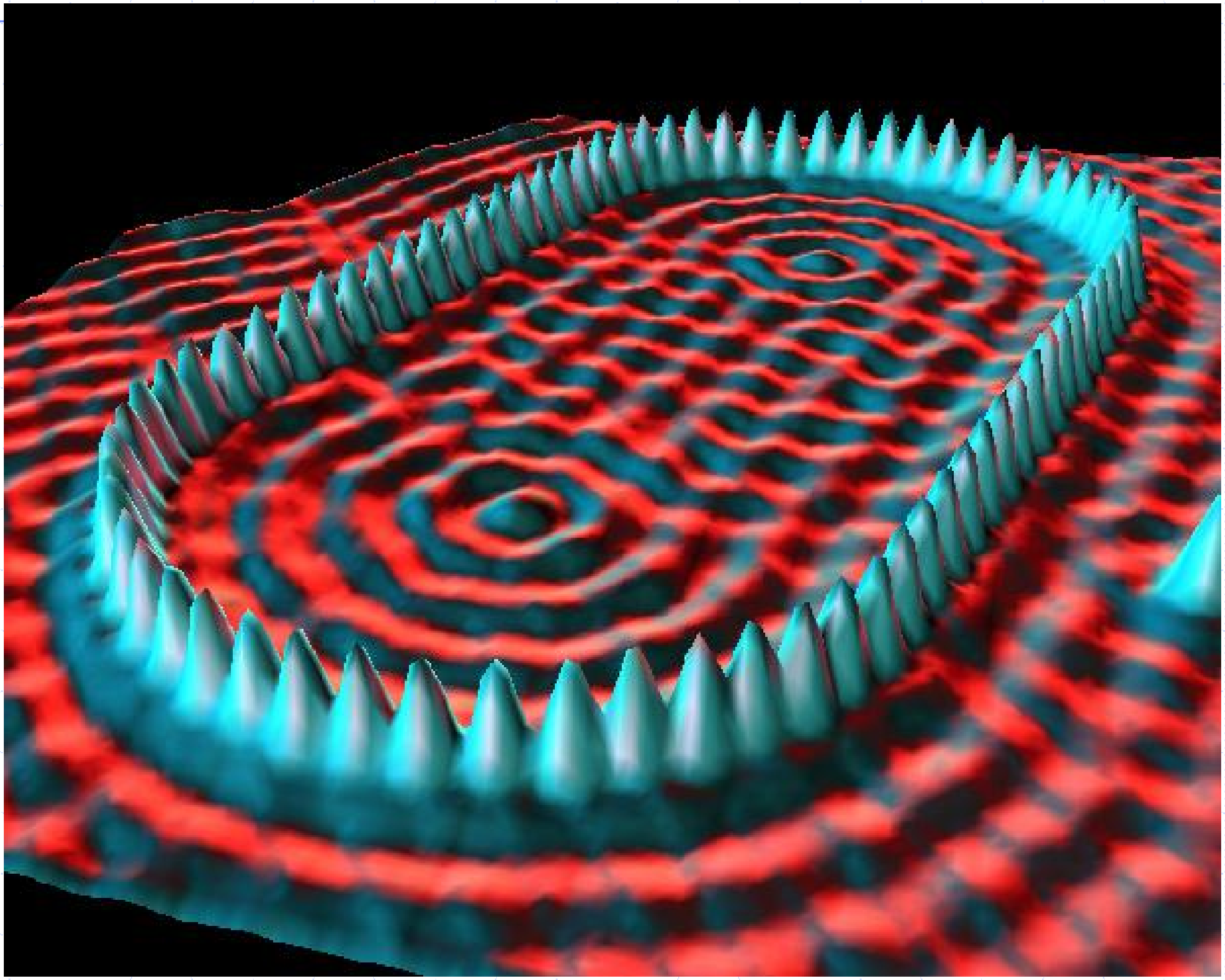
where we used $\sinh(x) = \left(e^{x} - e^{-x}\right)/2$

- For rough estimates we can further approximate this as (see example 6.15 in Thornton and Rex)

$$T = 2e^{-2\kappa L}$$

- The exponential shows the importance of the barrier width L over the barrier height V_0

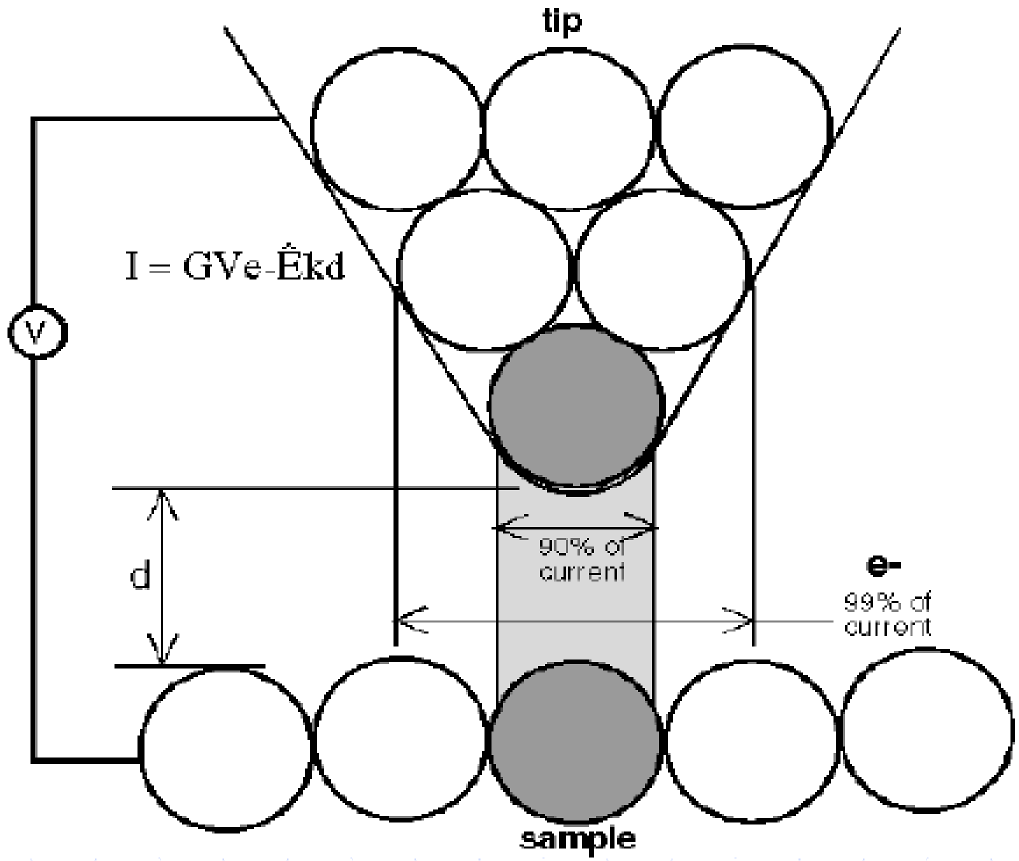
Scanning Tunneling Microscope



STM

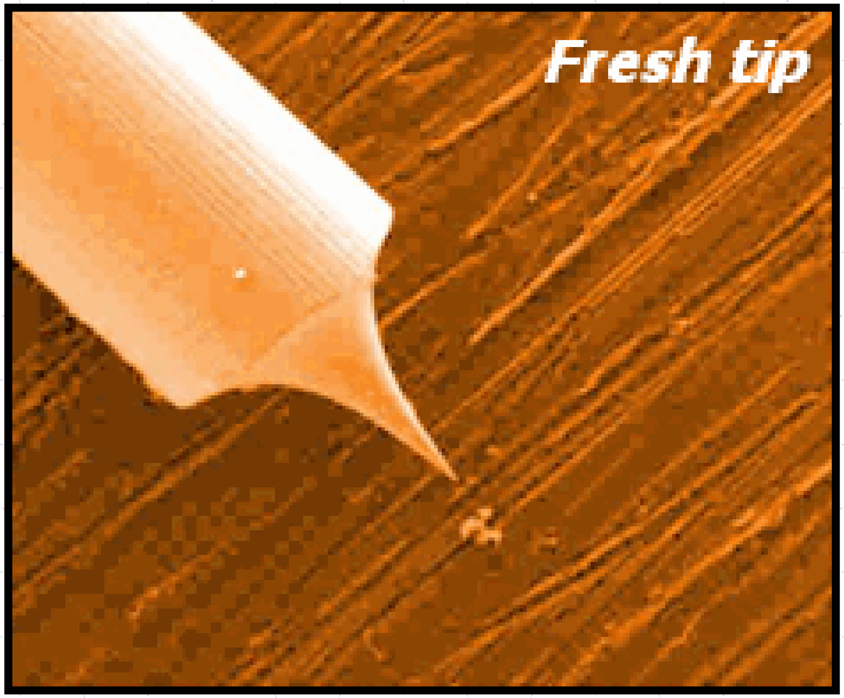
- Invented by Gerd Binnig and Heinrich Rohrer in 1982
- Nobel prize in 1986!
- The basic idea makes use tunneling
  - When a sharp needle tip is placed less than 1 nm from a conducting material surface and a voltage applied between them, electrons can tunnel between the tip and surface
  - Since the tunnel current varies exponentially with the tip-surface distance, sub-nm changes in distance can be detected

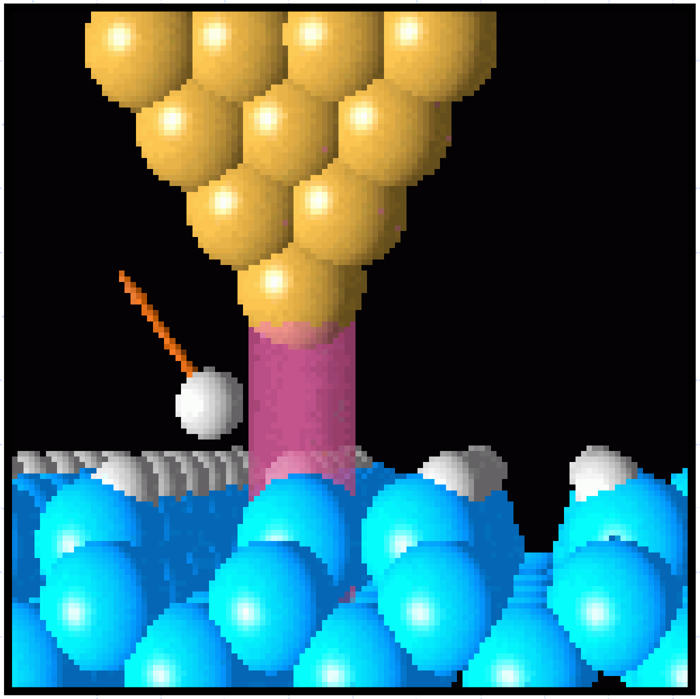
STM



STM

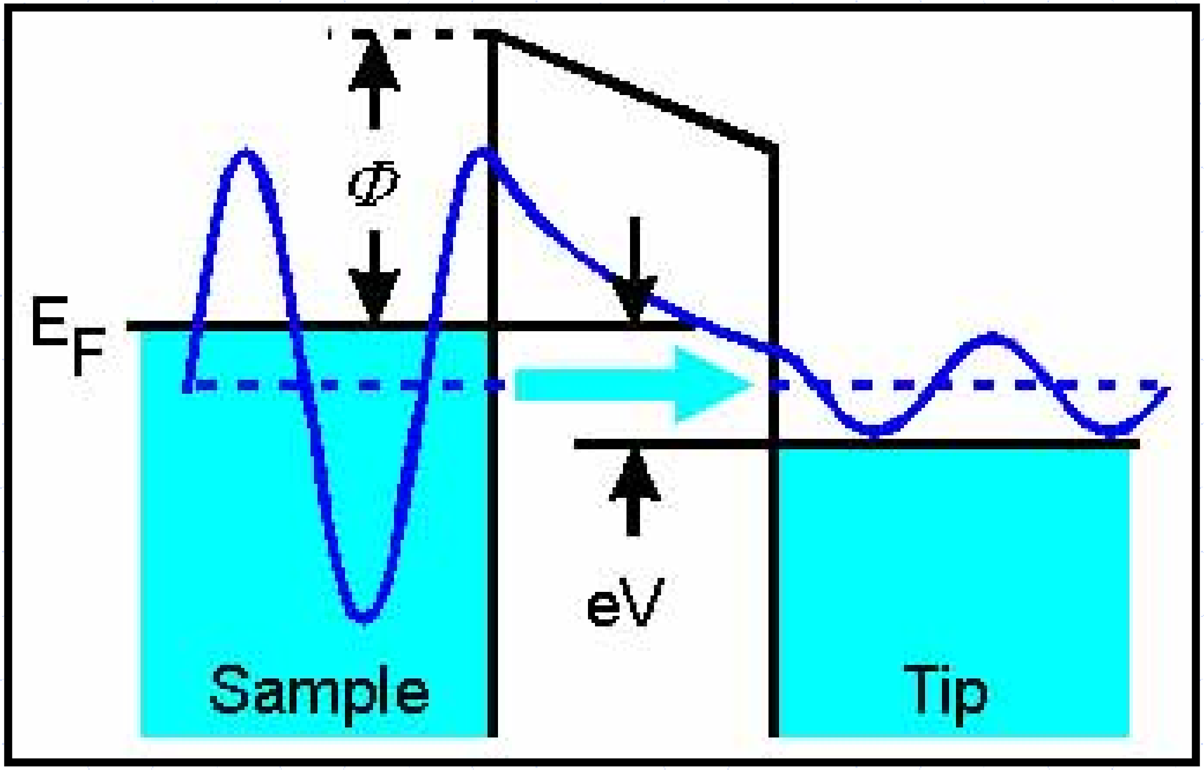
- STM tip





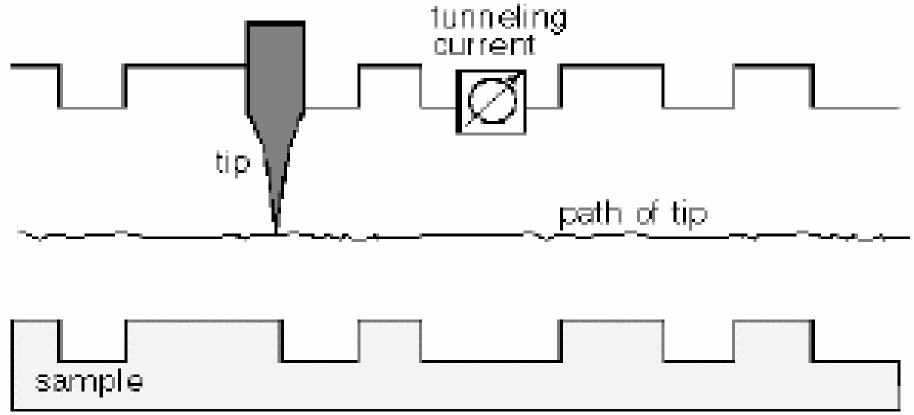
STM

- Tunneling through the potential barrier

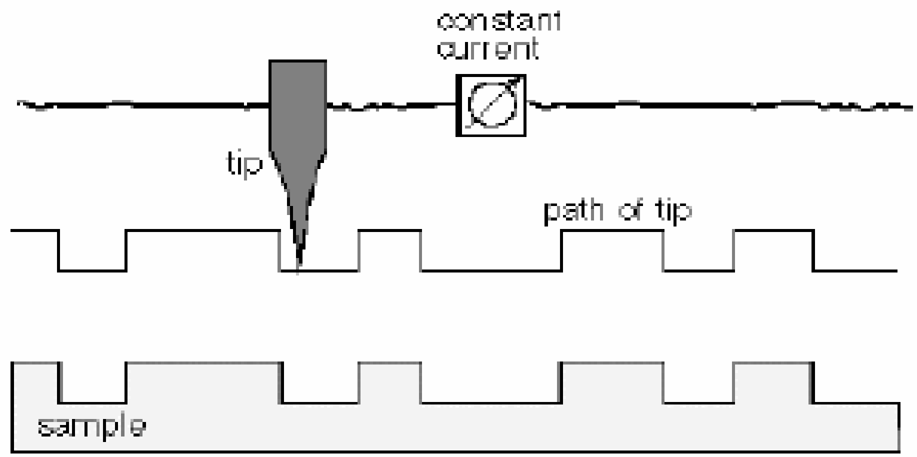


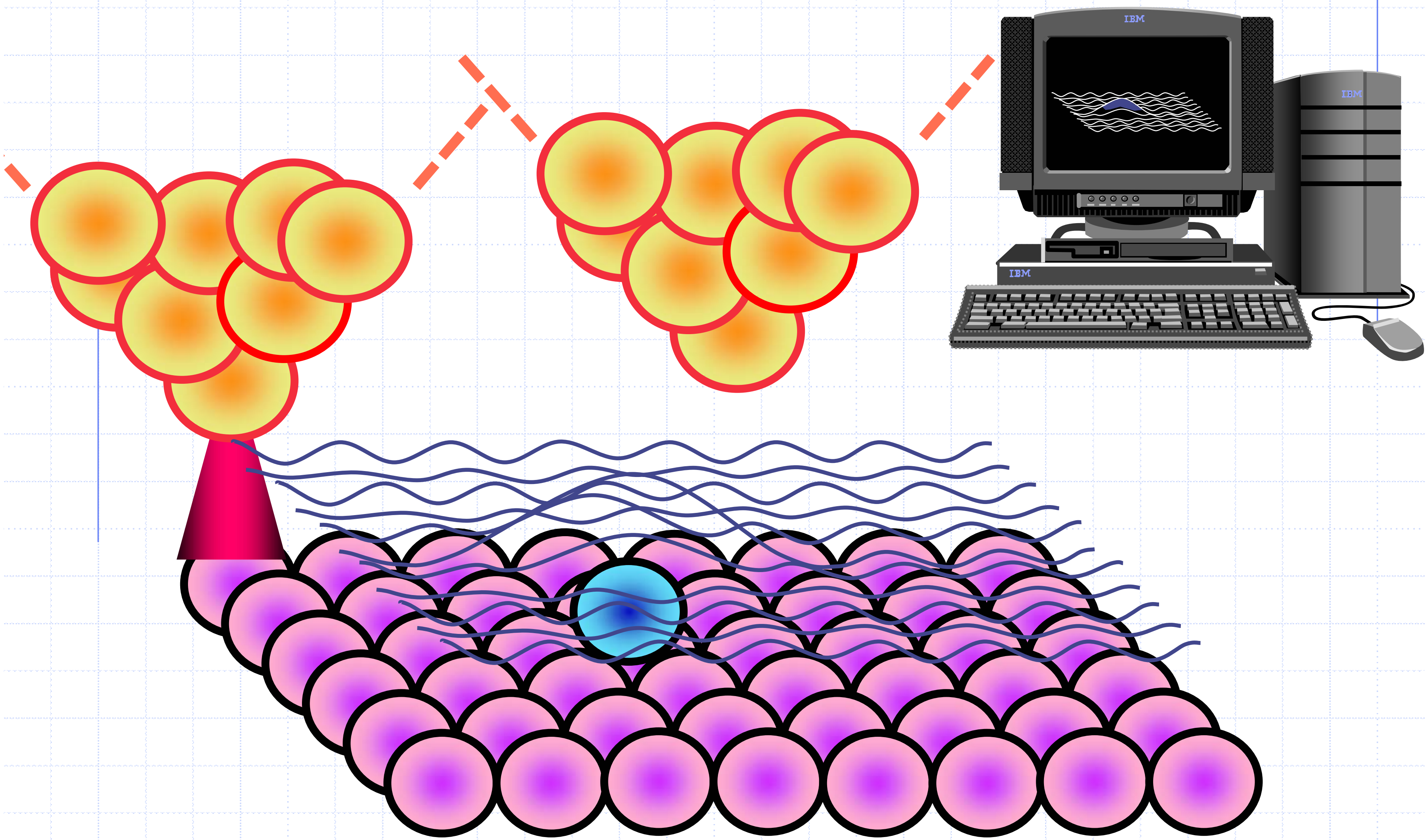
STM

- Raster scanning with constant Z



- Raster scanning with constant tunneling current

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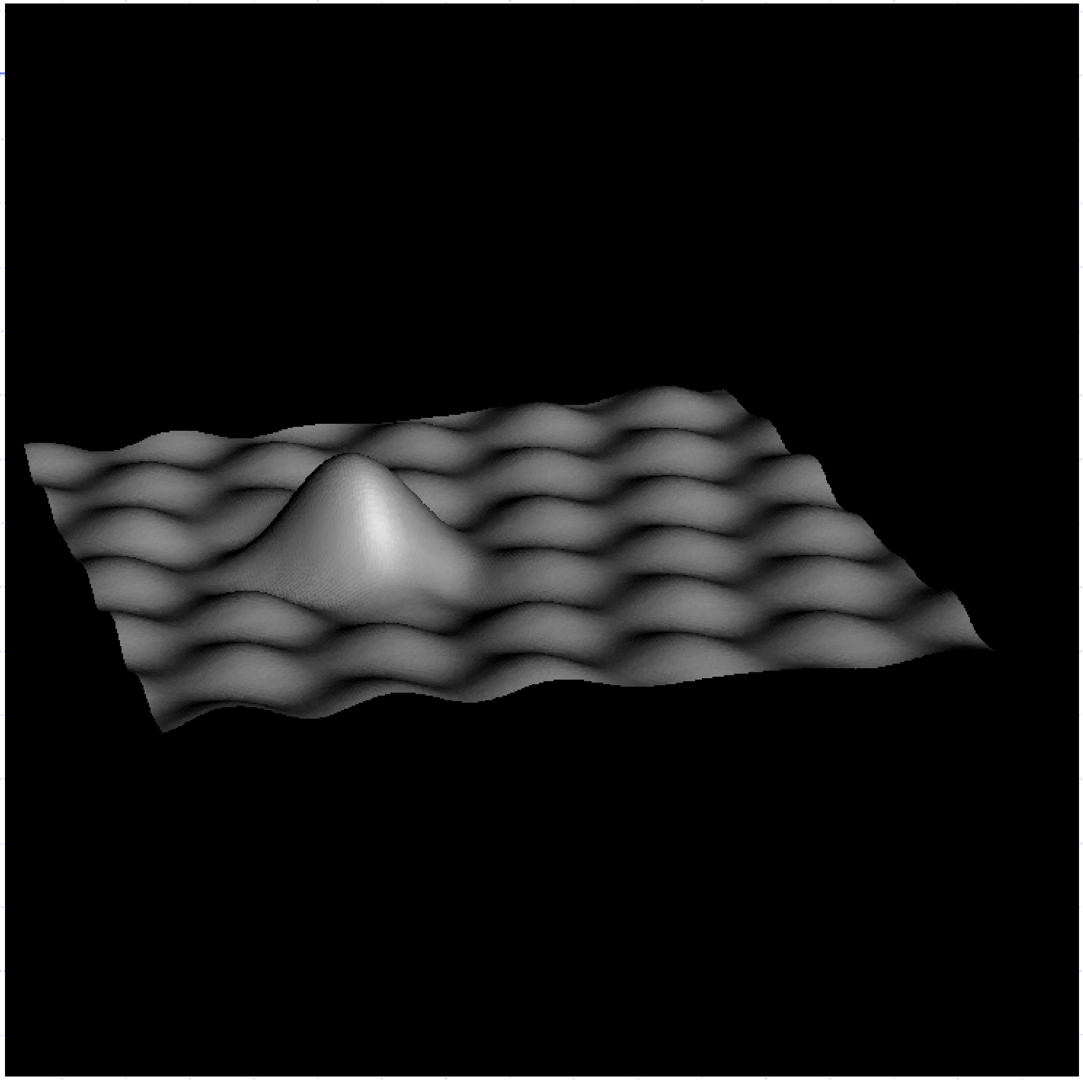
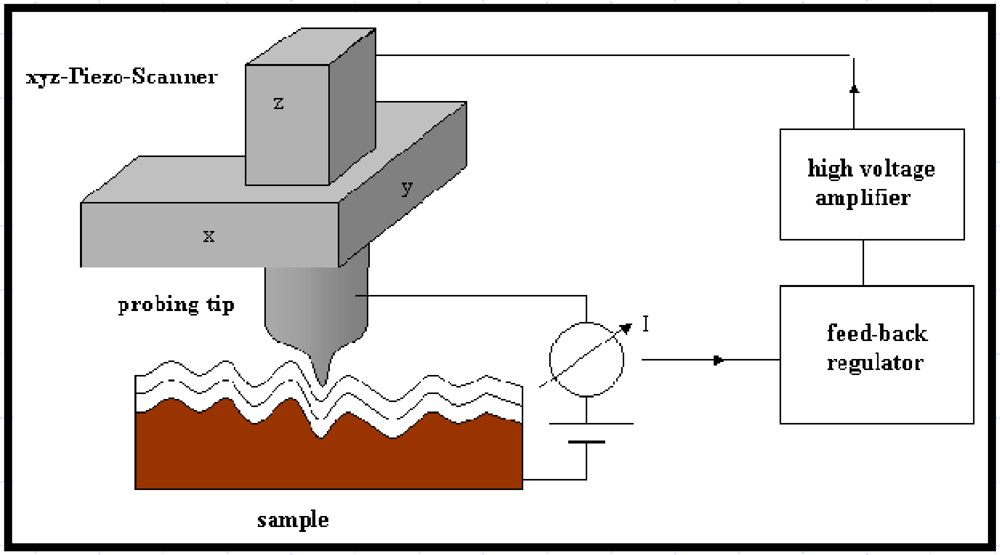
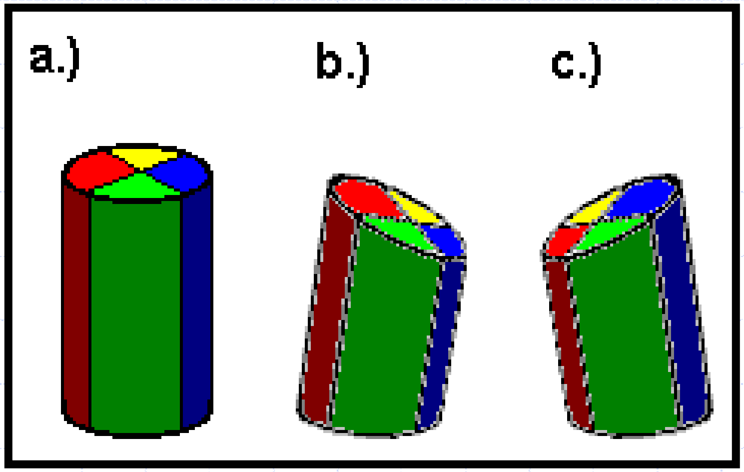
STM



STM

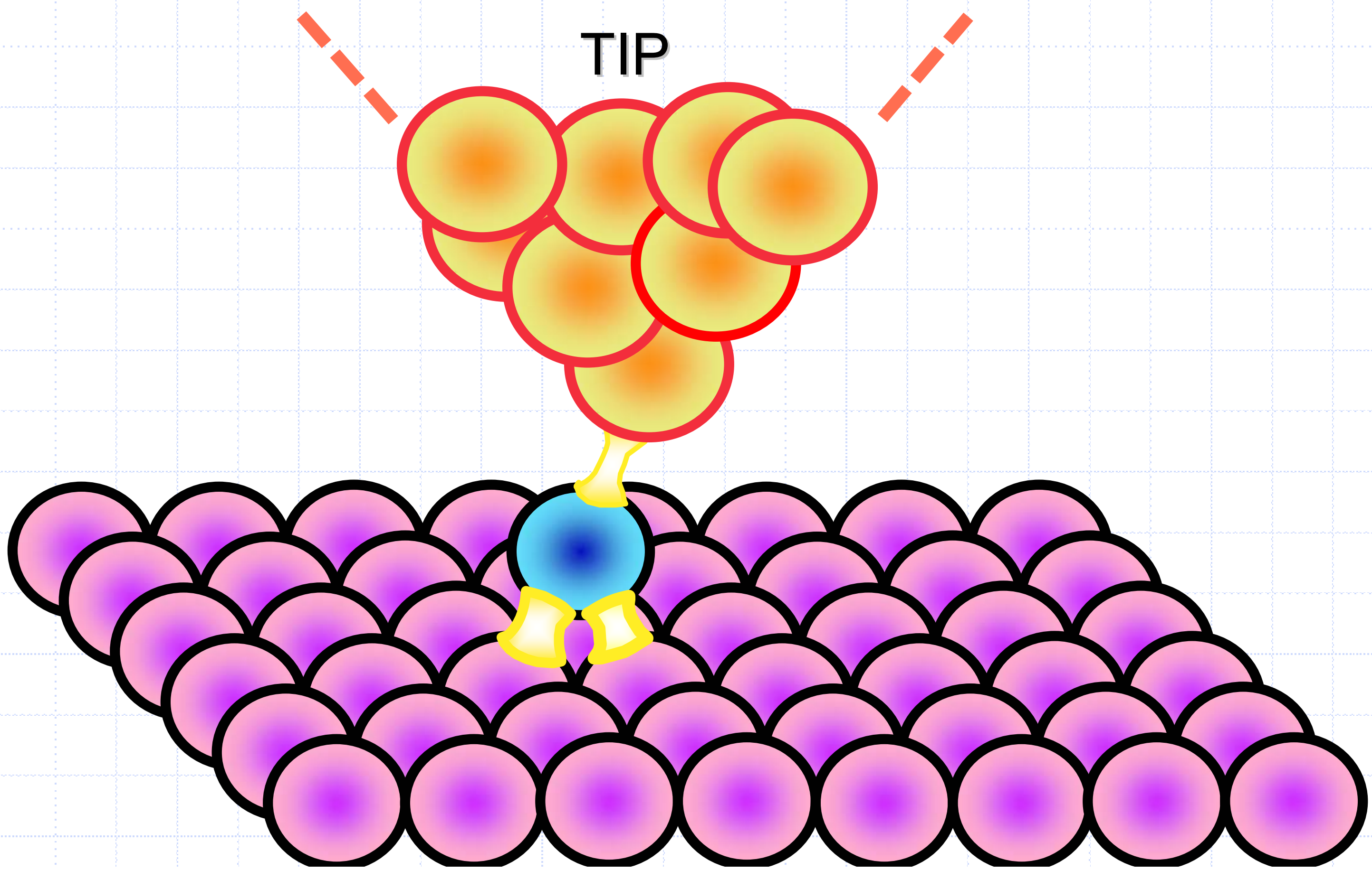
- The STM tip is attached to piezoelectric elements (usually a tube) that precisely control the position in x-y-z
  - Used to control tip-surface distance (z)
  - Used to raster scan (x-y)





STM

- STM can also be used to manipulate atoms via van der Waals, tunneling, or electric field forces



STM

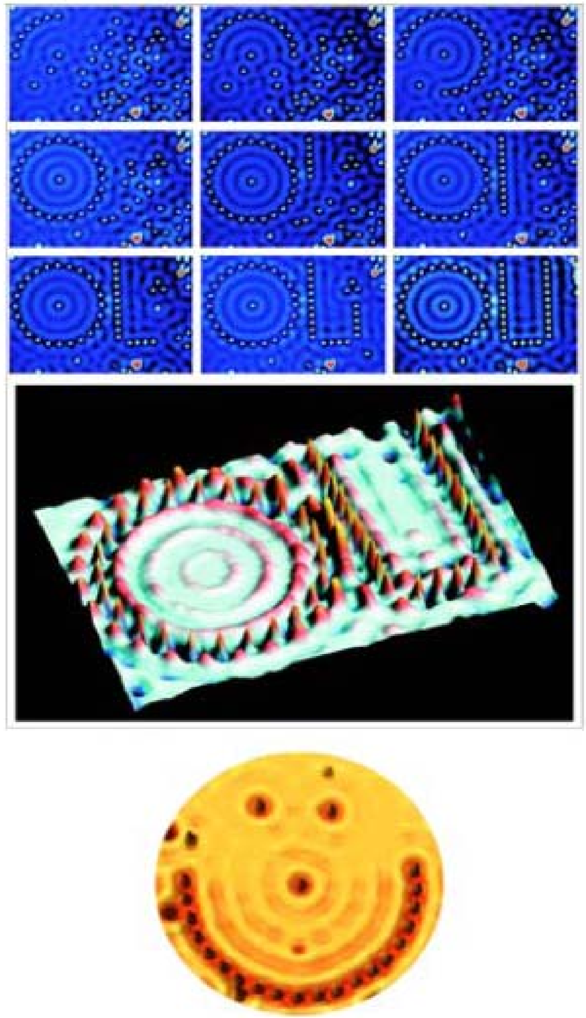
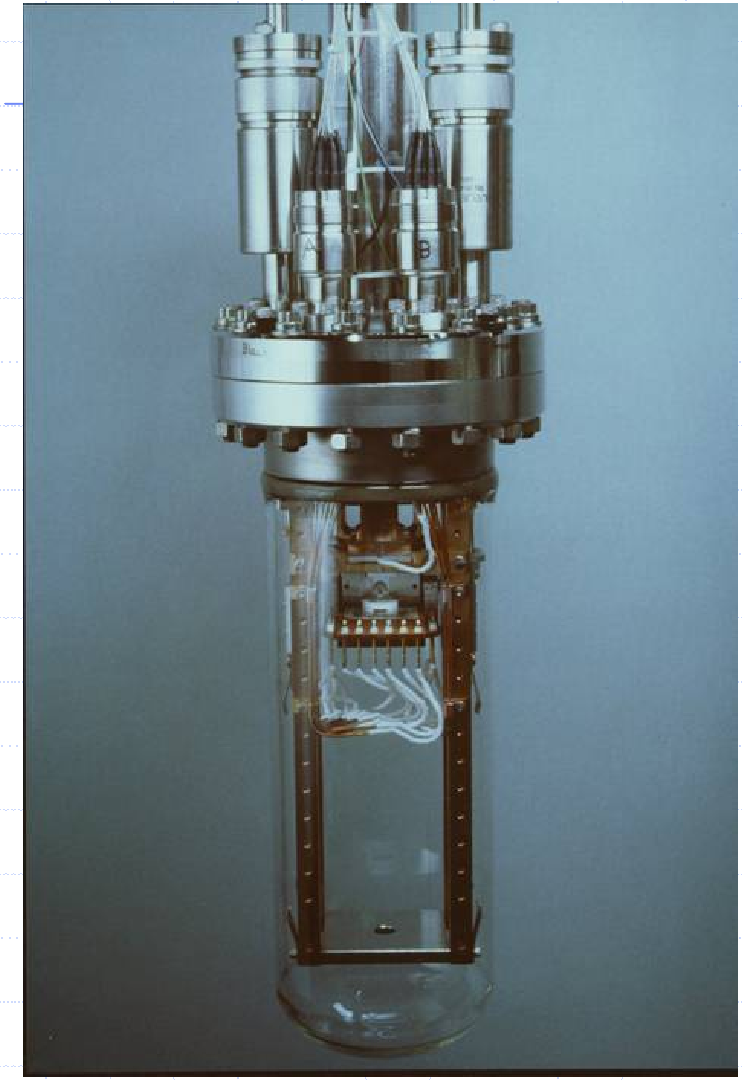


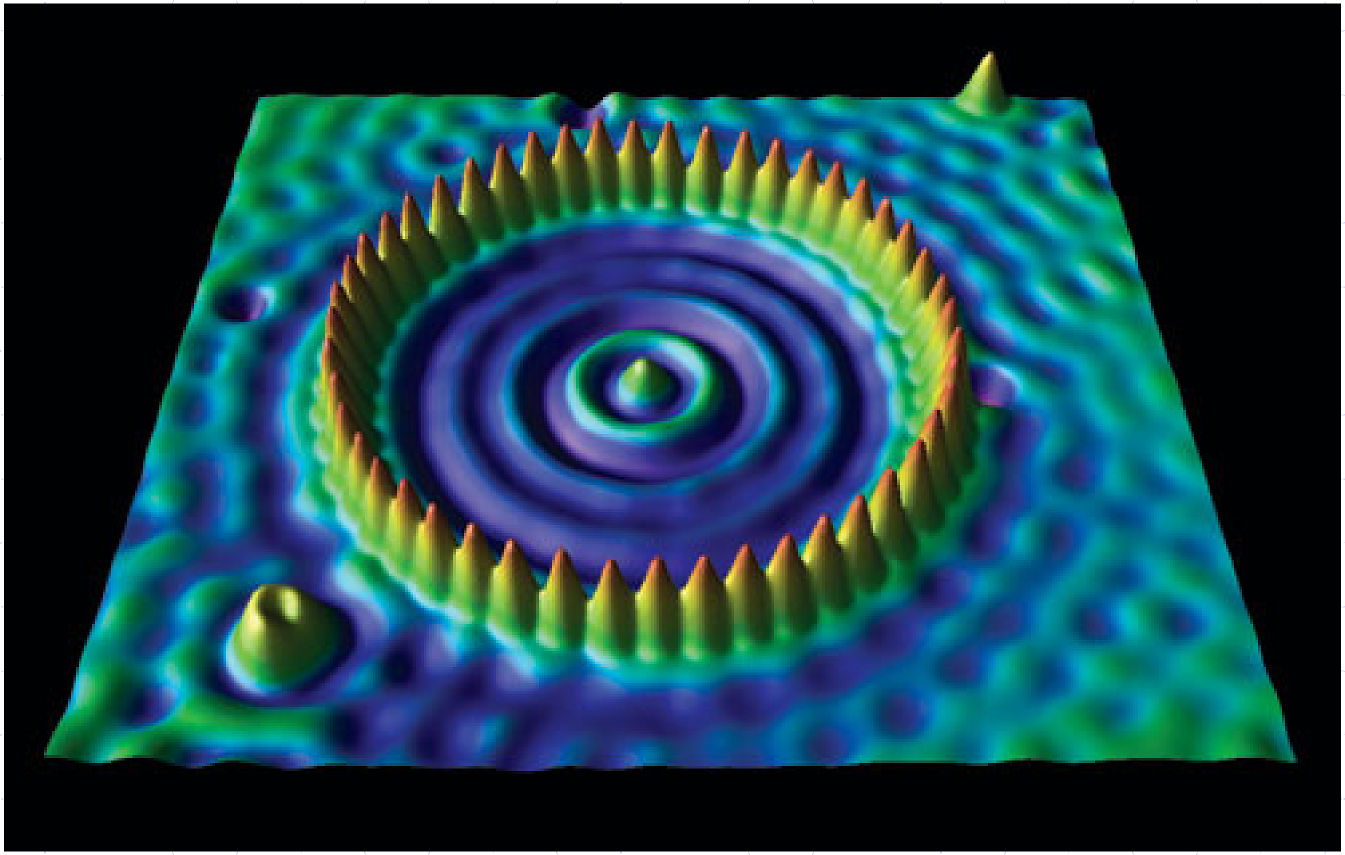
FIG. 7. OU logo writing sequence using individual silver atoms on a Ag(111) surface at 6 K (upper) and a three-dimensional representation (middle) (42 nm × 26 nm area, 51 silver atoms are used). "Atomic smiley" image is written by using silver atoms on a Ag(111) surface at 5 K (32 nm diameter).

STM



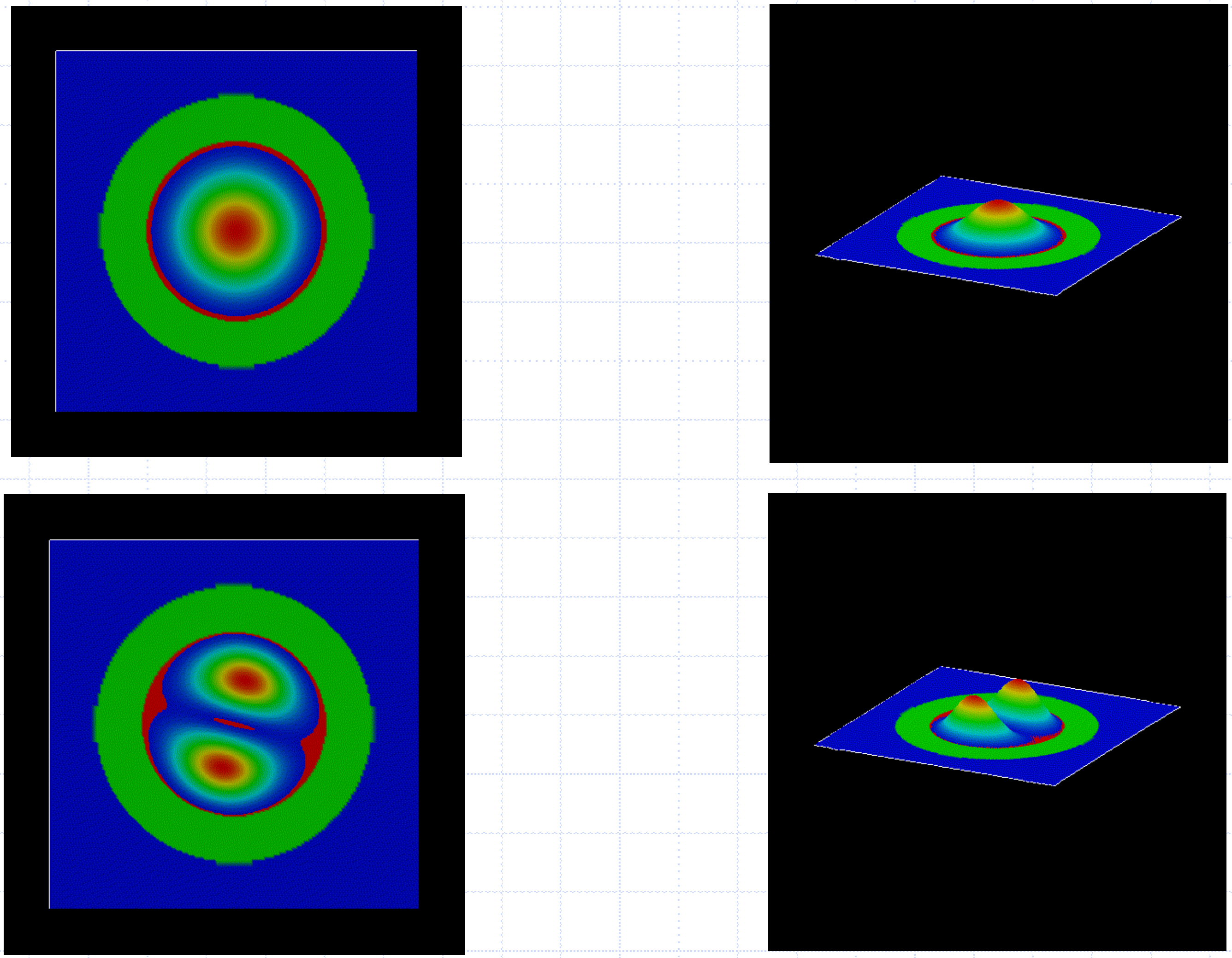
Quantum Corrals

- Electron in a corral of iron atoms on copper



Quantum Corrals

- Electron in a corral of iron atoms



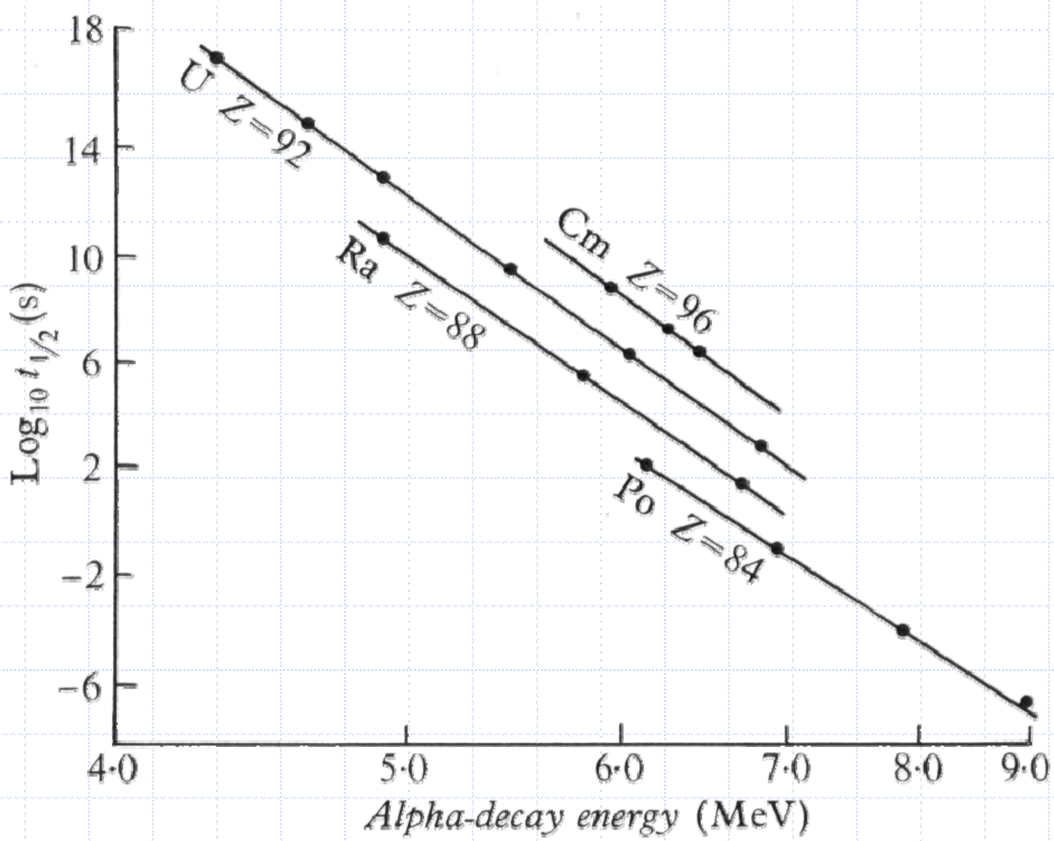
Alpha Decay

Geiger-Nuttall law

- Nuclei with A > 150 are unstable with respect to alpha decay
  - An alpha particle (α) consists of a bound state of 2 protons and 2 neutrons (^{4}He nucleus)
  - $A(Z,N) \rightarrow A(Z-2,N-2) + \alpha$
  - Effectively all of the energy released goes into the kinetic energy of the α

Alpha Decay

- Geiger-Nuttall law
  - Radioactive half lives vary from ~10^{-6}s to ~10^{17}s but the alpha decay energies only vary from 4 to 9 MeV



Alpha Decay

- Geiger-Nuttall law
  - The experimental data follow the Geiger-Nuttall law

$$\log_{10} W = C - \frac{D}{\sqrt{T_\alpha}} \text{ where}$$

W is the decay probability

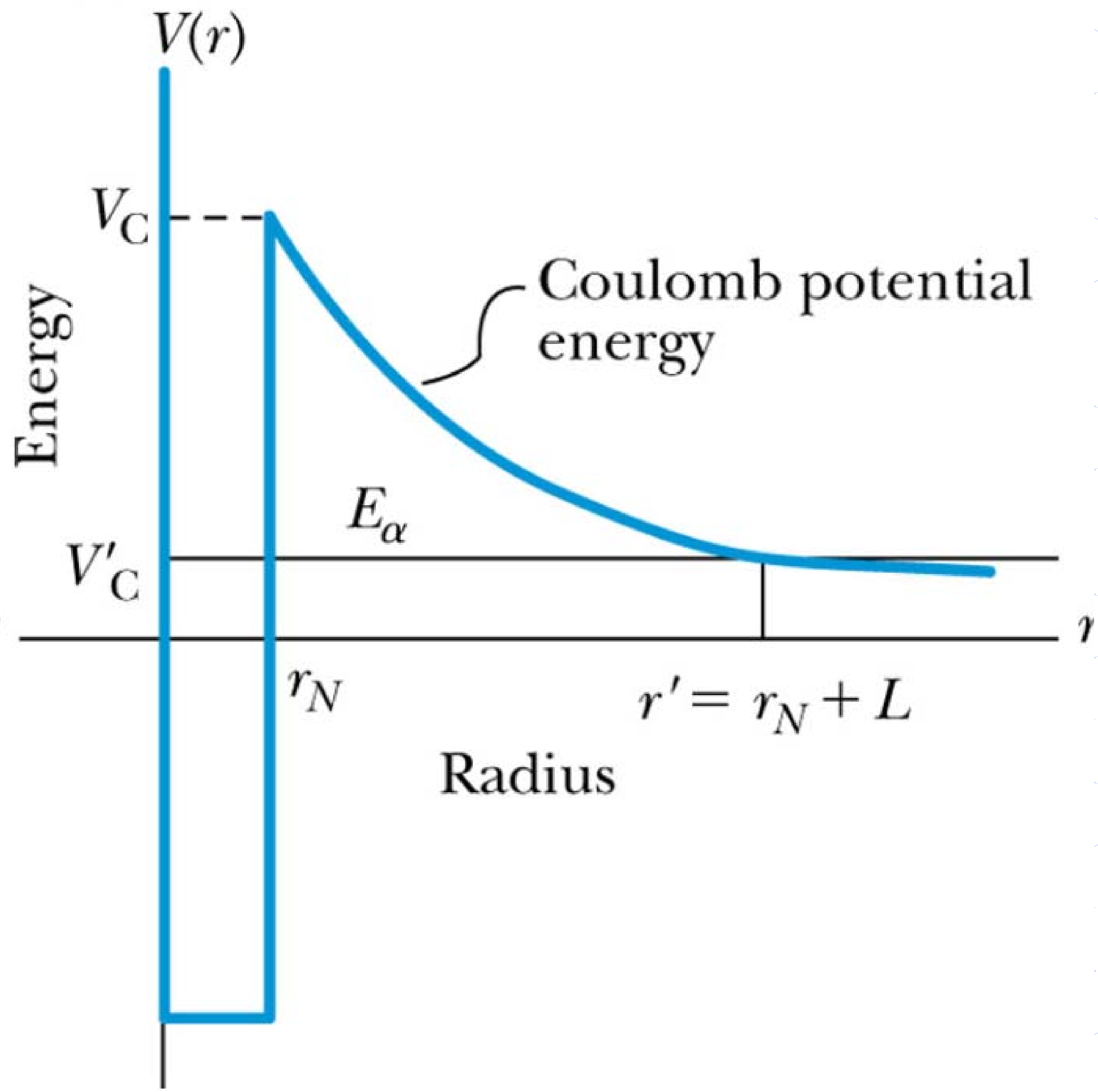
$\tau_{1/2} = \ln 2 / W$

C, D are constants

T_α is the alpha kinetic energy

  - A calculation of the quantum mechanic tunneling probability explained this law and was one of the early successes of quantum mechanics

Alpha Decay



Alpha Decay

- Calculation of the decay probability W

This is just a rough estimate

$$W = PvT$$

where

P is the probability of finding an alpha in a nucleus

v is the frequency that an alpha appears at the surface of the nucleus

T is the transmission probability

- Do this for ^{238}U alpha decay with r_N=7 F and T_α =4.2 MeV

Alpha Decay

- Calculation of P and ν

Guess $P = 0.1$

$$\nu = \frac{\mathrm{v}}{2R} = \frac{\sqrt{2T/M_\alpha}}{2R}$$

$$\nu = \frac{\sqrt{2 \times 4.2MeV/3727MeV/c^2}}{2 \times 7 \times 10^{-15}m}$$

$$\nu = 10^{21}/s$$

Alpha Decay

➢ Calculation of transmission probability T

- Preliminaries
  - Calculate the height of the Coulomb barrier

$$V_C = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 r_n} = 37 MeV$$

  - Calculate the tunneling distance

$$T = 4.2 MeV = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 r'}$$

$$r' = 6.2 \times 10^{-14} m = 62 F$$

$$L = 62 - 7 = 55 F$$

Alpha Decay

- The Coulomb barrier is not a square well
  - There is a way in quantum mechanics to calculate T correctly (called the WKB approximation) but for today we'll just estimate the equivalent height and width of a square well
  - Use V_C=20 MeV and r'=25 F

Alpha Decay

- Calculation of T

$$\kappa = \frac{\sqrt{2M_\alpha (V_C - E)}}{\hbar}$$

$$\kappa = \frac{\sqrt{2 \times 3727\, MeV / c^2 \times (20 - 4.2) MeV}}{6.58 \times 10^{-22}\, MeVs}$$

$$\kappa = 1.7 \times 10^{15} / m$$

then

$$T = 2e^{-2\kappa L} = 2e^{-(2)(1.7 \times 10^{15})(25 \times 10^{-15})} = 2.4 \times 10^{-37}$$

Alpha Decay

- Calculation of $t_{1/2}$

$$W = PvT$$

$$W = (0.1)(10^{21})(2.4\times10^{-37}) = 2.4\times10^{-17}\,/\,s$$

$$\tau_{1/2}(theory) = \frac{\ln 2}{W} = 2.8\times10^{16}\,s$$

$$\tau_{1/2}(\exp eriment) = 1.4\times10^{17}\,s$$