Double Slit Experiment

- The intensity $I(x) \sim |E(x)|^2$ only tells us probabilistically where the photon will strike the screen
  - We interpret E(x,t) as the probability amplitude for the photon
  - The probability is $\sim |E(x,t)|^2$
  - And the probability is all we can calculate
- E(x,t) satisfies Maxwell's equations which are linear and homogeneous in E(x,t)
  - This means if E_1 and E_2 are solutions then $a_1E_1 + a_2E_2$ is a solution
  - This principle of superposition is what produces the interference effects of waves

Wave Functions

- Now we are starting quantum mechanics
- Analogous to the last slide, we define the wave function $\Psi(x,t)$ that characterizes the quantum state of a particle
  - $\Psi(x,t)$ is a probability amplitude of the particle's presence
  - The wave equation for $\Psi(x,t)$ (yet to be determined) is homogeneous and linear
  - $\Psi(x,t)$ is complex
- Aside, $E(x,t)$ does not describe the quantum state of radiation (the photon)

Wave Functions

- In classical mechanics we talk about the trajectory of a particle
- In quantum mechanics we talk about the quantum state of a particle and wave propagation
  - The wave function Ψ(r,t) describes the particle's quantum state and contains all possible information about the particle

$\Psi(x,t)$ is the probability amplitude of the particle's presence

$|\Psi(x,t)|^2 dx$ is the probability of finding the particle between x and x + dx at time t

  - The probability interpretation is due to Born

Wave Functions

- Immediately you should see that you cannot predict with certainty the outcome of the simplest experiment
- Quantum mechanics tells you about the possible results and their associated probabilities
  - "God does place dice with the universe"
  - Or maybe QM is incomplete and this indeterminacy just reflects our ignorance
  - Actually there is experimental evidence that the second statement is false

Wave Functions

- Is the wave function real?
  - No, it's a complex function (still to be shown)
  - No, it is an abstraction and we can never observe it
- Then why is it useful?
  - It contains all of the information there is about a quantum system
  - We can observe the probability density $|\Psi(x,t)|^2$
  - We can (in principle) use the wave function to calculate any property of a physical system
  - Wave packets and collapse of the wave function represent our knowledge of the system, not the real physical system itself

Wave Functions

- Even though the probability amplitude Ψ(x,t) is complex, the probability density |Ψ(x,t)|^2 is real and positive definite

$$\left|\Psi\right|^2 = \Psi^*\Psi = (a+bi)(a-bi)$$

$$\left|\Psi\right|^2 = a^2 - i^2b^2 = a^2 + b^2 \geq 0$$

- For a single particle we must have

$$\int_{-\infty}^{\infty} \left|\Psi\right|^2 dx = \int_{-\infty}^{\infty} P(x)dx = 1$$

  - This means the wave function must be square integrable

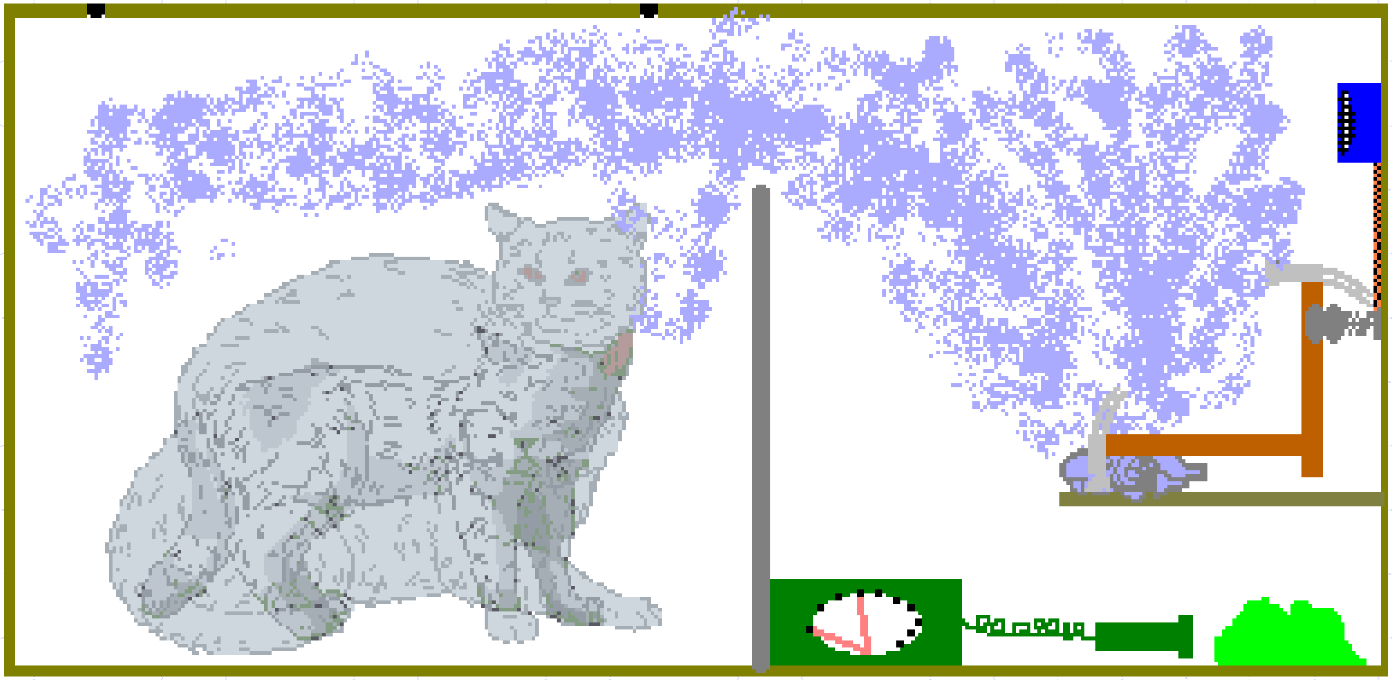
Copenhagen Interpretation

- Copenhagen interpretation of QM
  - The state of a quantum system is given by the wave function, which is a solution to Schrodinger's equation
  - The square of the wave function (wave amplitude) determines the probability of observing a certain result
    - Making a statistical statement is the best we can do
  - Quantum objects obey the Heisenberg uncertainty principle
  - Quantum objects have complementary wave and particle aspects

Copenhagen Interpretation

- Copenhagen interpretation of QM
  - Physical reality is the result of measurements
    - "Is the moon there if no one looks at it?"
  - The act of a measurement causes an instantaneous and discontinuous collapse of the wave function from a superposition of states to state corresponding to the observable measured
    - "Observations not only disturb what has to be measured, they produce it"
    - Schrodinger's cat
  - Quantum states cannot be separated from their measurement process
    - Einstein-Podolsky-Rosen paradox

Schrodinger's cat



Schrodinger's cat

- What is the state of the cat after 1 hour?
- QM says the wave function is a superposition of states

$$\Psi_{atom} = \frac{1}{\sqrt{2}}\left(\Psi_{decayed} + \Psi_{not\ decayed}\right)$$

$$\Psi_{cat} = \frac{1}{\sqrt{2}}\left(\Psi_{dead} + \Psi_{alive}\right)$$

- The act of making a measurement (opening the box) collapses the wave function into one state or another (dead cat/alive cat)
- This is nuts

Schrodinger's cat

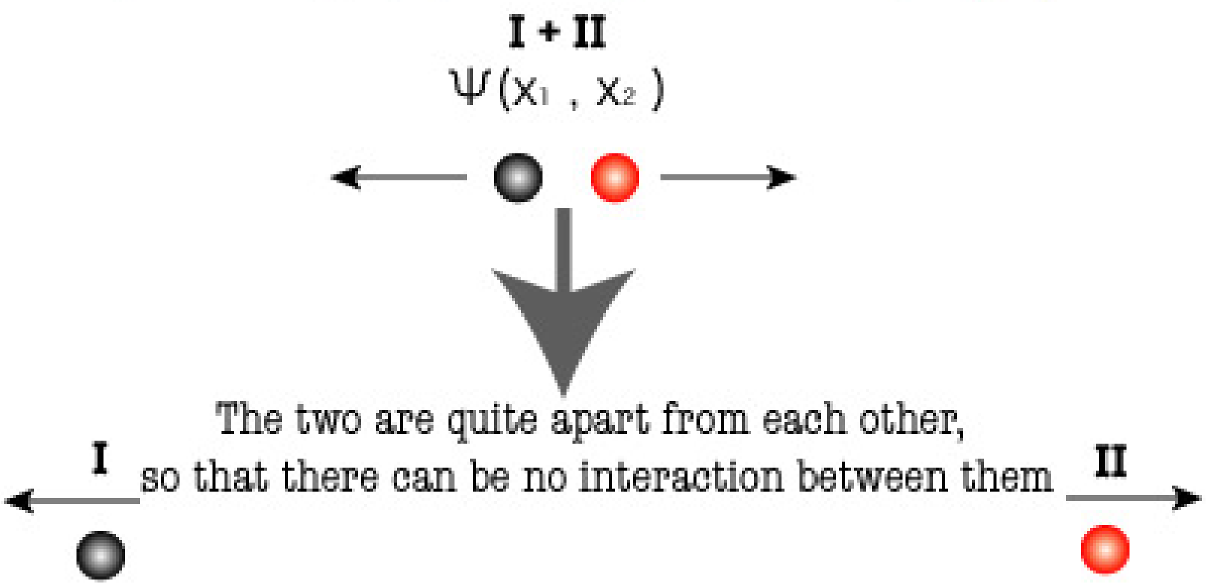
- We won't really resolve this paradox
  - One view is that the detection of the decaying nucleus is the measurement and causes the collapse of the wave function
  - Another view is that it makes no more sense to talk about one cat than it does one electron in the double slit experiment
  - Another view is that the cat is the observer (just kidding)
  - At least one experiment (using SQUID's not cats) that such macroscopic superposition of states are real

EPR Paradox

- EPR starts with two premises
  - In a complete theory, "every element of the physical reality must have a counterpart in the physical theory"
    - A complete theory must agree with experiment
  - "If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity"
    - If we don't disturb a system, the value of a physical quantity is real

EPR Paradox

EINSTEIN-PODOLSKY-ROSEN PARADOX, (1)



Suppose you measure the momentum of the black particle (I); then you can know the momentum of the red particle (II) as well. Likewise, if you measure the position of the black, then you can know the position of the red as well. In both cases, the measurement can be done **without disturbing** the red (since there can be **no interaction** between the black and the red).

EPR Paradox

EINSTEIN-PODOLSKY-ROSEN PARADOX, (2)

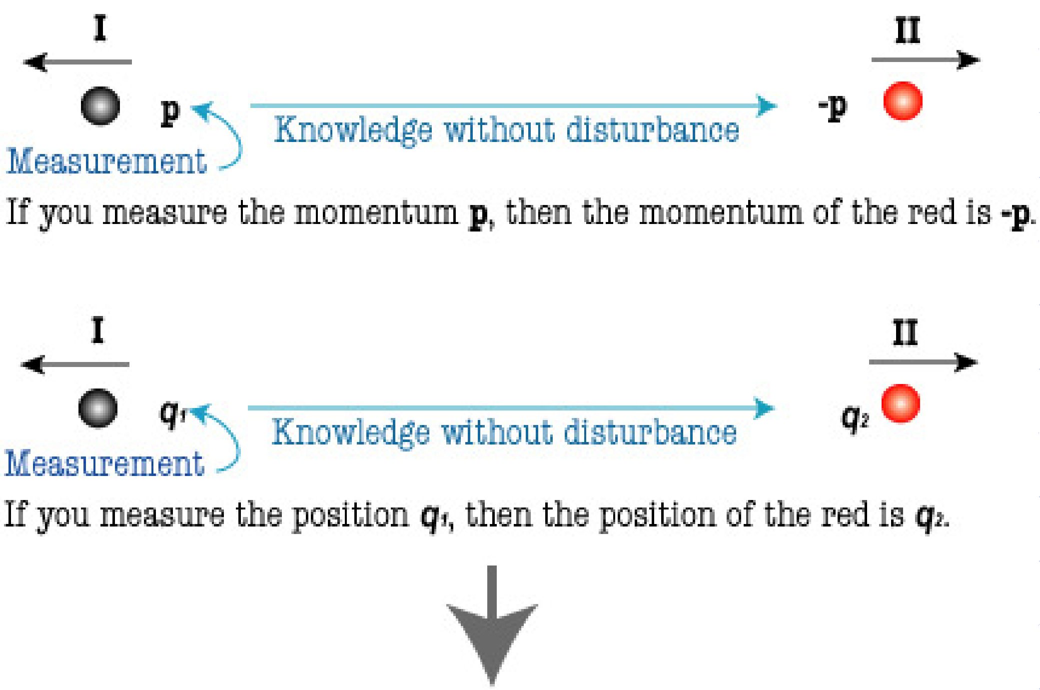
If you measure the momentum **p**, then the momentum of the red is **-p**. Since the momentum of the red was measured without disturbing it, that quantity must be regarded as **real**.

If you measure the position q_1, then the position of the red is q_2. Since the position of the red was measured without disturbing it, that quantity must be regarded as **real**.

Since both the momentum and the position of the red can be known without disturbing the red itself, both quantities must be regarded as real. BUT THE QUANTUM MECHANICS IMPLIES THAT THE TWO CANNOT BE REAL AT THE SAME TIME. This shows that something is wrong with the quantum mechanics.

EPR Paradox

Bohr Meets Einstein-Podolsky-Rosen



Bohr's reply: The experimental arrangement is essential in each case, and the measurement of momentum excludes the measurement of position, and vice versa. Although I and II do not physically interact, they are nevertheless correlated because of the experimental arrangement. Since this correlation is inherent in the description of "physical reality", the attribution of "reality" to each subsystem is unfounded.

EPR Paradox

- Bohr said position and momentum cannot be measured simultaneously
- EPR agreed but said by their definition, both the position and momentum of particle 2 are real (known precisely)
- Bohr said the wave functions of particle 1 and 2 are inseparable even though the particles are separated
- This means however that QM is non-local (spooky action-at-a distance) since the measurement of particle 1 is contributing to the physical reality of particle 2