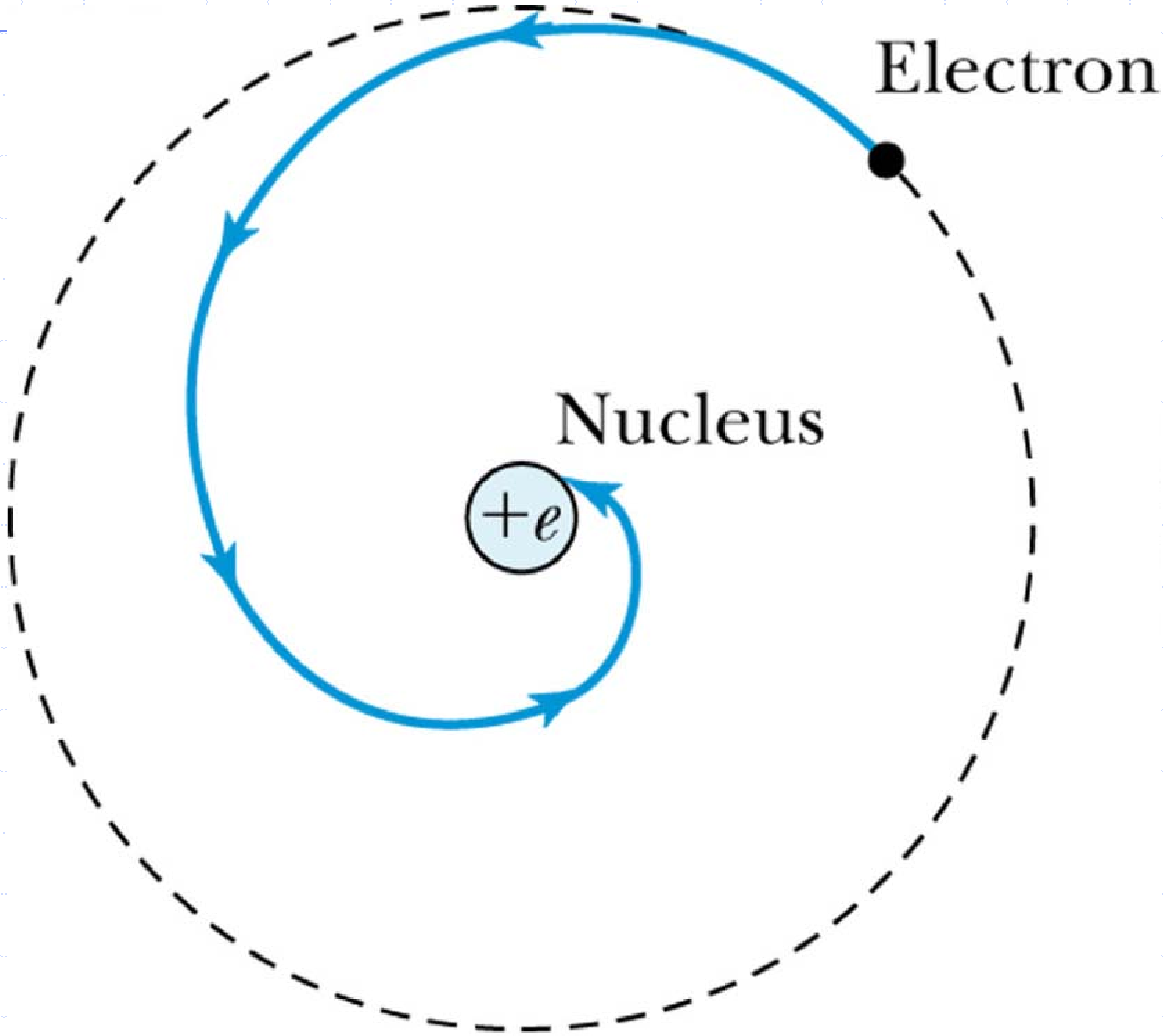
Bohr Model

- Rutherford established the atomic nucleus as a positive charge of radius ~ 1F
- At the same time, the radius of an atom was known to be ~ 10^{-10}m
- A natural model for the atom is a planetary model
- However electrons orbiting the nucleus are accelerating and hence radiate energy as electromagnetic waves

Bohr Model



Bohr Model

- Postulates of the Bohr model
  - An electron moves in a circular orbit about the nucleus acted on only by the Coulomb force and following classical mechanics
  - The only orbits possible are those whose angular momentum

$$L = \frac{nh}{2\pi} = n\hbar$$

  - The orbiting electron does not emit electromagnetic radiation
  - Electromagnetic radiation is emitted if an electron moves from one orbit to another. The frequency of the radiation is

$$f = \frac{(E_i - E_f)}{h}$$

Bohr Model

- Thus the Bohr model is a mixture of both classical and non-classical physics
- It described known atomic spectra precisely
- It had predictive power
- The math was easy

Bohr Model

- Assume initially that the nucleus is infinitely heavy
- Since the orbit of the electron is stable

$$k\frac{Ze^2}{r^2} = \frac{mV^2}{r}$$

- The orbital angular momentum L is a constant

$$L = mVr = n\hbar, n = 1,2,3...$$

$$V = \frac{n\hbar}{mr}$$

Bohr Model

- Then

$$kZe^2 = mrV^2 = \frac{mrn^2\hbar^2}{m^2r^2} = \frac{n^2\hbar^2}{mr}$$

- Thus

$$r = \frac{n^2\hbar^2}{kmZe^2} = \frac{n^2 a_0}{Z}$$

where the first Bohr radius $a_0 \equiv \frac{4\pi\varepsilon_0\hbar^2}{me^2} = 0.53\times10^{-10}m$

$$V = \frac{n\hbar}{mr}$$

Bohr Model

- Quantization of angular momentum restricts the possible orbits of the electron
  - For Z=1 and n=1 we find $r=0.53 \times 10^{-10}$m, in good agreement with the known atomic radius
- Evaluating the orbital velocity
  - For Z=1 and n=1, we find $v=2.2 \times 10^{6}$m/s
  - Relativistic mechanics is not needed (for small Z)

Bohr Model

- The total energy of the electron E = T+V

$$V = k\int_{\infty}^{r}\frac{Ze^2}{r^2}dr = -k\frac{Ze^2}{r}$$

$$T = \frac{1}{2}mv^2 = k\frac{Ze^2}{2r}$$

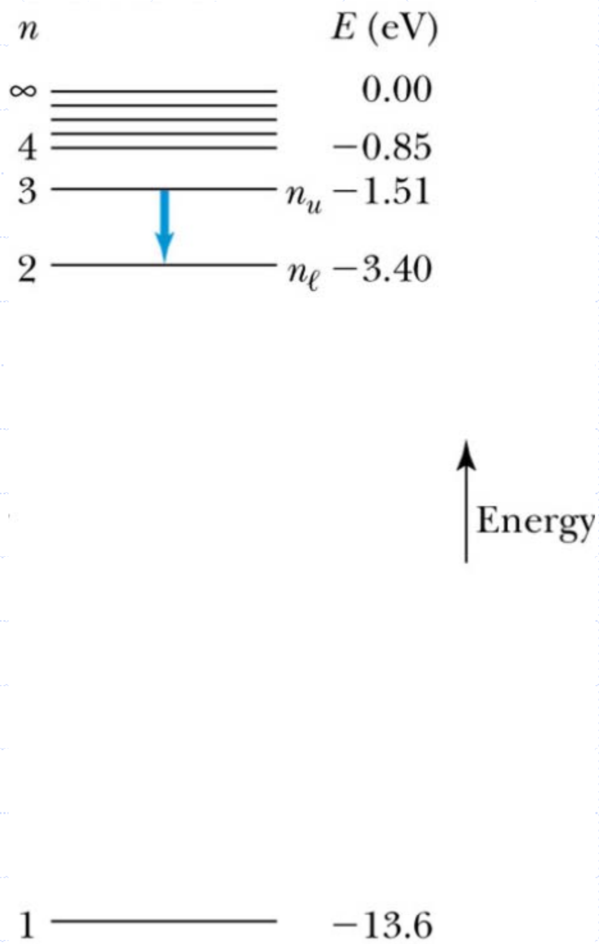
$$E = T + V = -k\frac{Ze^2}{2r} = -T$$

$$E_n = -\frac{Z^2e^2}{8\pi\varepsilon_0 a_0 n^2} \equiv -\frac{Z^2E_0}{n^2}$$

$$\text{where } E_0 = \frac{e^2}{8\pi\varepsilon_0 a_0} = \frac{me^4}{2\hbar^2(4\pi\varepsilon_0)^2} = 13.6eV$$

Bohr Model

- Energy levels in hydrogen



Bohr Model

- According to Bohr's postulate, emission of electromagnetic radiation occurs when an electron in a higher energy state E_i decays to a lower energy state E_f
- The frequency and wavelength of the emitted radiation is

$$hf = E_i - E_f$$

$$\frac{1}{\lambda} = \frac{E_i - E_f}{hc} = \frac{Z^2 E_0}{hc}\left(\frac{1}{n_f^2 - n_i^2}\right) \equiv Z^2 R_\infty \left(\frac{1}{n_f^2 - n_i^2}\right)$$

Line Spectra

Hydrogen Series of Spectral Lines

Discoverer (year)	Wavelength	n	k
Lyman (1916)	Ultraviolet	1	>1
Balmer (1885)	Visible, ultraviolet	2	>2
Paschen (1908)	Infrared	3	>3
Brackett (1922)	Infrared	4	>4
Pfund (1924)	Infrared	5	>5

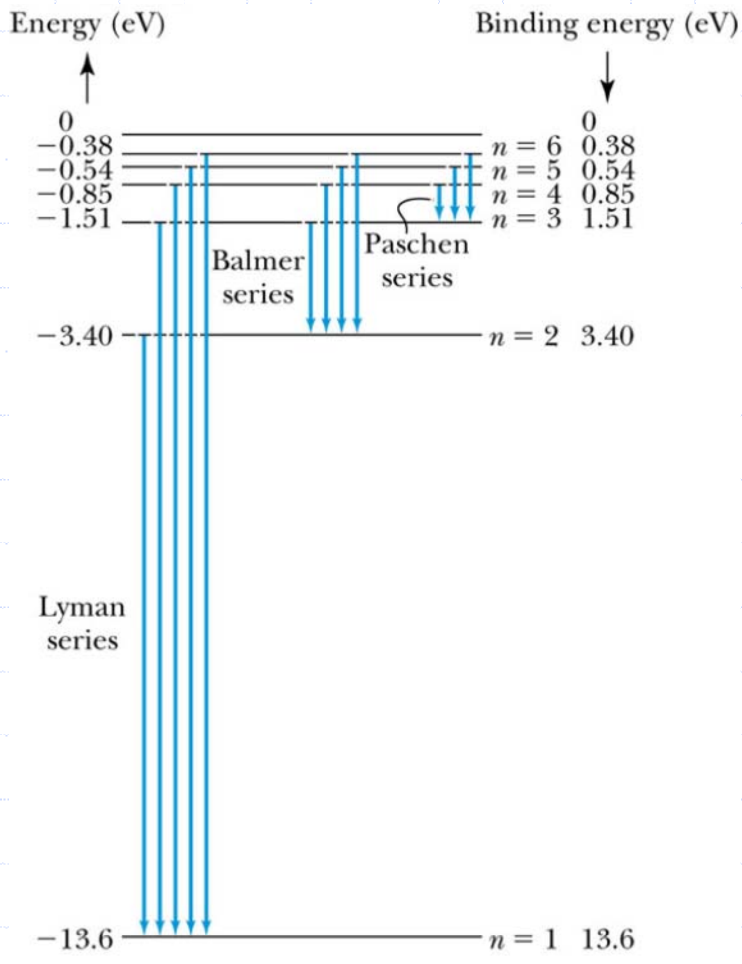
$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{k^2}\right), \quad R_H = 1.096776 m^{-1}$$

Bohr Model

- Thus the Bohr model correctly gives the wavelengths of (hydrogen) emission and absorption spectra
- Bohr's theory predicted the Lyman, Brackett and Pfund series before they were discovered
- It also predicted the spectra for He^+ before it was observed
- R_∞ depends only on fundamental constants (e, m_e, c, h-bar)
- R_∞ agrees well with R_H (1.097373 vs 1.096776 ($\times 10^7 m^{-1}$))

Bohr Model

- Radiative transitions in hydrogen



Bohr Model

- Suppose a He atom in the ground state absorbs a photon with λ=41.3nm. Will the He atom be ionized?

$$E = \frac{hc}{\lambda} = \frac{1240eV-nm}{41.3nm} = 30eV$$

$$E_1(He) = \frac{Z^2 E_0}{1^2} = (4)(13.6eV) = 54.4eV$$

Bohr Model

- We assumed the nucleus was infinitely heavy but in fact the electron and nucleus revolve around their mutual center-of-mass
  - If r is the distance between electron and nucleus

$$m(r-x) = Mx$$

$$x = \frac{mr}{m+M} \text{ and } r-x = \frac{Mr}{m+M}$$

  - The total angular momentum of the system is

$$L = m(r-x)^2\omega + Mr^2\omega$$

$$L = \frac{mM^2r^2\omega}{(m+M)^2} + \frac{Mm^2r^2\omega}{(m+M)^2}$$

$$L = \frac{mM}{m+M}r^2\omega$$

Bohr Model

- Summarizing

$$L = \frac{mM}{m+M} r^2 \omega$$

  - Check. As $M \to \infty$, $L \to mr^2\omega$ and $x \to 0$

- Following Bohr's postulate we let

$$L = \frac{mM}{m+M} r^2 \omega = \frac{mM}{m+M} Vr = n\hbar$$

- Our previous calculations then follow with the substitution

$$m \to \frac{mM}{m+M} \equiv \text{reduced mass } \mu$$

Bohr Model

- Thus

$$E_n = -\frac{Z^2 e^2}{8\pi\varepsilon_0 a_0 n^2} \equiv -\frac{Z^2 E_0}{n^2}$$

$$\text{where } E_0 = \frac{e^2}{8\pi\varepsilon_0 a_0} = \frac{\mu e^4}{2\hbar^2 (4\pi\varepsilon_0)^2} = 13.6 eV$$

$$\frac{1}{\lambda} = \frac{E_i - E_f}{hc} = \frac{Z^2 E_0}{hc}\left(\frac{1}{n_f^2 - n_i^2}\right) \equiv Z^2 R \left(\frac{1}{n_f^2 - n_i^2}\right)$$

- So $R = \left(\frac{\mu}{m}\right) R_\infty$

- And now R and R_H agree to within 3 parts in 100000

Bohr Model

- Calculate the three lowest energy states of positronium

$$\mu = \frac{m_e m_e}{m_e + m_e} = \frac{m_e}{2}$$

so

$$E_n = -\frac{Z^2 E_0}{n^2} = -\frac{Z^2 m e^4}{2\hbar^2 \left(4\pi\varepsilon_0\right)^2}$$

becomes

$$E_n = -\frac{Z^2 E_0}{2n^2}$$

$$E_1 = -6.8eV, E_2 = -1.7eV, \mathrm{E}_3 = -0.76eV$$

Bohr Model

- While the Bohr model simply and successfully described some of the features of atoms, it failed on other aspects
  - It was not applicable to multi-electron atoms
  - It was not able to predict the transition rate between atomic states
  - It could not account for the fine structure observed in spectral lines
  - It could explain how atoms bind into molecules
  - It was an ad-hoc mixture of ideas