Thomson Scattering

How does a photon (light) scatter from an electron?

Classically, the answer is Thomson scattering

- Assume
  - The wavelength of light is small compared to an atom
  - The energy of the light is large compared to the binding energy of atomic electrons
  - The energy of the light is smaller than $m_e c^2$
Thomson Scattering
Thomson Scattering

In Thomson scattering an electromagnetic (EM) wave of frequency $f$ is incident on an electron

- What happens to the electron?

Thus the electron will emit EM waves of the same frequency and in phase with the incident wave

The electron absorbs energy from the EM wave and scatters it in a different direction

In particular, the wavelength of the scattered wave is the same as that of the incident wave
Rayleigh Scattering

An aside, Rayleigh scattering is scattering of light from a harmonically bound electron.

You may recall the probability for Rayleigh scattering goes as $\frac{1}{\lambda^4}$.

- Why is the sky blue?
- Why are sunsets red?
- What color is the moon’s sky?
Compton Scattering

X-ray spectrum produced by bombarding a metal with electrons

- Line spectra correspond to atomic electron transitions in an excited atom
- Continuum corresponds to the emission of radiation from accelerated electrons (scattered by the Coulomb force of atomic nuclei)

\[
S = \frac{2}{3} \frac{q^2 a^2}{c^3}
\]
Cross Section

Incident photon

\[ E = hf \]
\[ \rho = \frac{h}{\lambda} \]

Target electron

\[ E_i = mc^2 \]

Recoil electron

\[ E_f = E_e \]

Scattered photon

\[ E = hf' \]
\[ \rho = \frac{h}{\lambda'} \]
Compton Effect

The change in wavelength can be found by applying:

- Energy conservation
  \[ hf + m_e c^2 = hf' + E_e = hf' + \left( p_e^2 c^2 + m_e^2 c^4 \right)^{1/2} \]

- Momentum conservation
  \[ \vec{p} = \vec{p}' + \vec{p}_e \]
  \[ p_e^2 = p^2 + p'^2 - 2 \vec{p} \cdot \vec{p}' = p^2 + p'^2 - 2 p \cdot p' \cos \theta \]
Compton Effect

➢ From energy conservation

\[ m_e^2 c^4 + (hf - hf')^2 + 2m_e c^2 (hf - hf') = m_e^2 c^4 + p_e^2 c^2 \]

\[ p_e^2 = \left( \frac{hf}{c} \right)^2 + \left( \frac{hf'}{c} \right)^2 - \frac{2hfhf'}{c^2} + 2m_e (hf - hf') \]

➢ From momentum conservation

\[ p_e^2 = p^2 + p'^2 - 2 \vec{p} \cdot \vec{p}' = p^2 - p'^2 - 2p \cdot p' \cos \theta \]

\[ p_e^2 = \left( \frac{hf}{c} \right)^2 + \left( \frac{hf'}{c} \right)^2 - 2 \frac{hf}{c} \frac{hf'}{c} \cos \theta \]

➢ Eliminating \( p_e^2 \)

\[ m_e c^2 (hf - hf') = hfhf' (1 - \cos \theta) \]
Compton Effect

➢ Continuing on

\[
\frac{f - f'}{ff'} = \frac{h}{m_e c^2} (1 - \cos \theta)
\]

➢ And using \( f = \frac{c}{\lambda} \) we arrive at the Compton effect

\[
\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)
\]

➢ And \( \frac{h}{mc} \) is called the Compton wavelength

\[
\lambda_C = \frac{h}{m_e c} = 2.43 \times 10^{-12} m
\]
Compton Effect

(a) Molybdenum Kα line, primary

(b) Modified from carbon at 135°

Unmodified

Glancing angle from calcite (proportional to wavelength)
Another process by which photons can interact with matter is electron-positron pair production

\[ \gamma \rightarrow e^+ + e^- \]

The process as shown cannot take place because energy and momentum are not simultaneously conserved

- Consider the center-of-momentum frame for the \( e^+ \) and \( e^- \). What is the momentum of the photon?

However energy and momentum are both conserved in the presence of a Coulomb field from an atomic nucleus or atomic electron
Pair Production

- **Energy and momentum conservation give**
  
  Energy: \( hf = E_+ + E_- \)

  Momentum (x): \( \frac{hf}{c} = p_- \cos \theta_- + p_+ \cos \theta_+ \)

  Momentum (y): \( 0 = p_- \sin \theta_- + p_+ \sin \theta_+ \)

- **Energy conservation can be re-written**

  \[ hf = \sqrt{p_-^2 c^2 + m^2 c^4} + \sqrt{p_+^2 c^2 + m^2 c^4} \]

- **But momentum conservation (x) shows**

  \[ hf_{\text{max}} = p_- c + p_+ c \]

- **Thus energy and momentum are not simultaneously conserved**
Pair Production

- Conservation of energy and momentum does hold in the presence of an atomic nucleus (or electron) where the recoil of the nucleus ensures momentum conservation.

In order for pair production to occur, the energy of the photon must be at least twice the electron rest mass:

\[ hf > 2m_e c^2 = 1.022 \text{ MeV} \]
Pair Production

(a) Free space (cannot occur)

(b) Beside nucleus
Pair Production

- A related process to electron-positron pair production is pair annihilation
  - \( e^+ + e^- \rightarrow \gamma \gamma \)
- A positron passing through matter will lose energy through collisions with atomic electrons
- It eventually slows down and annihilates with an electron (possibly first forming a bound system called positronium)
Pair annihilation is the basis of PET (Positron Emission Tomography) scanning

- PET scans are most often used to detect cancer and to examine the effects of cancer therapy by characterizing biochemical changes in the cancer
- PET scans are also used to study heart function and brain disorders
PET

PET scanning

- Positron emitting radioactive nuclei such as $^{11}$C, $^{13}$N, $^{15}$O, $^{18}$F are produced (at accelerators)
- The nuclei are incorporated into compounds used by the body such as sugar or ammonia
- Once taken into the body, positrons are emitted, lose energy in a few mm, and annihilate with electrons producing two 0.511 MeV photons that produced back-to-back
- Software reconstructs the point of origin of the annihilation producing a map showing tissues where the radiotracers have become concentrated
PET
Photon Interactions

Interactions of photons with matter

- The primary processes by which photons interact with matter are
- From low energy to high energy
  - Photoelectric effect
  - Compton scattering
  - Pair production
Photon Interactions