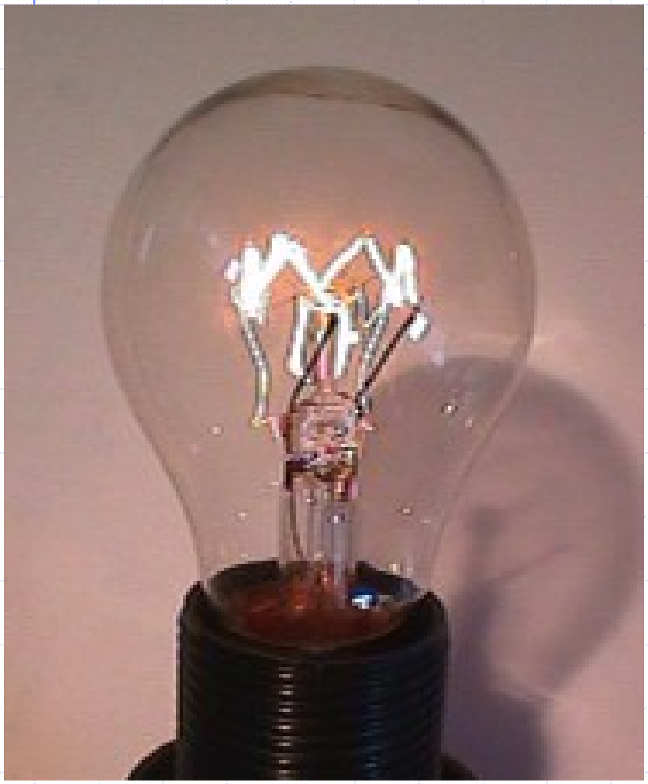
Blackbody Radiation

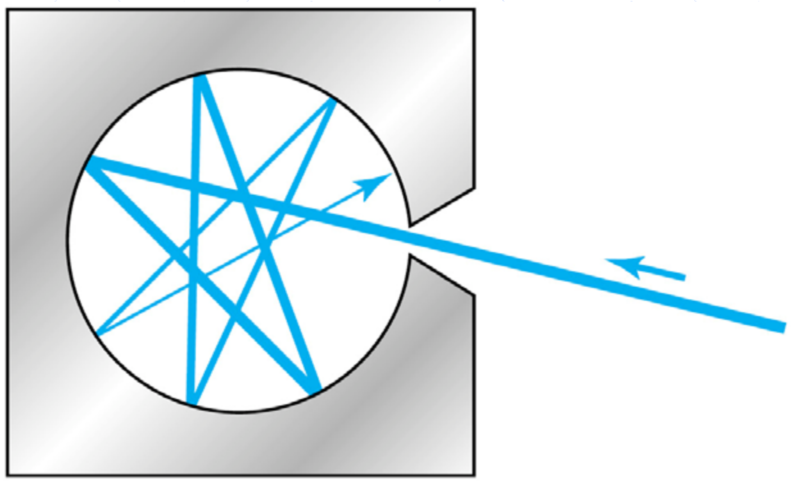
- All bodies at a temperature T emit and absorb thermal electromagnetic radiation
  - Blackbody radiation
  - In thermal equilibrium, the power emitted equals the power absorbed
- How is blackbody radiation absorbed and emitted?

Blackbody Radiation



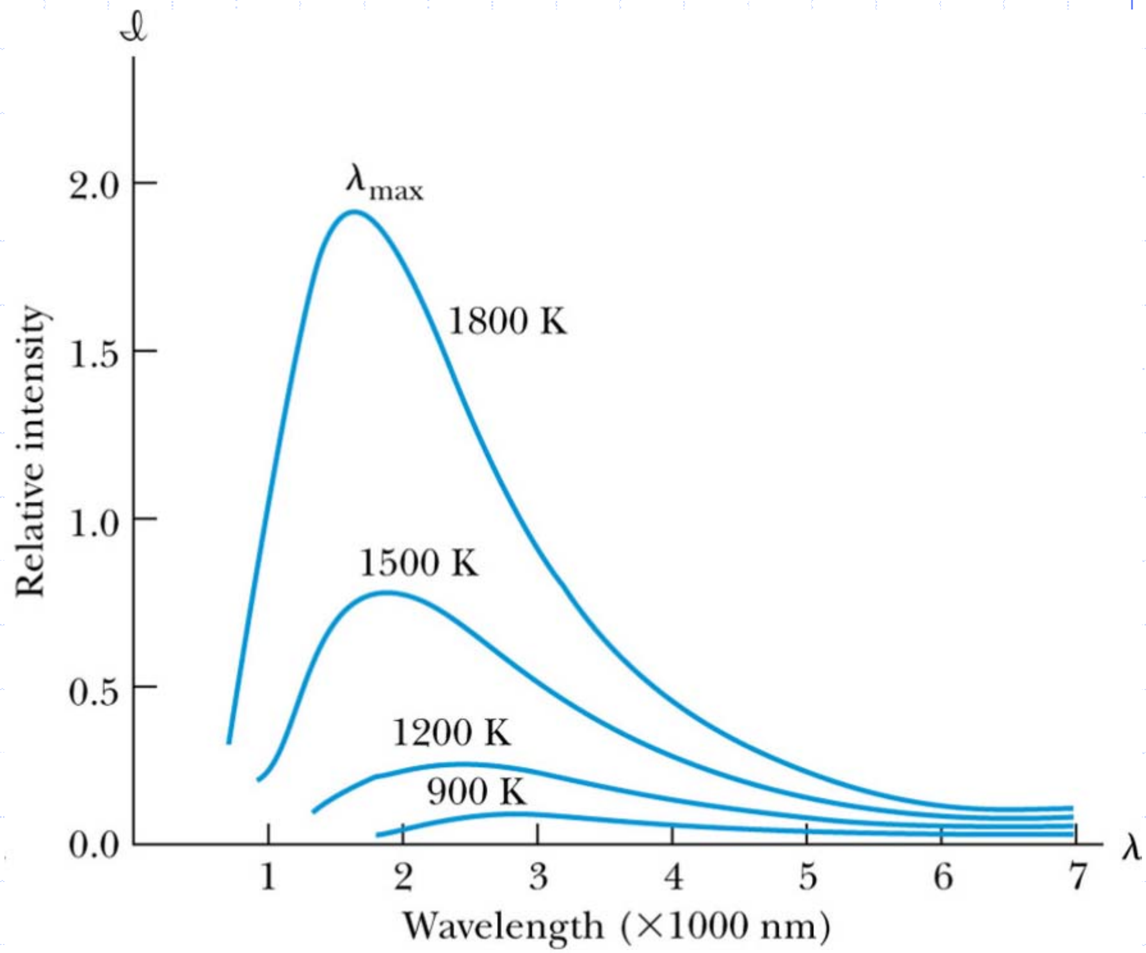
Blackbody Radiation

- A blackbody is a perfect absorber of radiation
- A simple blackbody is given by a hole in a wall of some enclosure
- Both absorption and can occur
- The radiation properties of the cavity are independent of the enclosure material

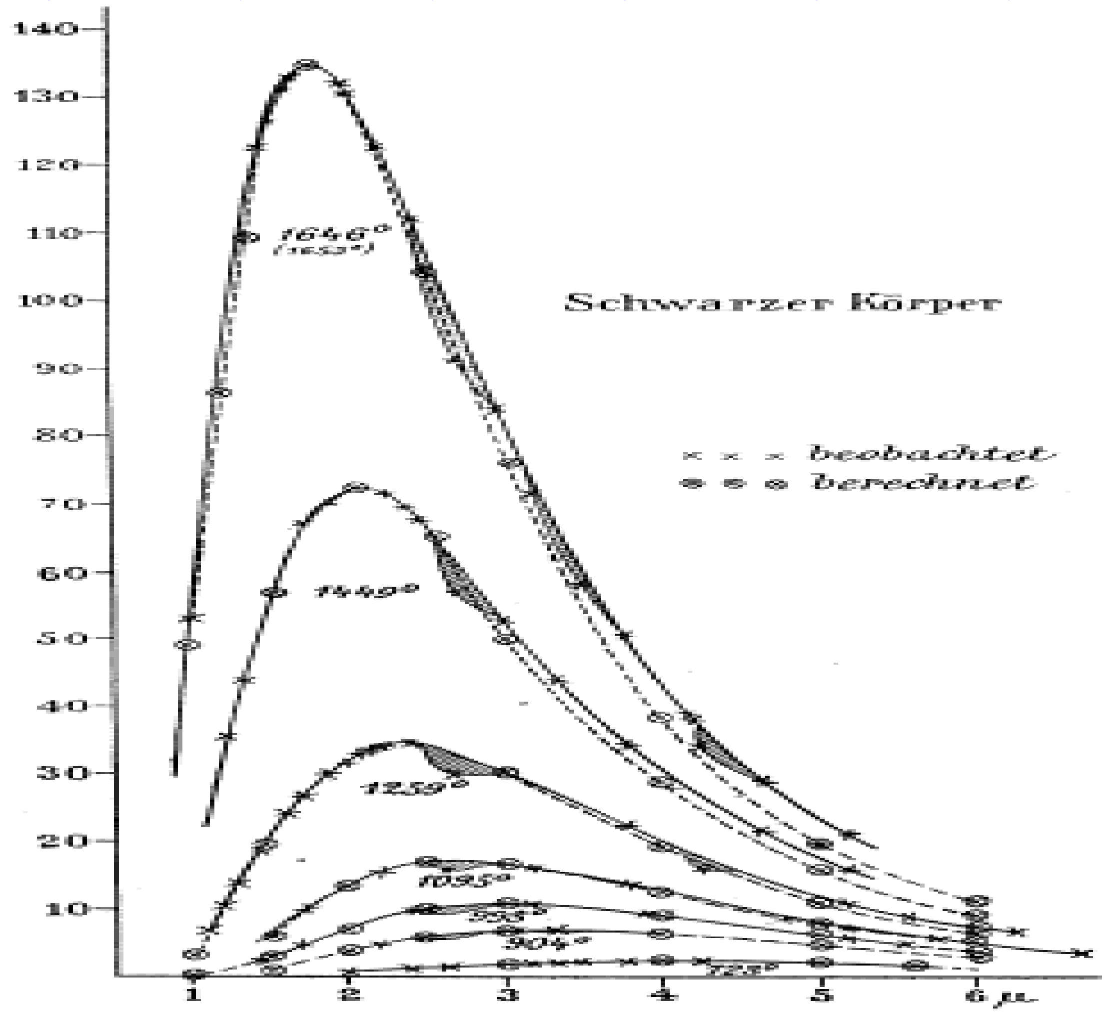


Blackbody Radiation

- The y axis is power per area per wavelength



Blackbody Radiation



Blackbody Radiation

- Wien's displacement law

$$\lambda_{\max} T = 2.898 \times 10^{-3} mK$$

  - Wavelength decreases as T increases

- Stefan-Boltzmann law

$$R(T) = \varepsilon \sigma T^4$$

$$\sigma = 5.6705 \times 10^{-8} W / \left(m^2 K^4 \right)$$

  - Total power / area radiated increases as T^4

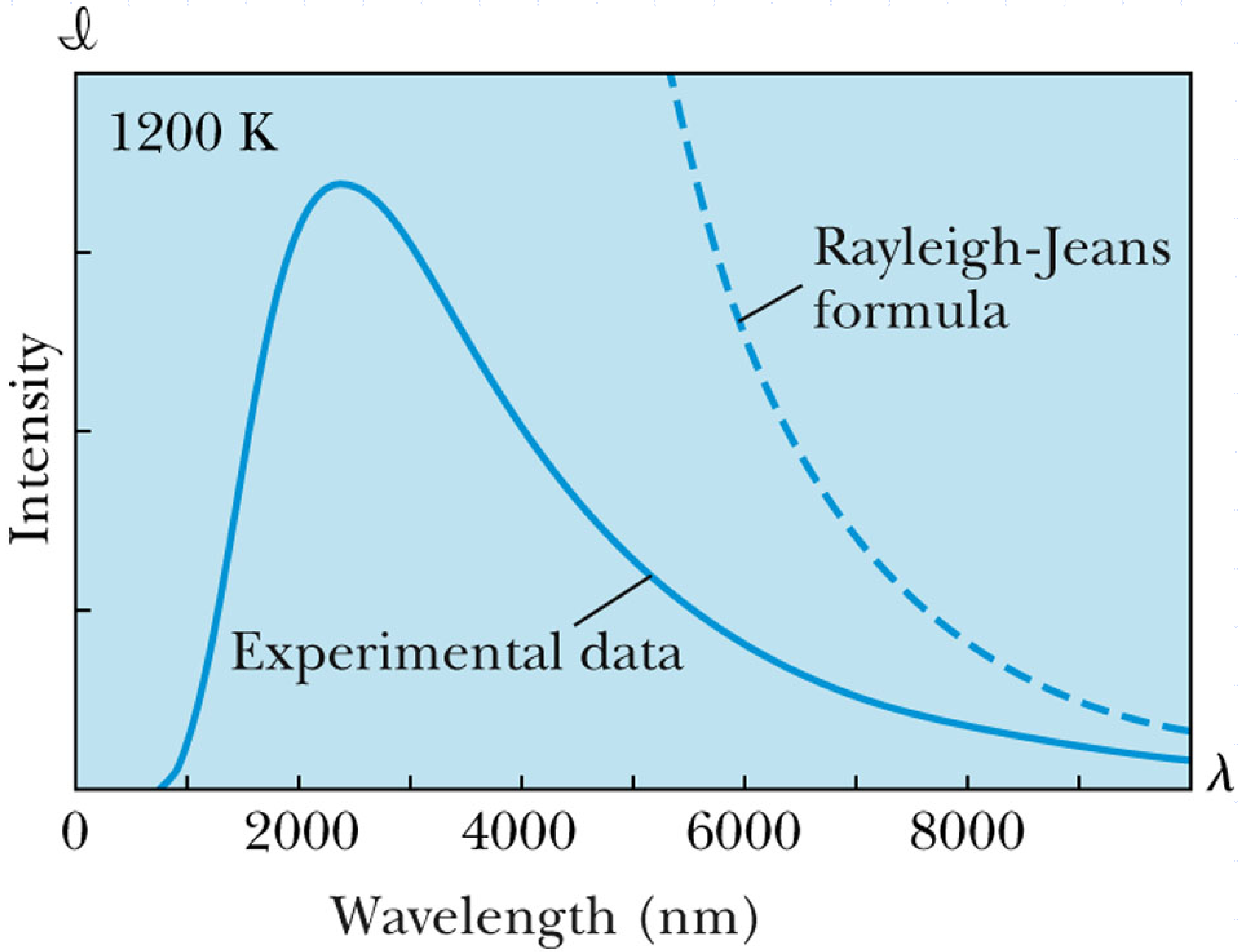
Blackbody Radiation

- Attempts to calculate the spectral distribution of blackbody radiation from first principles failed
- The best description was given by the Rayleigh-Jeans formula

$$I(\lambda, T) = \frac{2\pi ckT}{\lambda^4}$$

- This described the distribution at long wavelengths but increased without limit as $\lambda \to 0$
  - Ultraviolet catastrophe

Blackbody Radiation



Blackbody Radiation

- Planck was able to calculate the correct distribution by assuming energy was quantized (he was desperate)
  - Microscopic (atomic) oscillators can only have certain discrete energies

$$E_n = nhf$$

$$h = 6.6261 \times 10^{-34}\,Js$$

  - The oscillators can only absorb or emit energy in multiples of

$$\Delta E = hf$$

Blackbody Radiation

- Planck's radiation law agreed with data

$$I(\lambda,T)=\frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

- It leads directly to Wien's displacement law and the Stefan-Boltzmann law
- It agrees with Rayleigh-Jeans formula for large wavelengths
  - See derivations for both in Thornton and Rex

Blackbody Radiation

- We may derive the radiation law later in the course but for now consider
  - The electromagnetic energy inside the (metallic) cavity must exist in the form of standing waves with nodes at the surfaces
  - Classically, the equipartition theorem gives the average energy of a standing wave

$$\bar{\varepsilon} = kT$$

    - Note the same value is predicted for all standing waves independent of their frequency
  - The amount of radiation coming out of the cavity is then the number of different wave modes X the energy per mode

Blackbody Radiation

- Let E=nhv, then the average energy is

$$\bar{\varepsilon} = \frac{\sum_{n=0}^{\infty} \varepsilon P(\varepsilon)}{\sum_{n=0}^{\infty} P(\varepsilon)} = \frac{\sum_{n=0}^{\infty} \frac{nhf}{kT} e^{-\frac{nhf}{kT}}}{\sum_{n=0}^{\infty} \frac{1}{kT} e^{-\frac{nhf}{kT}}} = \frac{kT \sum_{n=0}^{\infty} n\alpha e^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}}$$

where $\alpha = \dfrac{hf}{kT}$

and $\dfrac{1}{kT} e^{-\frac{\varepsilon}{kT}}$ is the Boltzmann distribution P(ε)

Kinetic Theory of Gases

- Based on "atomic" theory of matter
- Results include
  - Speed of a molecule in a gas
    - $v_{rms} = (\langle v^2 \rangle)^{1/2} = (3kT/m)^{1/2}$
  - Equipartion theorem
    - Internal energy $U = f/2\ NkT = f/2\ nRT$
  - Heat capacity
    - $C_V = (dU/dT)_V = f/2\ R$
  - Maxwell speed distribution

$$f(v) = 4\pi N \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT}$$

Blackbody Radiation

- To evaluate this we use standard "tricks" from statistical mechanics

$$-\alpha \frac{d}{d\alpha} \ln \sum_{n=0}^{\infty} e^{-n\alpha} = \frac{-\alpha \frac{d}{d\alpha} \sum_{n=0}^{\infty} e^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}} = \frac{-\sum_{n=0}^{\infty} \alpha \frac{d}{d\alpha} e^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}}$$

$$= \frac{\sum_{n=0}^{\infty} n\alpha e^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}}$$

Blackbody Radiation

- And note

$$\sum_{n=0}^{\infty} e^{-n\alpha} = 1 + e^{-\alpha} + e^{-2\alpha} + e^{-3\alpha} + \ldots$$

$$= 1 + X + X^2 + X^3 + \ldots$$

$$= (1 - X)^{-1}$$

Blackbody Radiation

- Putting these together we have

$$\bar{\varepsilon} = kT\left(-\alpha \frac{d}{d\alpha} \ln \sum_{n=0}^{\infty} e^{-n\alpha}\right) = -hf \frac{d}{d\alpha} \ln \sum_{n=0}^{\infty} e^{-n\alpha}$$

$$\bar{\varepsilon} = -hf \frac{d}{d\alpha} \ln\left(1 - e^{-\alpha}\right)^{-1} = \frac{-hf}{\left(1 - e^{-\alpha}\right)^{-1}} (-1)\left(1 - e^{-\alpha}\right)^{-2} e^{-\alpha}$$

$$\bar{\varepsilon} = \frac{hfe^{-\alpha}}{1 - e^{-\alpha}} = \frac{hf}{e^{\alpha} - 1} = \frac{hf}{e^{\frac{hf}{kT}} - 1} = \frac{hc}{\lambda} \frac{1}{e^{\frac{hc}{\lambda kT}}}$$

Blackbody Radiation

- To finish the calculation we'd have to multiply the average energy of a wave of frequency f x the number of waves with frequency f
- This would give us the Planck distribution

$$I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Blackbody Radiation

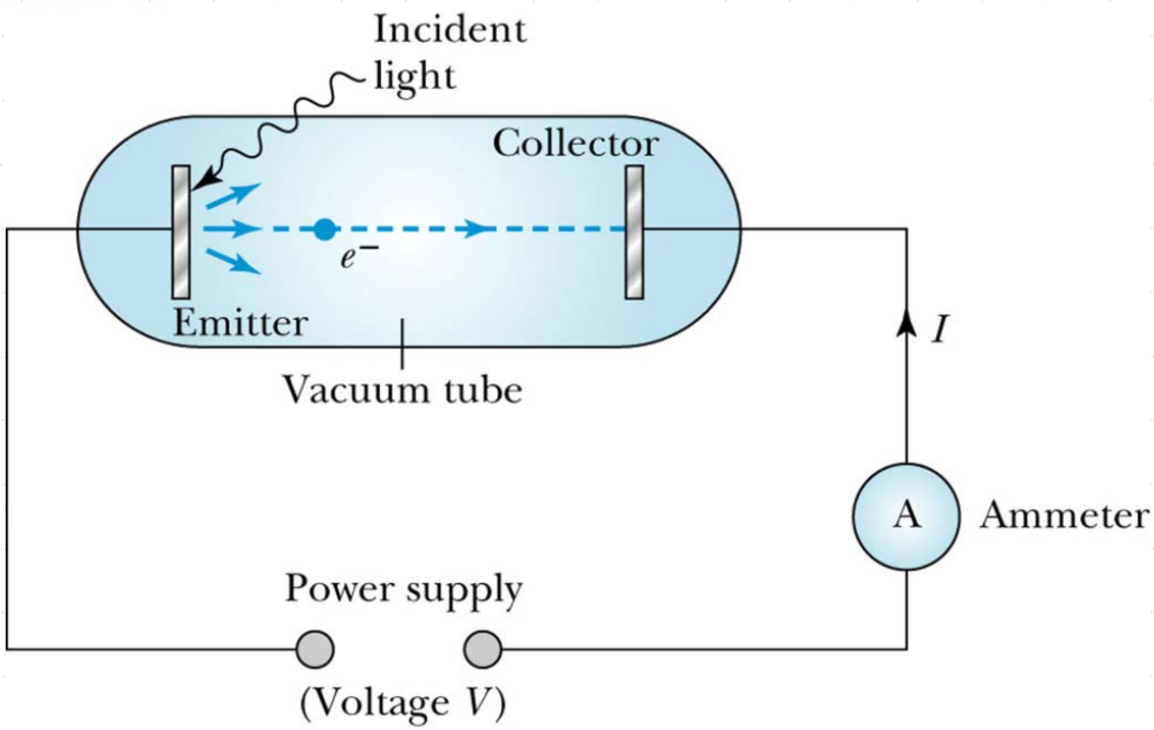
- You can how Planck avoided the ultraviolet catastrophe
  - Because the energy is proportional to the frequency
    - The average energy is kT when the possible energies are small compared to kT
    - The average energy is extremely small when the possible energies are large compared to kT (because P(ε) is extremely small)

Blackbody Radiation

- Planck's paper is generally considered to be the birthplace of quantum mechanics
  - Revisionist history?
  - Planck did not pay too much attention to energy quantization
  - Neither did anyone else
  - There is controversy of whether he even intended the energy of an oscillator to be nhf

Photoelectric Effect

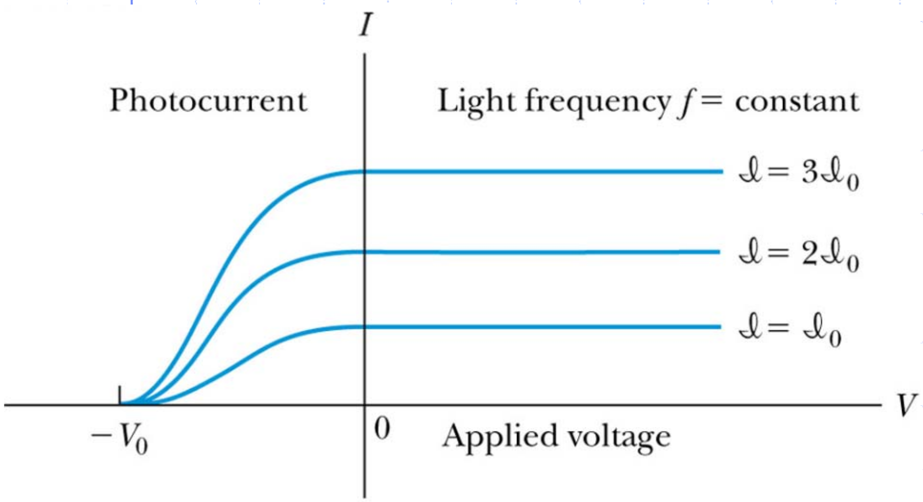
- Light incident on a metal will eject electrons
  - Aside, other means of doing this are with temperature, electric fields, and particle bombardment

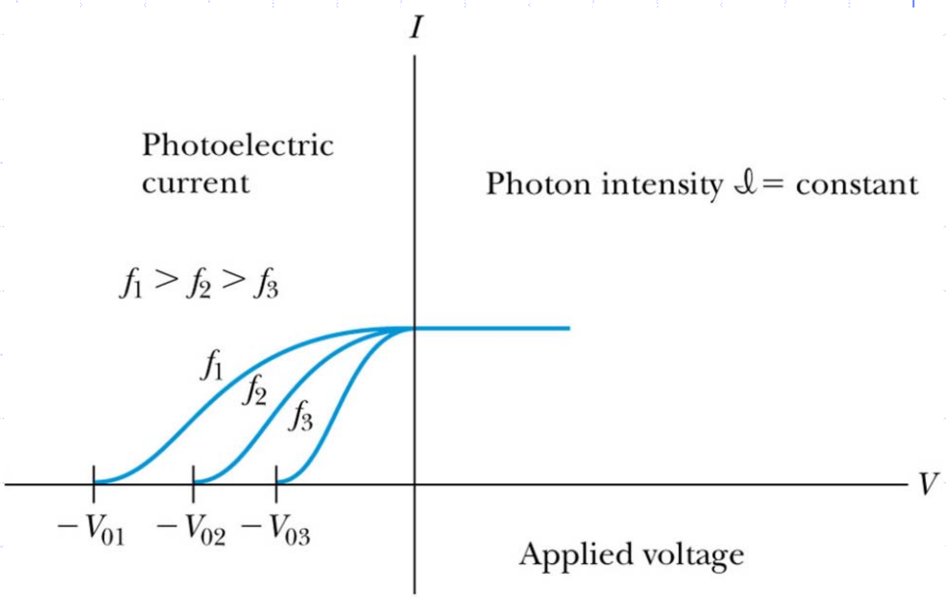


Photoelectric Effect

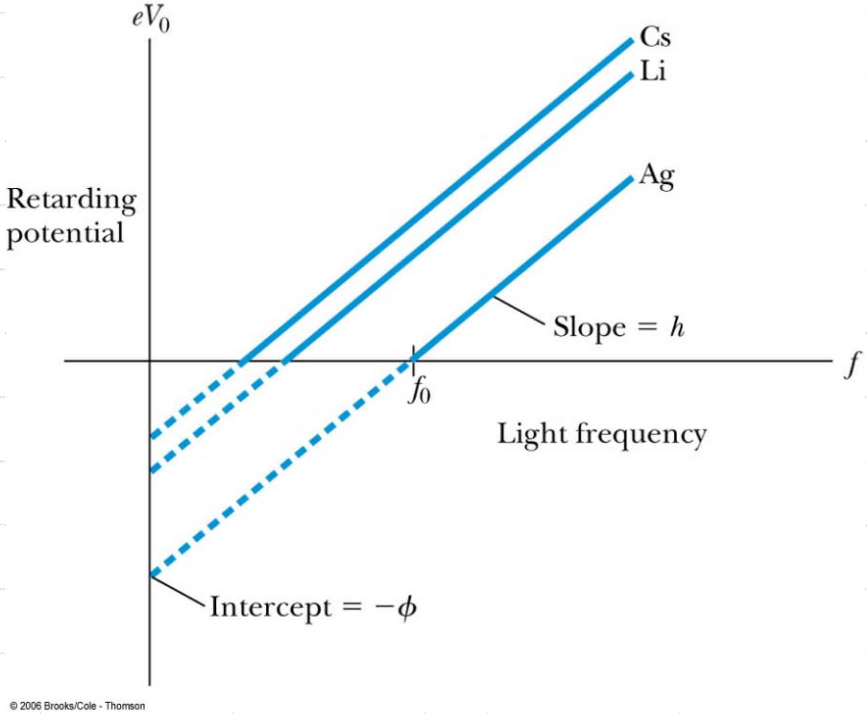
- Experiments showed
  - Kinetic energy of the photoelectrons are independent of the light intensity
  - The maximum kinetic energy of the photoelectrons depends on the light frequency
  - The smaller the work function φ the smaller the threshold frequency to produce photoelectrons
  - The number of photoelectrons is proportional to light intensity

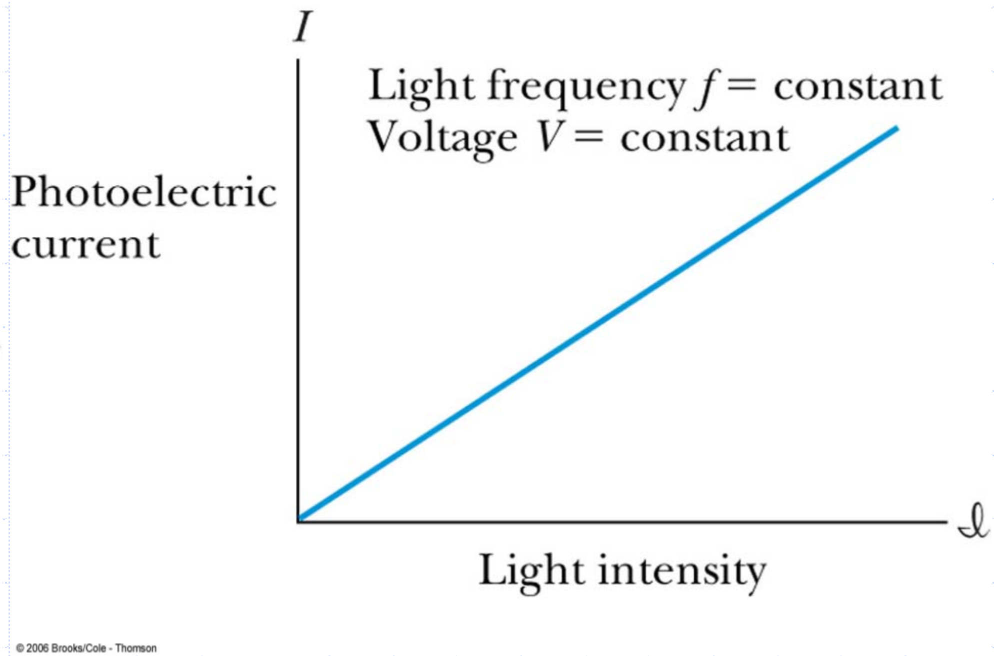
Photoelectric Effect





Photoelectric Effect





Photoelectric Effect

- Explained by Einstein in one of his annus mirabilis papers
- In his paper he assumed
  - Electromagnetic field was quantized
  - Light quanta were localized in space (like particles) == photons
  - Energy $E = hf$
  - In the photoelectric process, the energy quanta (photons) are completely absorbed

Photoelectric Effect

- Thus photons penetrate the surface of the metal and are absorbed by electrons
- The electrons overcome attractive forces that normally hold them in the material and escape
- Conservation of energy gives
  - $hf = 1/2mv^2_{max} + \varphi$
- And consequently he predicted
  - $1/2mv^2_{max} = eV_0 = hf - \varphi$
  - Note h/e can be measured from the slope

Photoelectric Effect

- This was strange since it involved Planck's constant h
- This was a difficult experiment to carry out
  - It took almost a decade to verify
  - Millikan was the principle experimenter (who tried to prove Einstein's theory wrong)
- The end result was proof that light energy is quantized and E=hf

Photoelectric Effect