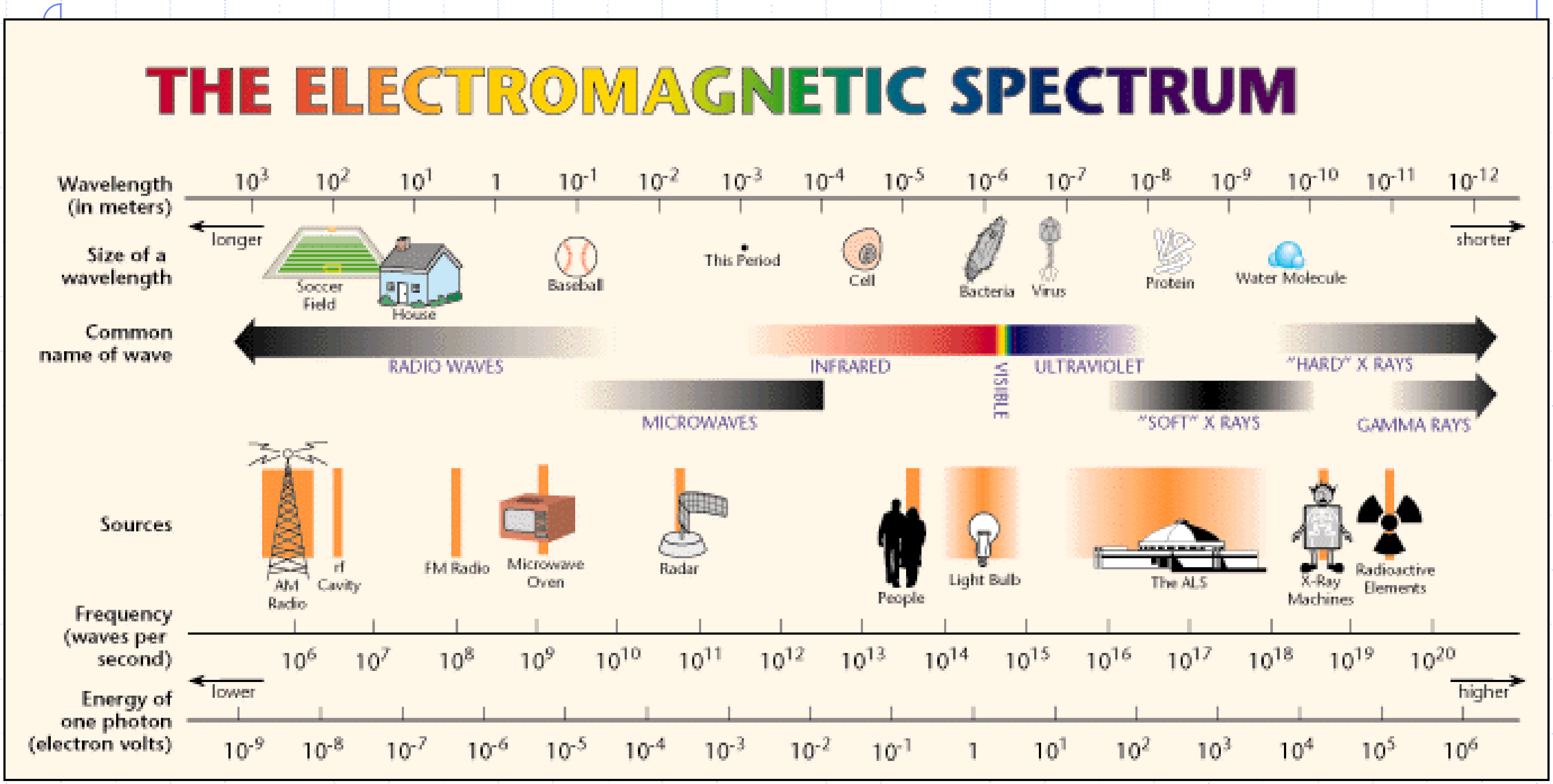
Thomson Scattering

- How does a photon (light) scatter from an electron?
- Classically, the answer is Thomson scattering
  - Assume
    - The wavelength of light is small compared to an atom
    - The energy of the light is large compared to the binding energy of atomic electrons
    - The energy of the light is smaller than $m_e c^2$

Thomson Scattering



Thomson Scattering

- In Thomson scattering an electromagnetic (EM) wave of frequency f is incident on an electron
  - What happens to the electron?
- Thus the electron will emit EM waves of the same frequency and in phase with the incident wave
- The electron absorbs energy from the EM wave and scatters it in a different direction
- In particular, the wavelength of the scattered wave is the same as that of the incident wave

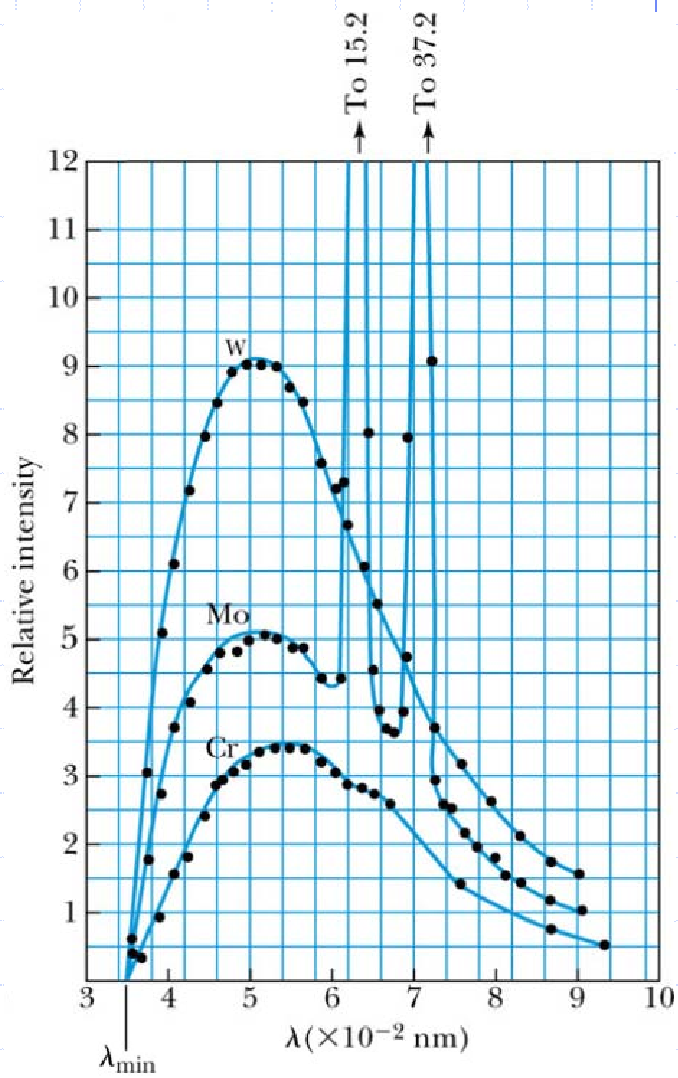
Rayleigh Scattering

- An aside, Rayleigh scattering is scattering of light from a harmonically bound electron
- You may recall the probability for Rayleigh scattering goes as $1/\lambda^4$
  - Why is the sky blue?
  - Why are sunsets red?
  - What color is the moon's sky?

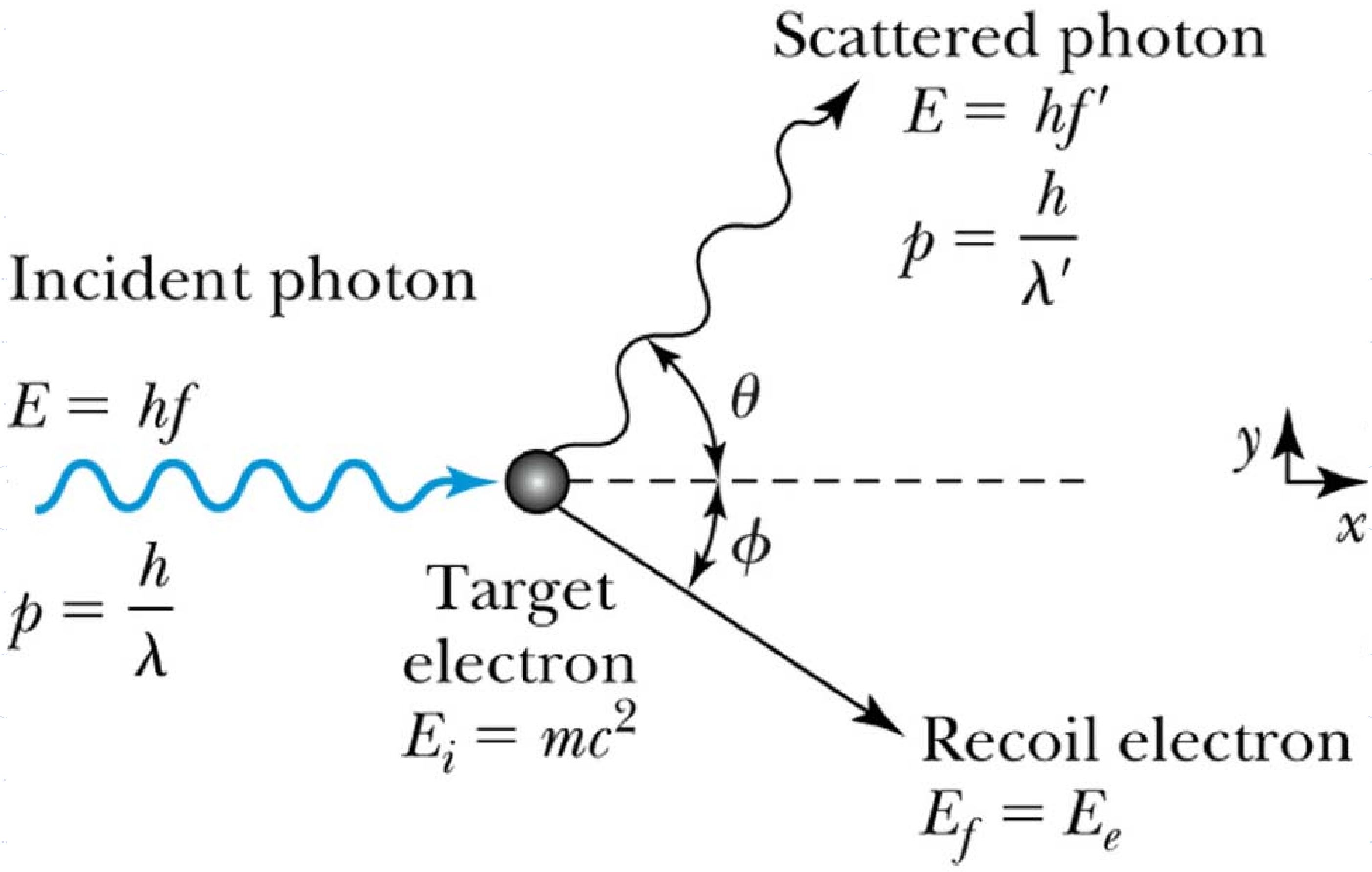
Compton Scattering

- X-ray spectrum produced by bombarding a metal with electrons
  - Line spectra correspond to atomic electron transitions in an excited atom
  - Continuum corresponds to the emission of radiation from accelerated electrons (scattered by the Coulomb force of atomic nuclei)

$$S = \frac{2}{3}\frac{q^2 a^2}{c^3}$$



Cross Section



Compton Effect

- The change in wavelength can be found by applying
  - Energy conservation

$$hf + m_e c^2 = hf' + E_e = hf' + \left(p_e^2 c^2 + m_e^2 c^4\right)^{1/2}$$

  - Momentum conservation

$$\vec{p} = \vec{p}' + \vec{p}_e$$

$$p_e^2 = p^2 + p'^2 - 2\vec{p}\cdot\vec{p}' = p^2 + p'^2 - 2p\cdot p'\cos\theta$$

Compton Effect

- From energy conservation

$$m_e^2c^4 + (hf - hf')^2 + 2m_ec^2\left(hf - hf'\right) = m_e^2c^4 + p_e^2c^2$$

$$p_e^2 = \left(\frac{hf}{c}\right)^2 + \left(\frac{hf'}{c}\right)^2 - \frac{2hfhf'}{c^2} + 2m_e\left(hf - hf'\right)$$

- From momentum conservation

$$p_e^2 = p^2 + p'^2 - 2\vec{p}\cdot\vec{p}' = p^2 - p'^2 - 2p\cdot p'\cos\theta$$

$$p_e^2 = \left(\frac{hf}{c}\right)^2 + \left(\frac{hf'}{c}\right)^2 - 2\frac{hf}{c}\frac{hf'}{c}\cos\theta$$

- Eliminating p_e^2

$$m_ec^2\left(hf - hf'\right) = hfhf'\left(1 - \cos\theta\right)$$

Compton Effect

- Continuing on

$$\frac{f-f'}{ff'}=\frac{h}{m_e c^2}(1-\cos\theta)$$

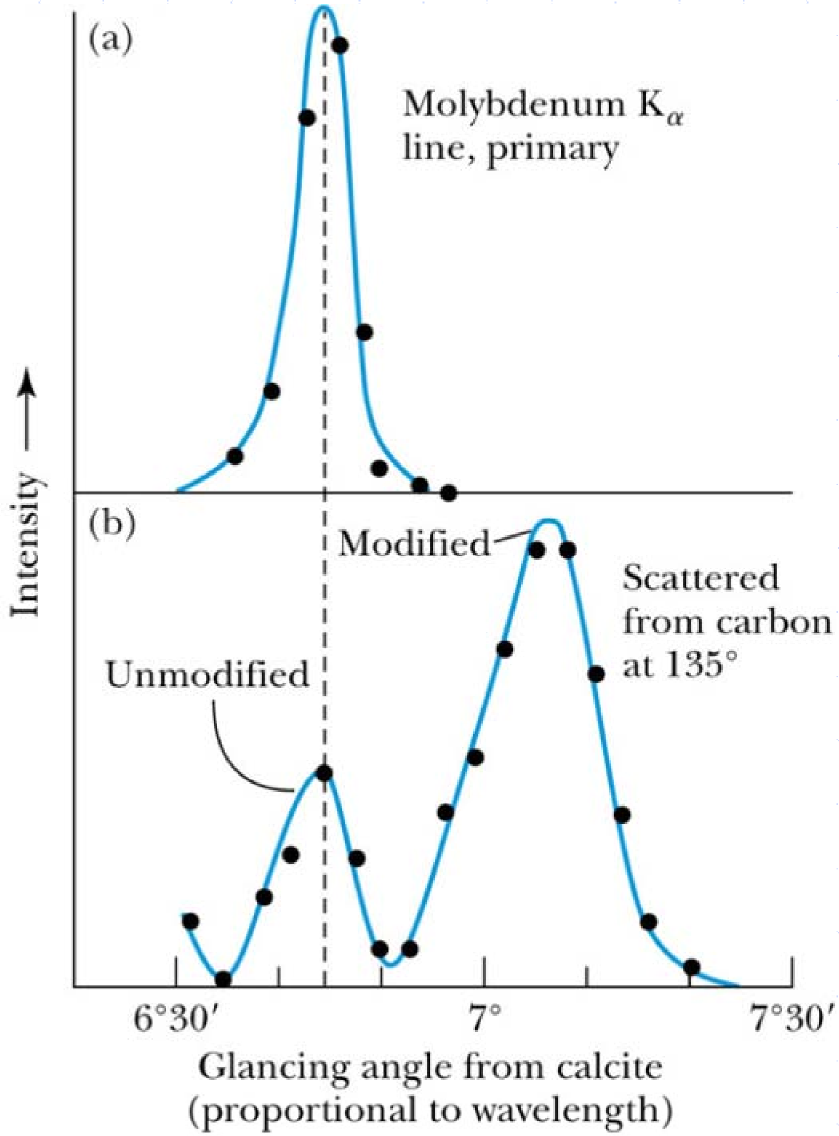
- And using *f=c/λ* we arrive at the Compton effect

$$\lambda'-\lambda=\frac{h}{m_e c}\left(1-\cos\theta\right)$$

- And *h/mc* is called the Compton wavelength

$$\lambda_C=\frac{h}{m_e c}=2.43\times10^{-12}m$$

Compton Effect



Pair Production

- Another process by which photons can interact with matter is electron-positron pair production
  - $\gamma \rightarrow e^{+}+e^{-}$
- The process as shown cannot take place because energy and momentum are not simultaneously conserved
  - Consider the center-of-momentum frame for the e^{+} and e^{-}. What is the momentum of the photon?
- However energy and momentum are both conserved in the presence of a Coulomb field from an atomic nucleus or atomic electron

Pair Production

- Energy and momentum conservation give

  Energy $hf = E_- + E_+$

  Momentum (x) $\frac{hf}{c} = p_- \cos\theta_- + p_+ \cos\theta_+$

  Momentum (y) $0 = p_- \sin\theta_- + p_+ \sin\theta_+$

- Energy conservation can be re-written

$$hf = \sqrt{p_-^2 c^2 + m^2 c^4} + \sqrt{p_+^2 c^2 + m^2 c^4}$$

- But momentum conservation (x) shows

$$hf_{\max} = p_- c + p_+ c$$

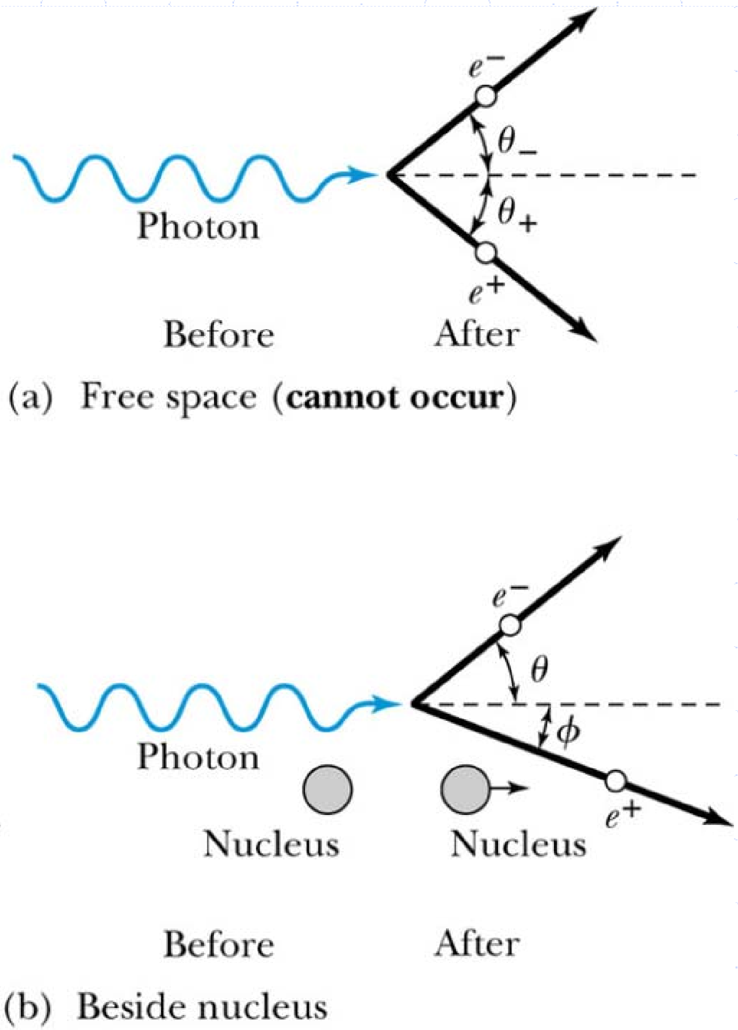
- Thus energy and momentum are not simultaneously conserved

Pair Production

- Conservation of energy and momentum does hold in the presence of an atomic nucleus (or electron) where the recoil of the nucleus ensures momentum conservation
- In order for pair production to occur, the energy of the photon must be at least twice the electron rest mass

$$hf > 2m_e c^2 = 1.022\ \mathrm{MeV}$$

Pair Production



Pair Production

- A related process to electron-positron pair production is pair annihilation
  - $e^{+}+e^{-} \rightarrow \gamma\,\gamma$
- A positron passing through matter will lose energy through collisions with atomic electrons
- It eventually slows down and annihilates with an electron (possibly first forming a bound system called positronium)

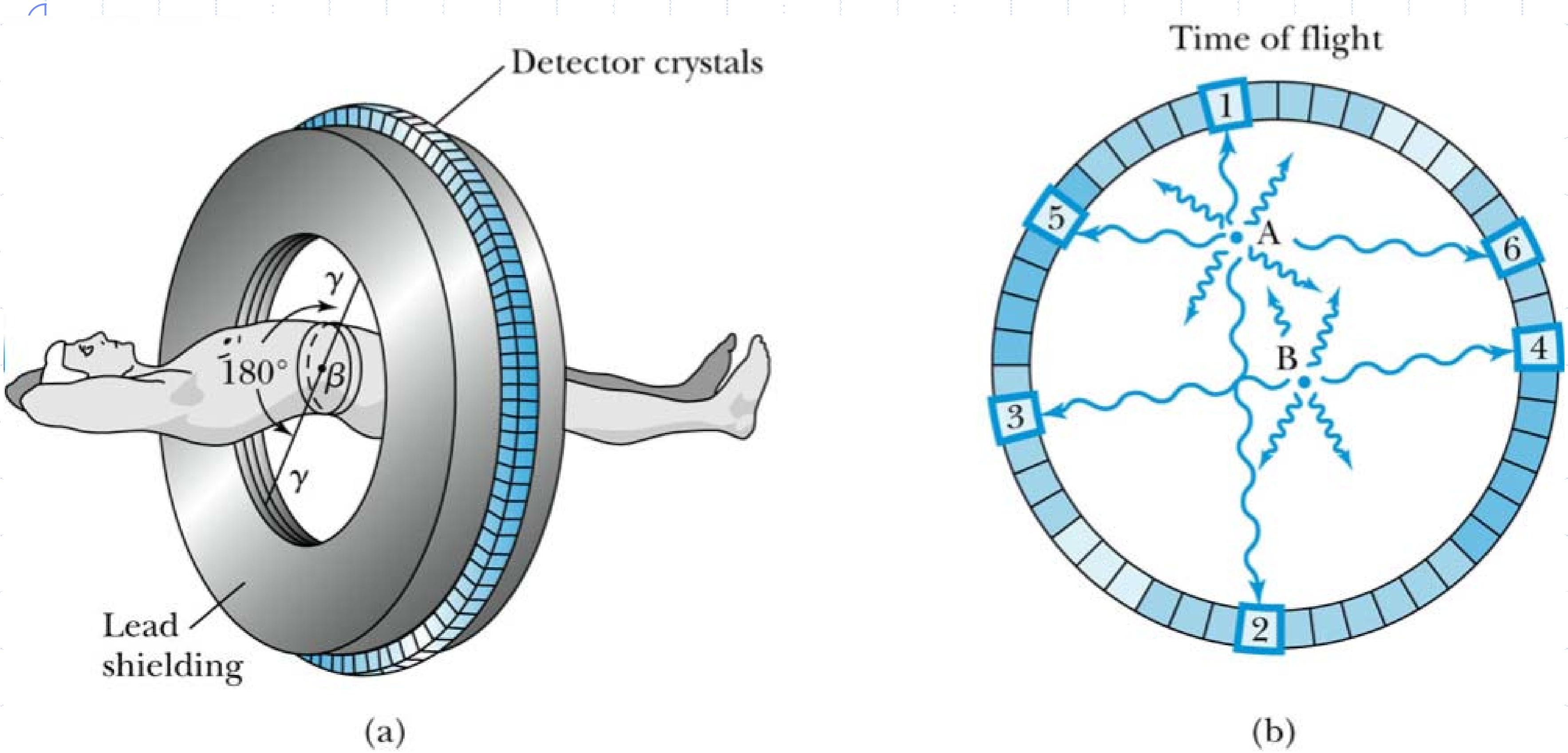
PET

- Pair annihilation is the basis of PET (Positron Emission Tomography) scanning
  - PET scans are most often used to detect cancer and to examine the effects of cancer therapy by characterizing biochemical changes in the cancer
  - PET scans are also used to study heart function and brain disorders

PET

- PET scanning
  - Positron emitting radioactive nuclei such as ^{11}C, ^{13}N, ^{15}O, ^{18}F are produced (at accelerators)
  - The nuclei are incorporated into compounds used by the body such as sugar or ammonia
  - Once taken into the body, positrons are emitted, lose energy in a few mm, and annihilate with electrons producing two 0.511 MeV photons that produced back-to-back
  - Software reconstructs the point of origin of the annihilation producing a map showing tissues where the radiotracers have become concentrated

PET


Detector crystals
Lead shielding
180°
β
γ
γ
(a)
Time of flight
1
2
3
4
5
6
A
B
(b)

PET

Photon Interactions

- Interactions of photons with matter
  - The primary processes by which photons interact with matter are
  - From low energy to high energy
    - Photoelectric effect
    - Compton scattering
    - Pair production

Photon Interactions