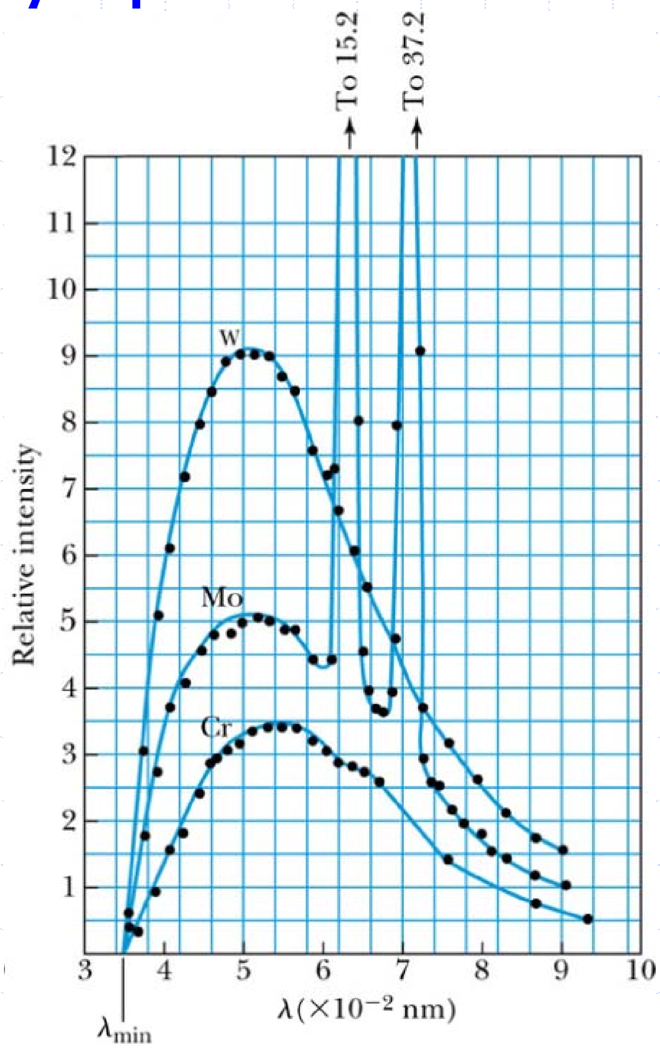
Bohr Model

- In addition to the atomic line spectra of single electron atoms, there were other successes of the Bohr Model
  - X-ray spectra
  - Frank-Hertz experiment

X-ray Spectra

- Recall the x-ray spectra shown a few lectures ago



X-ray Spectra

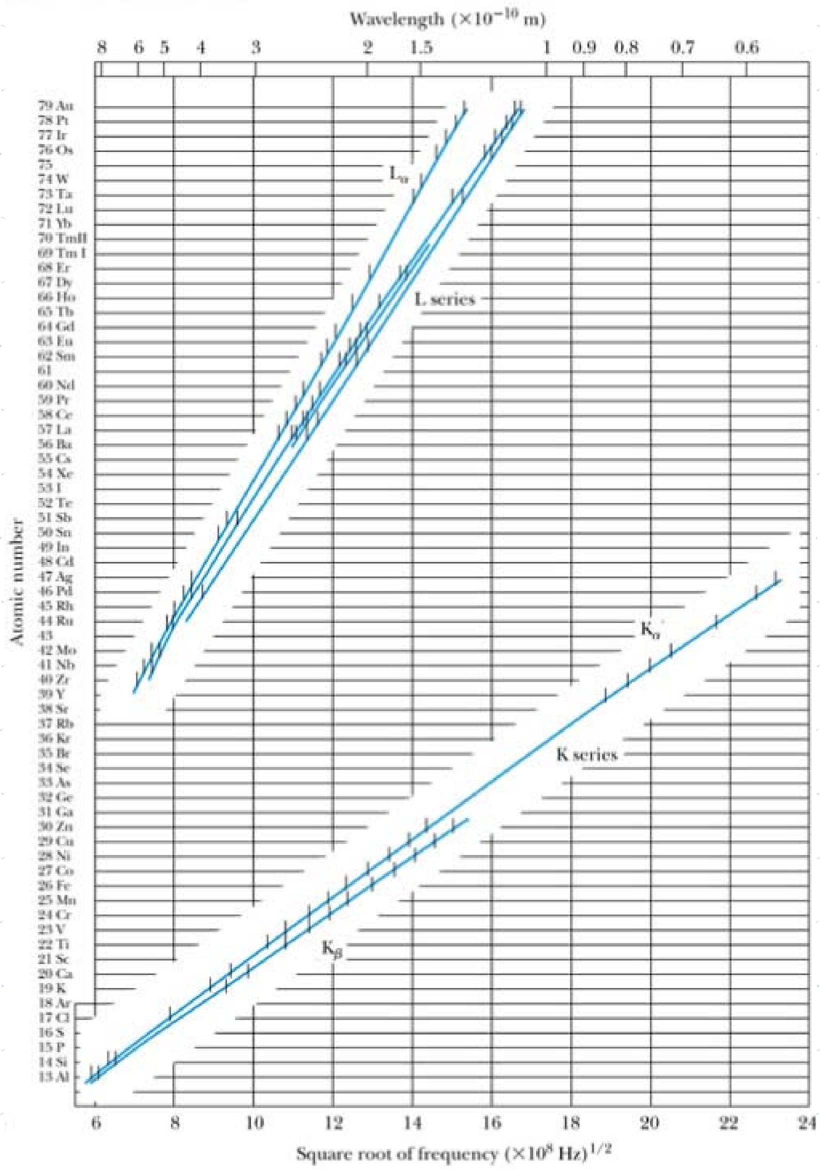
- Moseley found experimentally that the wavelengths of characteristic x-ray lines of elements followed a regular pattern

$$f_{K_\alpha} = \frac{3cR}{4}(Z-1)^2$$

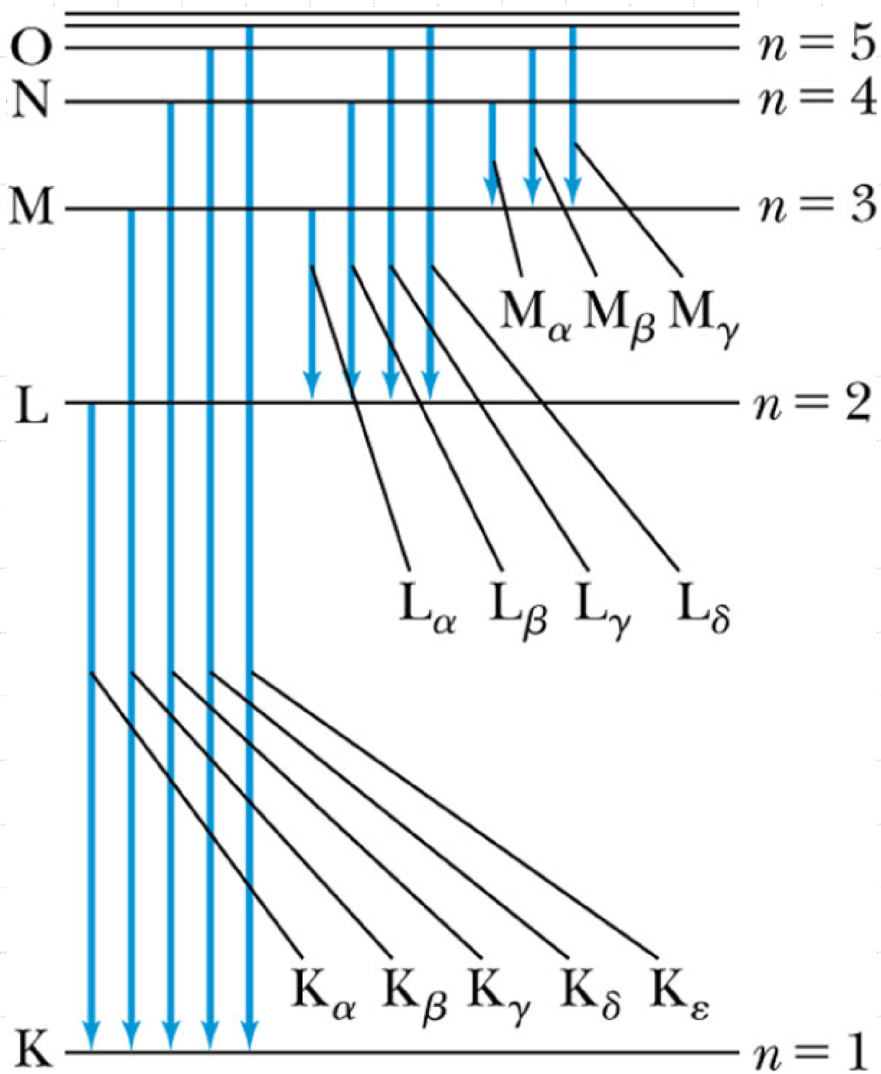
- A similar formula described the L-series x-rays

$$f_{L_\alpha} = \frac{5cR}{36}(Z-7.4)^2$$

X-ray Spectra



X-ray Spectra



X-ray Spectra

- Moseley's law can be easily understood in terms of the Bohr model
  - If a K (n=1) shell electron is ejected, an electron in the L (n=2) shell will feel an effective charge of Z-1 (Z from the nucleus – 1 electron remaining in the K shell)
  - We have then for the n=2 to n=1 transition

$$\frac{1}{\lambda} = (Z-1)^2 R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$f_{K_\alpha} = \frac{c}{\lambda} = \frac{3cR}{4}(Z-1)^2$$

  - Exactly agreeing with Moseley's law

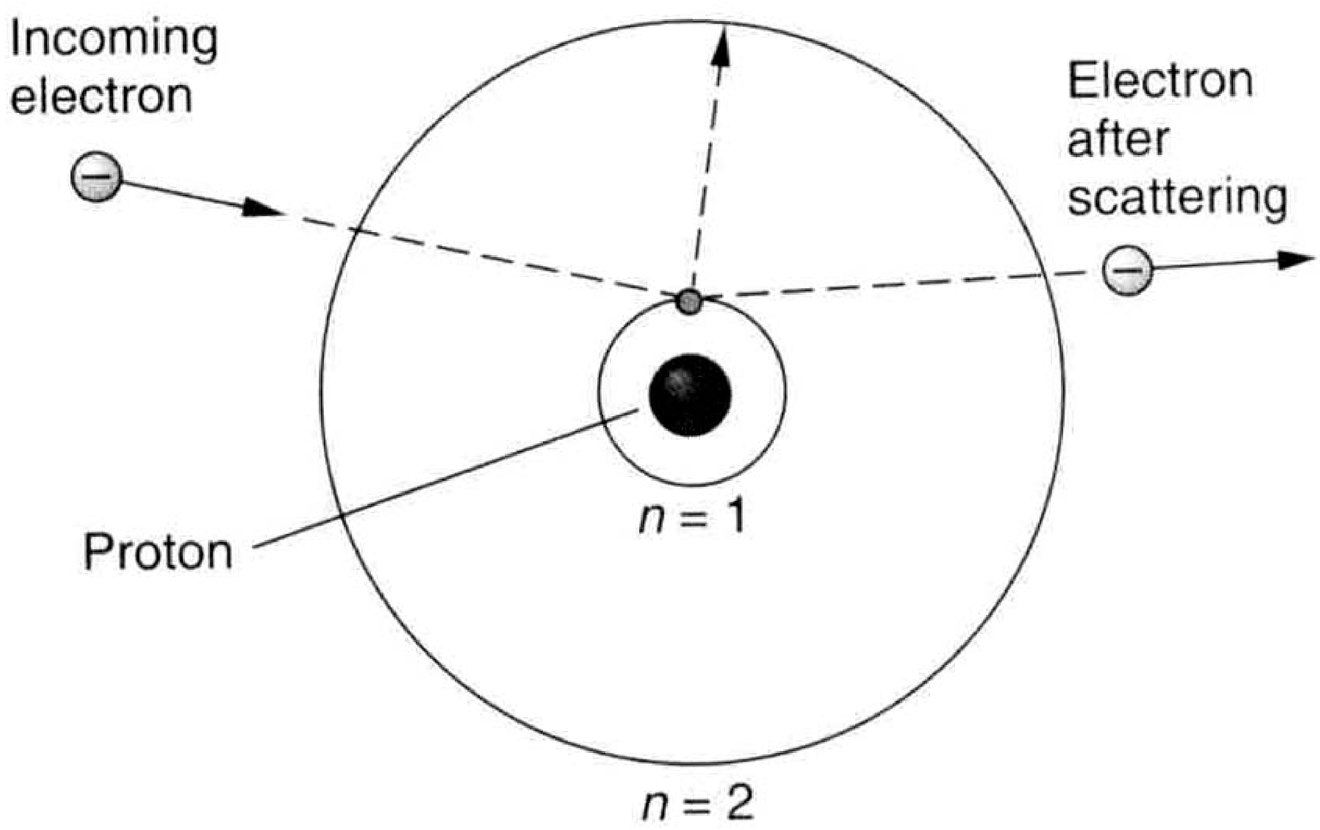
X-ray Spectra

- Applying the Bohr model to the L-series

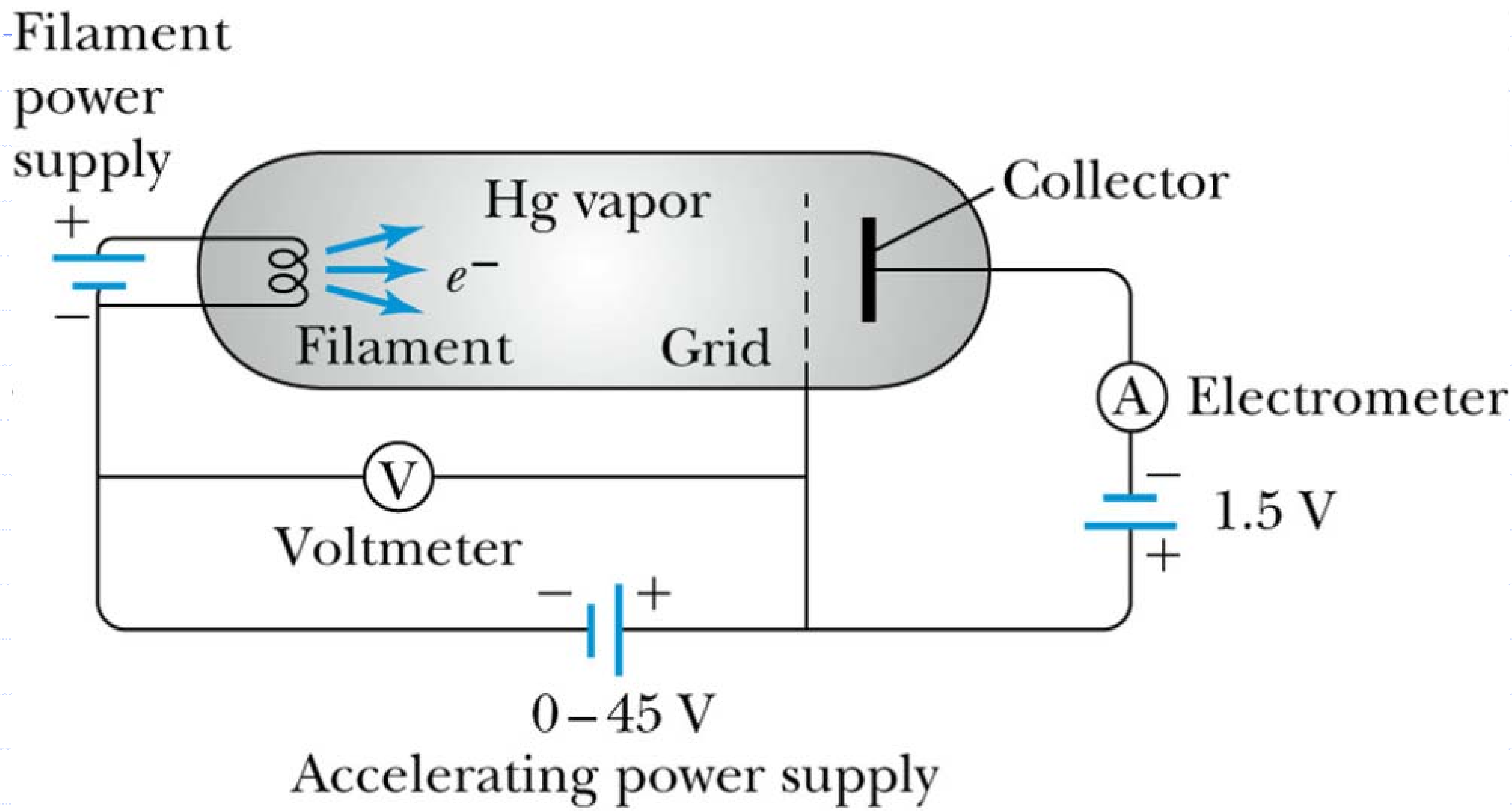
$$f_{L_\alpha} = cRZ_{eff}^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{5cR}{36}Z_{eff}^2$$

- Now there are two electrons in the K shell and several in the L shell thus we might expect a $(Z\text{-}2\text{-several})^2$ dependence
  - The data show $Z_{eff} = (Z - 7.4)$
- Aside, based on the regular patterns in his data he showed
  - The periodic table should be ordered by Z not A
  - Elements with Z=43,61, and 75 were missing

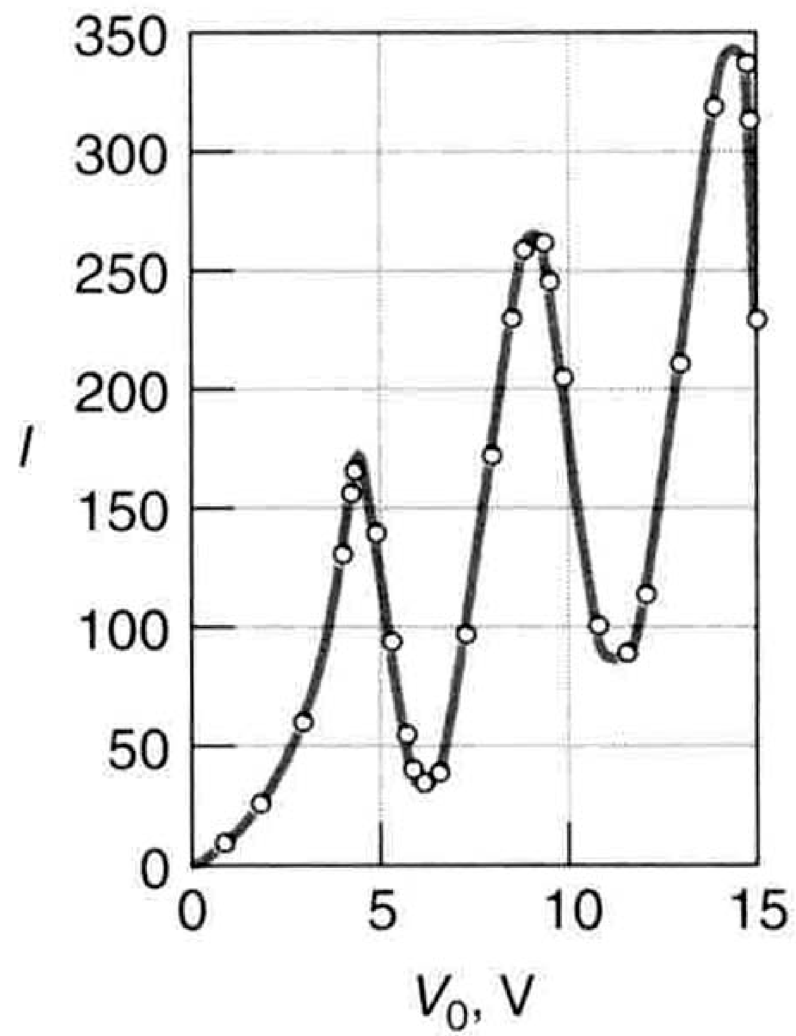
Franck-Hertz Experiment



Franck-Hertz Experiment



Franck-Hertz Experiment



Franck-Hertz Experiment

- The data show
  - The current increases with increasing voltage up to V=4.9V followed by a sudden drop in current
    - This is interpreted as a significant fraction of electrons with this energy exciting the Hg atoms and hence losing their kinetic energy
    - We would expect to see a spectral line associated with de-excitation of

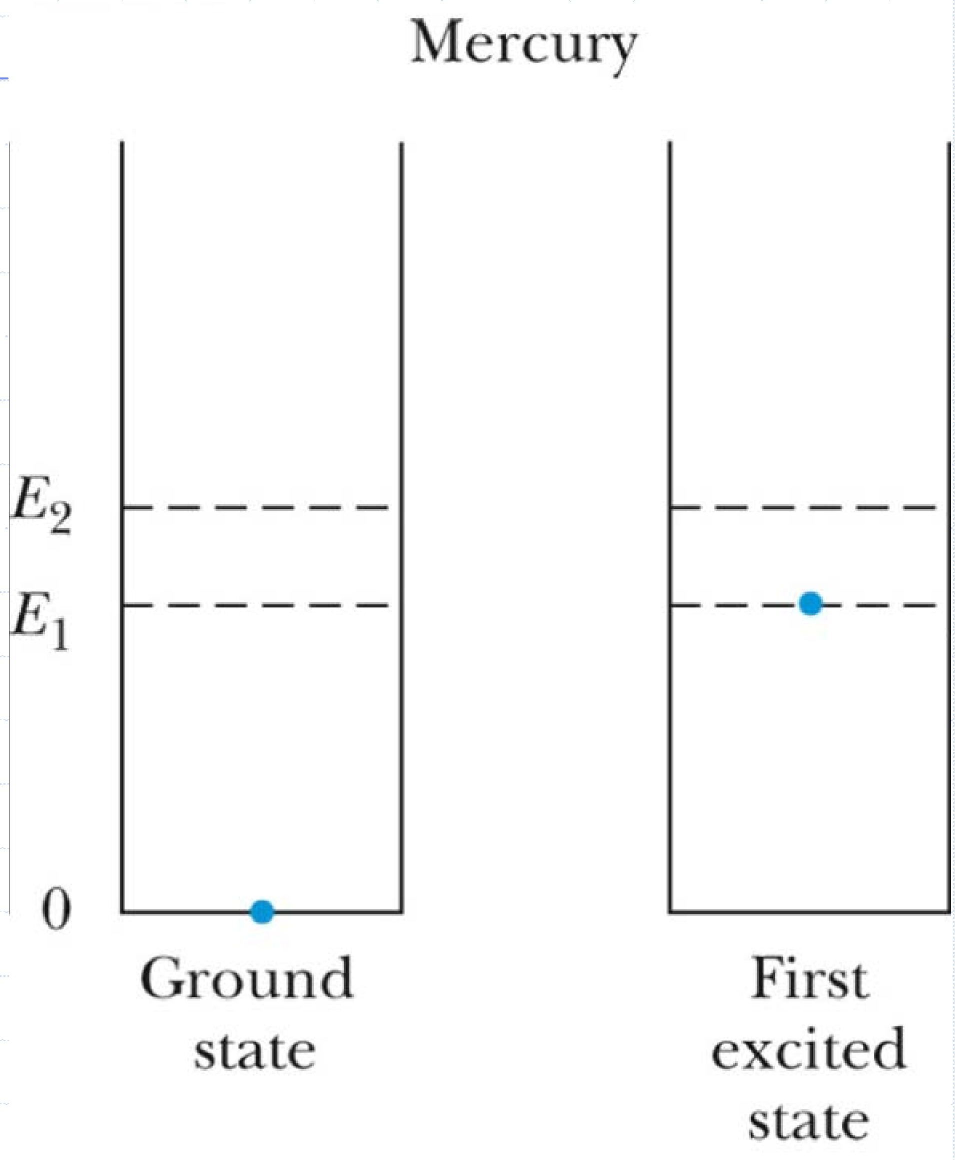
$$\lambda = \frac{hc}{eV} = \frac{1.24 \times 10^4 eV - A}{4.9 eV} = 253 nm$$

Franck-Hertz Experiment

- As the voltage is further increased there is again an increase in current up to V=9.8V followed by a sharp decrease
  - This is interpreted as the electron possessing enough kinetic energy to generate two successive excitations from the Hg ground state to the first excited state
  - Excitations from the ground state to the second excited state are possible but less probable

➢ The observation of discrete energy levels was an important confirmation of the Bohr model

Frank-Hertz Experiment



Correspondence Principle

- There were difficulties in reconciling the new physics in the Bohr model and classical physics
  - When does an accelerated charge radiate?
- Bohr developed a principle to try to bridge the gap
  - The predictions of quantum theory must agree with the predictions of classical physics in the limit where the quantum numbers n become large
  - A selection rule holds true over the entire range of quantum numbers n (both small and large n)

Correspondence Principle

- Consider the frequency of emitted radiation by atomic electrons
  - Classical

$$f_{classical} = \frac{\omega}{2\pi} = \frac{V}{2\pi r} = \frac{1}{2\pi}\left(\frac{ke^2}{mr^3}\right)^{1/2}$$

$$\text{using } r_n = n^2 a_0 = \frac{n^2\hbar^2}{kme^2}$$

$$\text{we find } f_{classical} = \frac{me^4}{4\varepsilon_0^2 h^3}\frac{1}{n^2}$$

Correspondence Principle

- Quantum

$$f_{Bohr} = \frac{E_0}{h}\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$f_{Bohr} = \frac{E_0}{h}\left(\frac{2n+1}{n^2(n+1)^2}\right)$$

which for n large becomes

$$f_{Bohr} \approx \frac{2nE_0}{hn^4} = \frac{2E_0}{hn^3}$$

substituting for $E_0 = \dfrac{me^4}{2\hbar^2(4\pi\varepsilon_0)^2}$

$$f_{Bohr} = \frac{me^4}{4\varepsilon_0^2 h^3}\frac{1}{n^3} = f_{classical}$$

Wilson-Sommerfeld Quantization

- Quantization appears to play an important role in this "new physics"
  - Planck, Einstein invoked energy quantization
  - Bohr invoked angular momentum quantization
- Wilson and Sommerfeld developed a general rule for the quantization of periodic systems

Wilson-Sommerfeld Quantization

$$\oint P dq = nh$$

where P is some component of momentum

and q is the corresponding coordinate

and $\oint$ means integrate over one cycle

Wilson-Sommerfeld Quantization

- For a particle moving in a central field (like the Coulomb field)
  - P = L
  - q = φ
- Then

$$\oint P dq = \oint L d\phi = nh$$

$$L \oint d\phi = nh$$

$$L = \frac{nh}{2\pi} = n\hbar$$

- Which is just the Bohr condition

Wilson-Sommerfeld Quantization

- For a particle undergoing simple harmonic motion
  - $P = p_x$
  - $q = x$

$$\text{Newton's law } m\frac{d^2x}{dt^2} = -kx$$

$$\text{has solution } x = A\sin\omega t$$

$$\text{then } dx = \omega A\cos\omega t\,dt$$

$$\text{and } p_x = m\frac{dx}{dt} = m\omega A\cos\omega t$$

Wilson-Sommerfeld Quantization

- Continuing on

$$\oint P dq = \oint p_x dx = \oint m\omega^2 A^2 \cos^2 \omega t dt = nh$$

Recalling from a simple harmonic oscillator

$$E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2$$

$$2E\oint \cos^2 \omega t dt = nh$$

$$\frac{2E}{\omega}\int_0^{2\pi} \cos^2 \theta d\theta = \frac{2E}{\omega}\pi = nh$$

$$E = \frac{nh\omega}{2\pi} = nhf$$

- Which is just the Planck condition