Heisenberg Uncertainty

- Heisenberg uncertainty principle

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- This means you can't simultaneously measure x and p_x to an arbitrarily small precision
- This also means you can't use classical physics because you can't specify (exactly) the initial conditions

Waves

The relations

$$\Delta x \Delta k > \frac{1}{2} \text{ and } \Delta \omega \Delta t > \frac{1}{2}$$

are general properties of Fourier transforms

Quantum mechanics enters the picture because we associate de Broglie waves with material particles via $p = \hbar k$ and $E = \hbar \omega$

Heisenberg Uncertainty

- Is this a significant?
  - No, if you are macroscopic
  - Yes, if you are an electron
- Note this refers to simultaneous measurements of p_x and x

Consider a 50 g bullet and an electron each with velocity 300m/s $\pm$ 0.03m/s.

What is the minimum position uncertainty?

For the bullet

$$\Delta x = \frac{\hbar}{2}\frac{1}{\Delta p} = \frac{\hbar}{2}\frac{1}{m\Delta v} = \frac{1.0546\times 10^{-34}\,Js}{(2)(0.05kg)(0.03m/s)}$$

$$\Delta x = 3.5\times 10^{-32}\,m$$

For the electron

$$\Delta x = \frac{\hbar}{2}\frac{1}{\Delta p} = \frac{\hbar}{2}\frac{1}{m\Delta v} = \frac{1.0546\times 10^{-34}\,Js}{(2)(9.1\times 10^{-31}kg)(0.03m/s)}$$

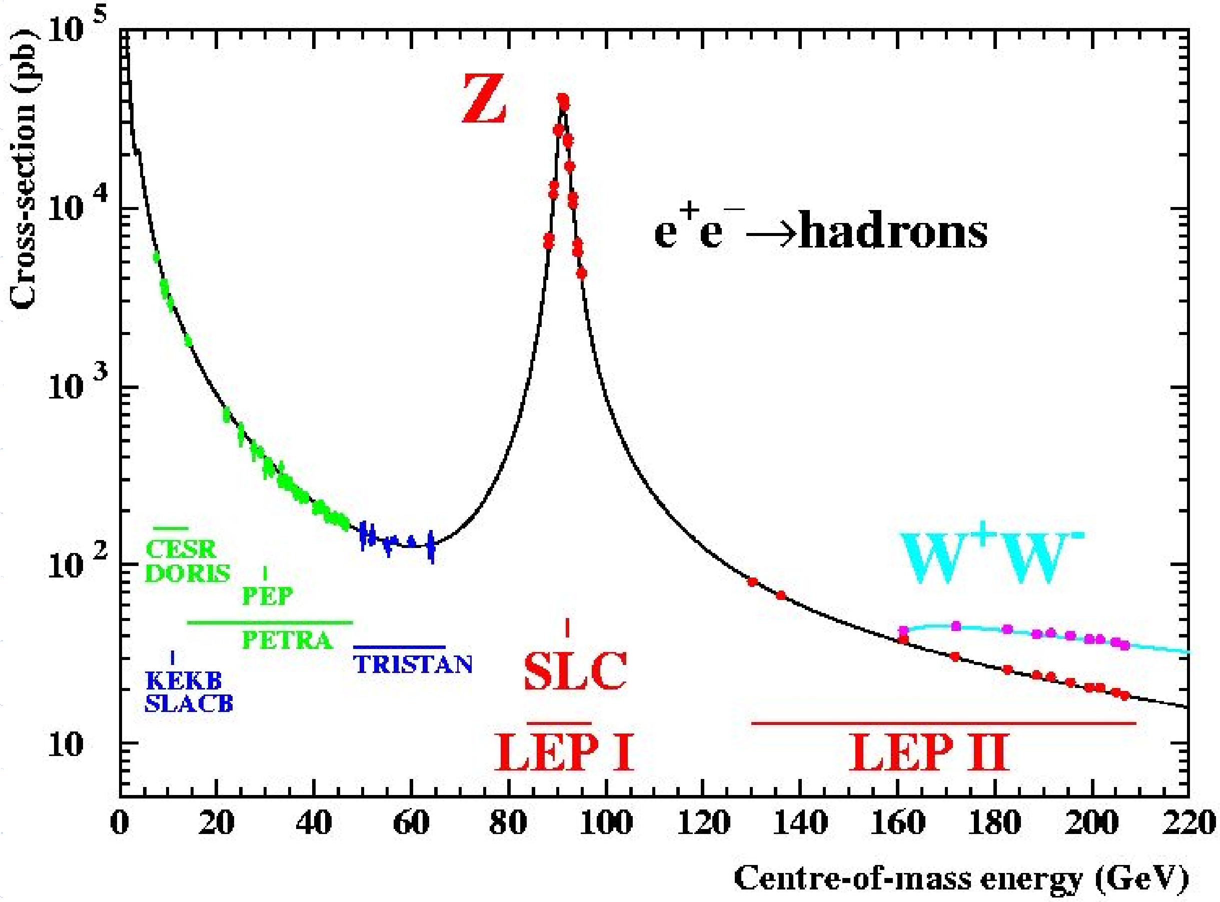
$$\Delta x = 1.9mm$$

Heisenberg Uncertainty

- Consequences of the energy-time uncertainty relation are
  - A particle that decays does not have a well-defined mass
  - Atomic transitions do not have a well-defined energy
    - This is called their natural line width

Heisenberg Uncertainty

- Mass of the Z-boson particle



Heisenberg Uncertainty

Consider an atomic de-excitation that occurs

in 10^{-8}s emitting radiation with $\lambda = 300\text{nm}$

$$\Delta E = h\Delta f = \frac{hc\Delta\lambda}{\lambda^2} = \frac{\hbar}{2}\frac{1}{\Delta t}$$

$$\Delta\lambda = \frac{\hbar\lambda^2}{2hc\Delta t} = \frac{\lambda^2}{4\pi c\Delta t} = 2.4\times10^{-6}\,nm$$

$$\frac{\Delta\lambda}{\lambda} = 7.9\times10^{-9}$$

this width is small but measurable

Heisenberg Uncertainty

- A useful relation is

$$\overline{(\Delta x)^2} = \overline{(x-\bar{x})^2} = \overline{\left(x^2 - 2x\bar{x} + \bar{x}^2\right)} = \overline{x^2} - \bar{x}^2$$

- Sometimes the mean (x-bar) is 0 by symmetry

$$\overline{(\Delta x)^2} = \overline{x^2}$$

- Hence in some cases the uncertainty in Δx can be used to measure the mean of x^2
- Similar arguments apply to Δp, ΔE, and Δt

Heisenberg Uncertainty

- Use the uncertainty principle to estimate the kinetic energy of an electron localized in a hydrogen atom

$$\Delta x \Delta p = \frac{\hbar}{2}$$

$$\Delta p = \frac{\hbar}{2\Delta x} \approx \frac{\hbar}{2a_0}$$

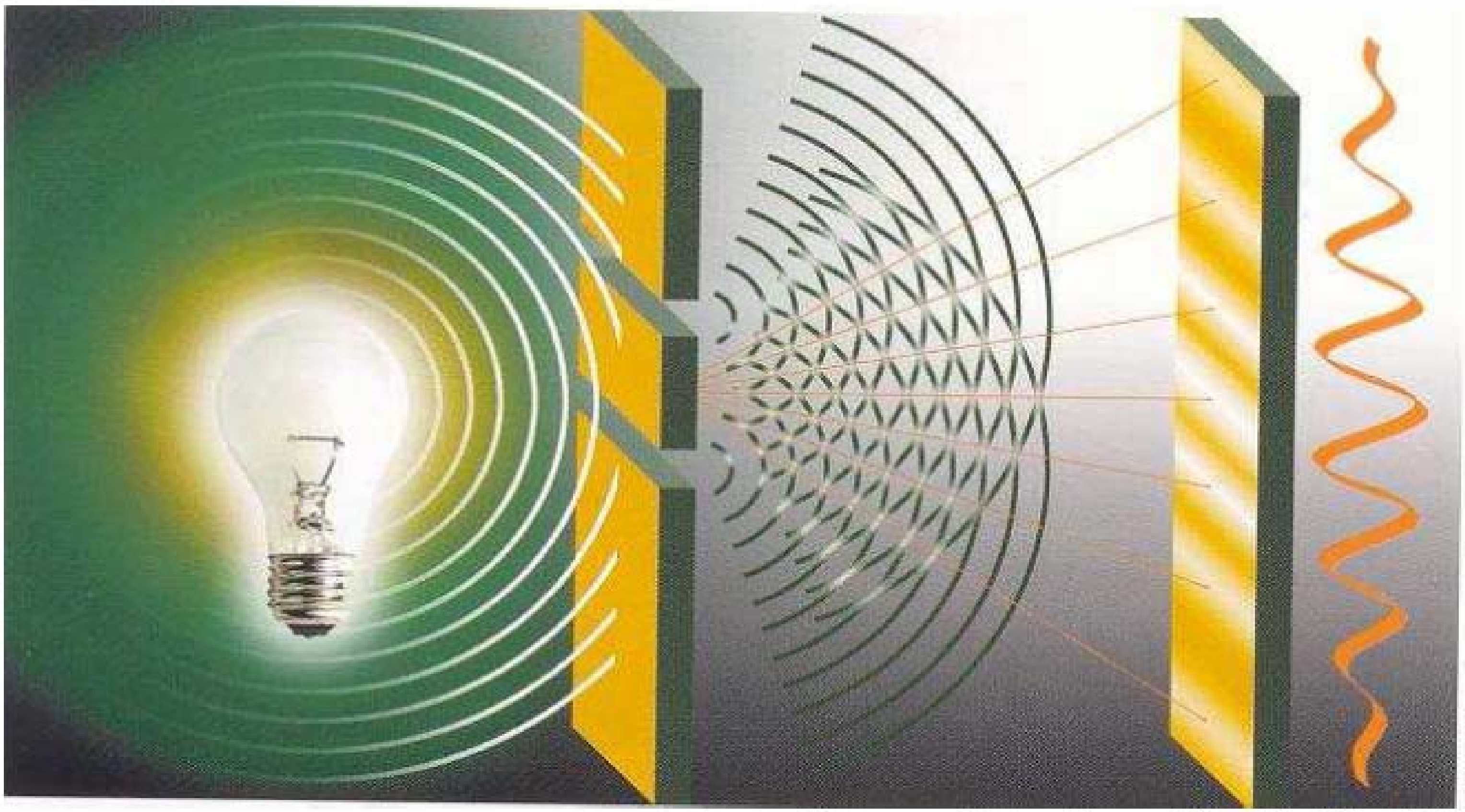
$$T = \frac{p^2}{2m} = \frac{\overline{p^2}}{2m} = \frac{(\Delta p)^2}{2m} = \frac{(\hbar c)^2}{8mc^2 a_0^2}$$

$$T = \frac{(197.3eVnm)^2}{(8)(0.511\times 10^6 eV)(.053nm)^2} = 3.4eV$$

Same order of magnitude as T in the Bohr model

Recall $T = -E = 13.6eV$

Double Slit Experiment



From: Quantum (J. Al-Khalili)

Double Slit Experiment

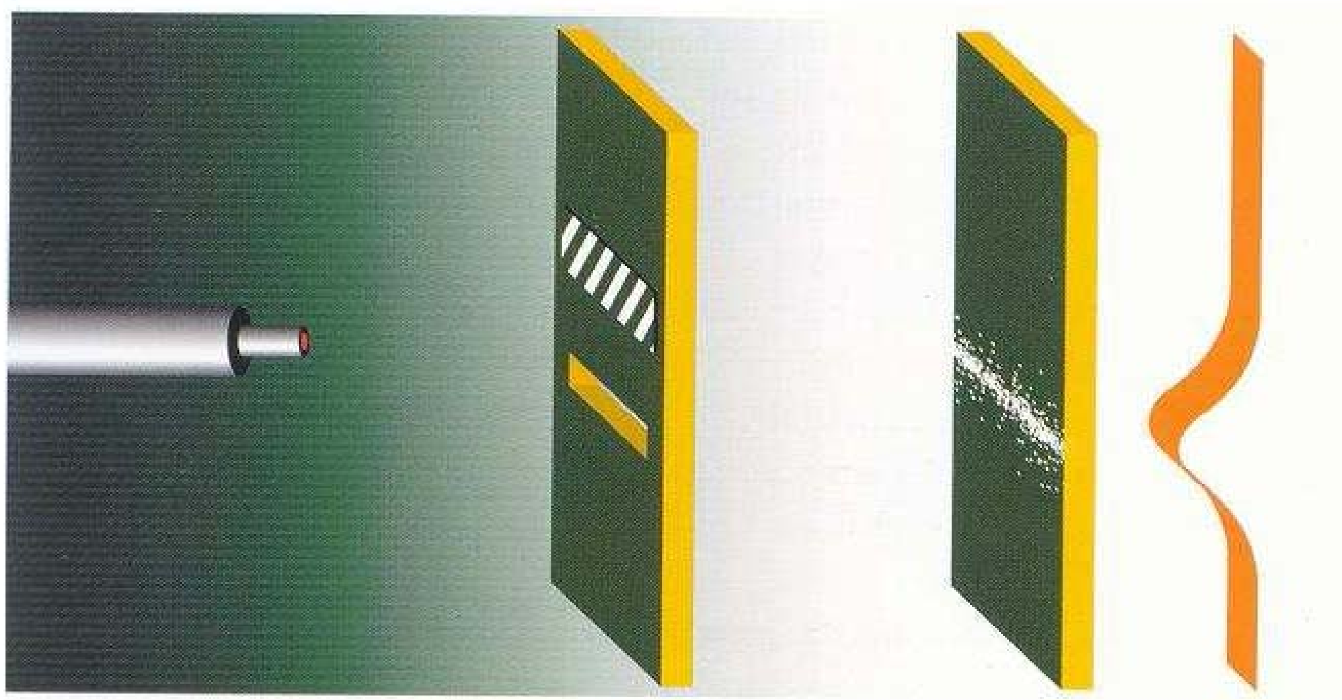
- What do we actually observe on the screen?
  - The light intensity $I(x)$
  - Clearly $I(x) \neq I_1(x) + I_2(x)$
  - $I(x) \sim |E(x)|^2 = |E_1(x)+E_2(x)|^2$
  - $I(x) \sim |E_1|^2 + |E_1|^2 + 2\mathrm{Re}|E_1{}^{*}E_2|$
- The interference term, which depends on the phase difference between E_1 and E_2, produces the interference pattern

Double Slit Experiment

- What happens if we close one or the other slits?
- What happens if we send one photon at a time towards the two slits?
- What happens if we monitor which slit the single photon entered?
- What happens if we use electrons instead of photons?

Double Slit Experiment

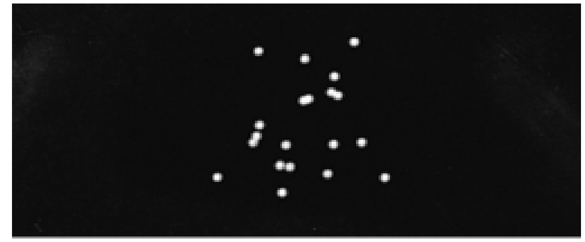
- What happens if we close one of the slits?



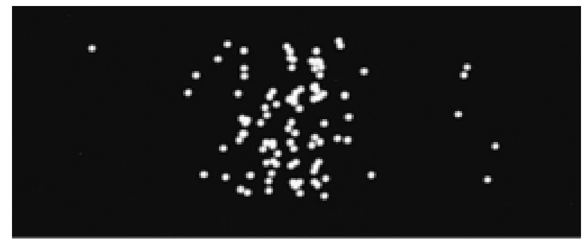
- No interference pattern. Just the diffraction pattern from the single open slit.

Double Slit Experiment

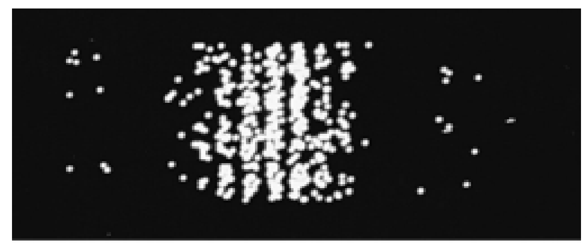
- What happens if we send one photon at a time towards the two slits?
- We see individual “hits” corresponding to each photon
- But as the photons arrive one by one over time, they build up an interference pattern



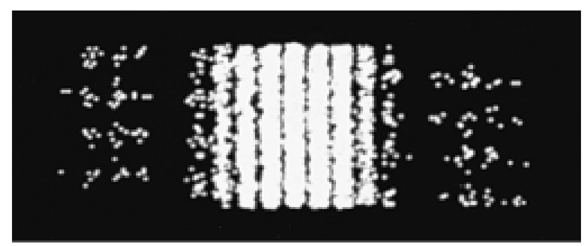
(a) 20 counts



(b) 100 counts



(c) 500 counts



(d) ~4000 counts

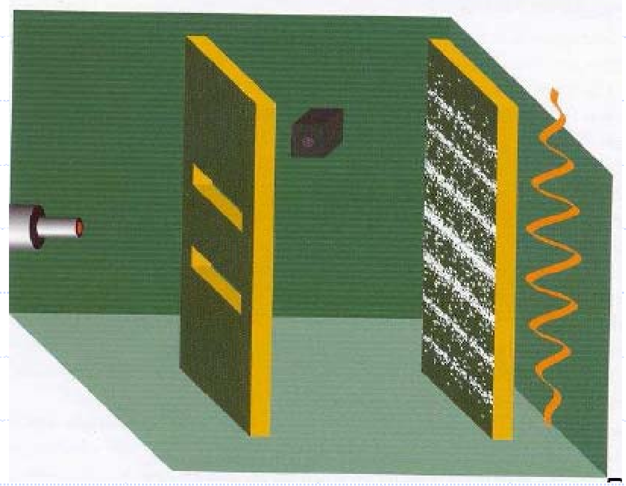
Double Slit Experiment

- What happens if we monitor which slit the single photon entered?

Detector on

No interference pattern

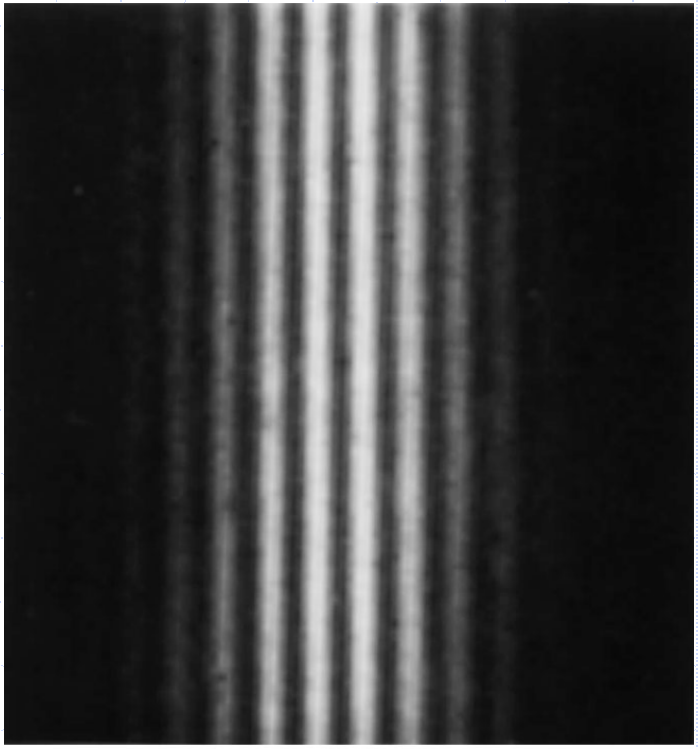
Detector off



Interference pattern

Double Slit Experiment

- What happens if we use electrons?



- No difference in any of the preceding discussion

Double Slit Experiment

- There is a lot going on here!
- Consider the example where the photon or electron is measured in order to determine through which slit it passed
  - If the photon through slit 1 is detected with a photodetector it is removed (and equivalent to blocking slit 1)
  - If an electron through slit 1 is measured using light (e.g. Compton scattering), again the interference pattern vanishes

Double Slit Experiment

- The act of observing the electron has changed the experiment

$$p_{electron} \approx \frac{h}{\lambda_{electron}} \sim \frac{h}{d}; \text{ where } d \text{ is the slit separation}$$

$$p_{photon} \approx \frac{h}{\lambda_{photon}} > \frac{h}{d}$$

- Thus the photon momentum is at least as large as that of the electron and will change the direction of the electron (destroying the interference pattern)

Double Slit Experiment

- We can also interpret this in terms of the Heisenberg uncertainty principle

To locate the electron's position we need

$$\Delta y < \frac{d}{2}$$

An uncertainty arises in the electron's momentum because of scattering with the photon. The uncertainty is

$$\Delta p_y > \frac{\hbar}{d}$$

This gives rise to an uncertainty in the electron's angle

$$\Delta\theta \approx \frac{\Delta \mathrm{p_y}}{\mathrm{p}} = \frac{(\Delta \mathrm{p_y})\lambda}{h} > \frac{\hbar\lambda}{hd} = \frac{\lambda}{2\pi d}$$

Interference maxima and minima are

$$\sin\theta_{\max} \approx \theta = \frac{\lambda}{d} \text{ and } \theta_{\min} = \frac{\lambda}{2\mathrm{d}}$$

Since $\Delta\theta \sim \theta_{\min}$ the inteference pattern is washed out

Double Slit Experiment

- When we treat the electron as a wave we observe an interference pattern (in the double slit experiment)
- When we treat the electron as a particle (in trying to localize it) we do not observe an interference pattern
- This is an example of Bohr's principle of complementarity
  - Wave and particle models are complementary. If a measurement proves the wave character of radiation or matter then it is impossible to prove the particle character in the same measurement. And vice-versa.

Double Slit Experiment

- Now consider the example where one photon passes through the slit at a time
  - Light acts as a particle since there is one "hit" on the screen for each particle
  - Light acts as a wave because if we accumulate hits the interference pattern appears
- Both particle and wave aspects are needed to explain the experiment
  - Neither particle nor wave theory can explain the observation alone
- This is called particle-wave duality

Double Slit Experiment

- Einstein linked the wave and particle aspects of the photon by equating the square of the electric field strength (averaged over one cycle) with the radiant energy in a unit volume

$$I = \frac{1}{\mu_0 c}\overline{E^2} = Nhf$$

where

I is the intensity of radiation

E is the electric field strength

N is the average number of photons per unit time crossing a unit area perpendicular to the direction of propagation

Thus $\overline{E^2}$ (amplitude squared) is a probability measure of the photon density

Double Slit Experiment

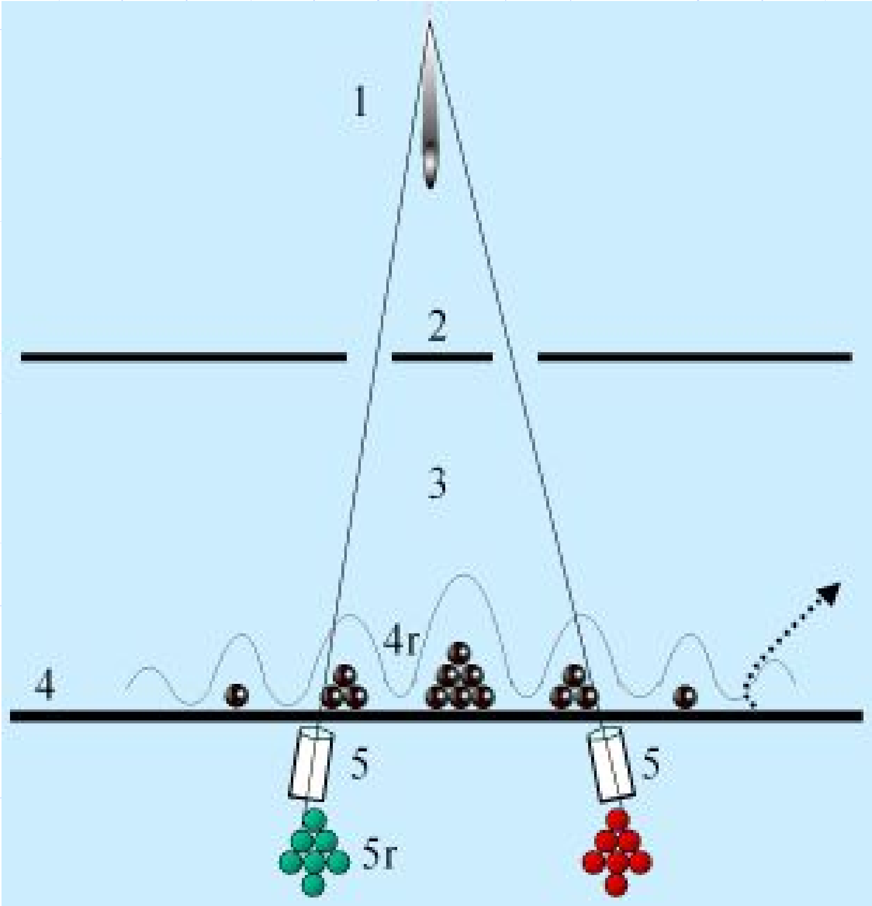
- The single photon experiment contains another disturbing aspect
  - We see that each photon must be considered separately (since each photon obviously passed through one slit and produced a "hit")
  - But how does the photon know where to go to produce an interference pattern (which is observed only after a sufficiently large number of hits)?
  - And how does the photon know whether the other slit is open or closed?
  - For a photon passing through one of the slits why should the state of the other slit matter at all?
- Welcome to quantum mechanics

Double Slit Experiment

- The road out of this paradox is to realize
  - We cannot know through which slit the photon passed without destroying the interference pattern
    - There is nothing preventing us from saying the photon went through both slits and interfered with itself
  - We cannot know exactly where the photon is going; its direction is probabilistic and the probability is proportional to $|E(x)|^2 = I(x)$

Double Slit Experiment

- Consider the “delayed choice” double slit experiment (Wheeler)



Double Slit Experiment

- After the photon has passed the slits (region 3) we decide to use the screen or not
  - If we decide to use the screen we observe an interference problem and the photon passed through both slits
  - If we decide to not use the screen we observe a hit in one or the other telescopes and the photon passed through one or the other slits
- An equivalent experiment was carried out in the lab (1987) and gives the above results

Double Slit Experiment

- In the delayed choice experiment the photon seems to have responded instantly to our choice
  - Einstein commented quantum mechanics contains "spooky action-at-a-distance" phenomena
- In the delayed choice experiment our observation (choice) brings about the results that have occurred and we have apparently determined what happened in the past
- From Bohr through Johns the advice given is
  - Don't think, calculate