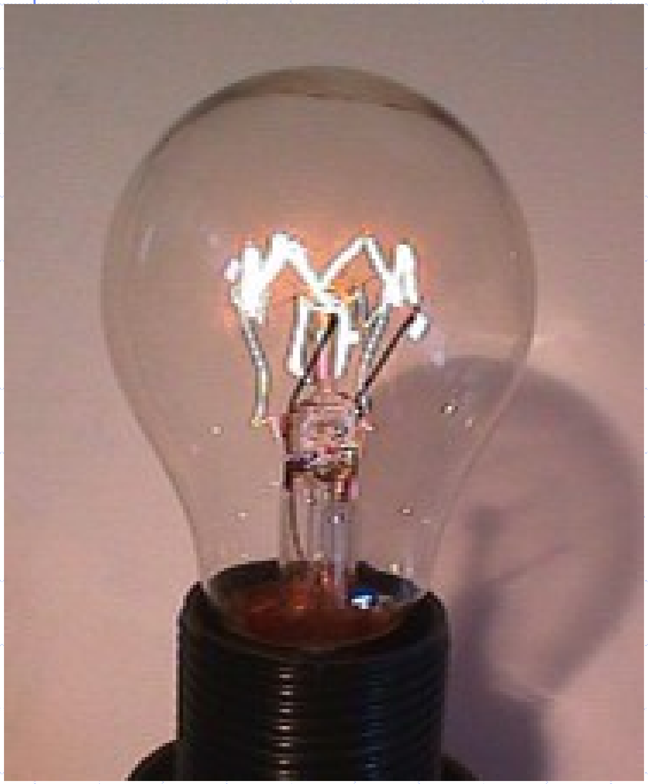
Blackbody Radiation

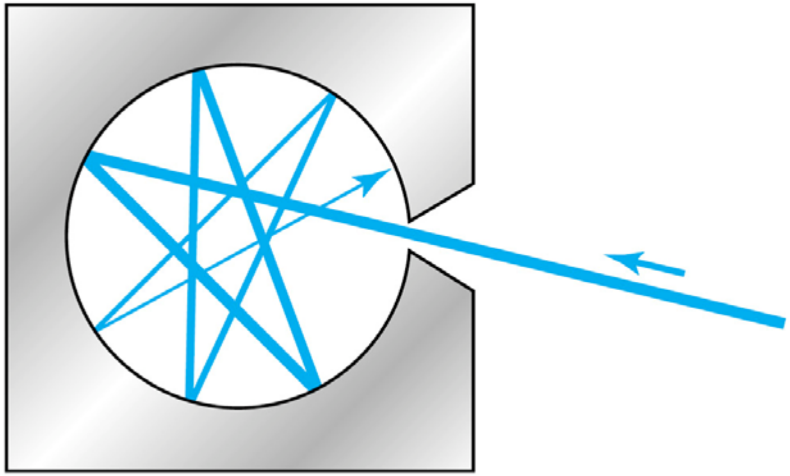
- All bodies at a temperature T emit and absorb thermal electromagnetic radiation
  - Blackbody radiation
  - In thermal equilibrium, the power emitted equals the power absorbed
- How is blackbody radiation absorbed and emitted?

Blackbody Radiation



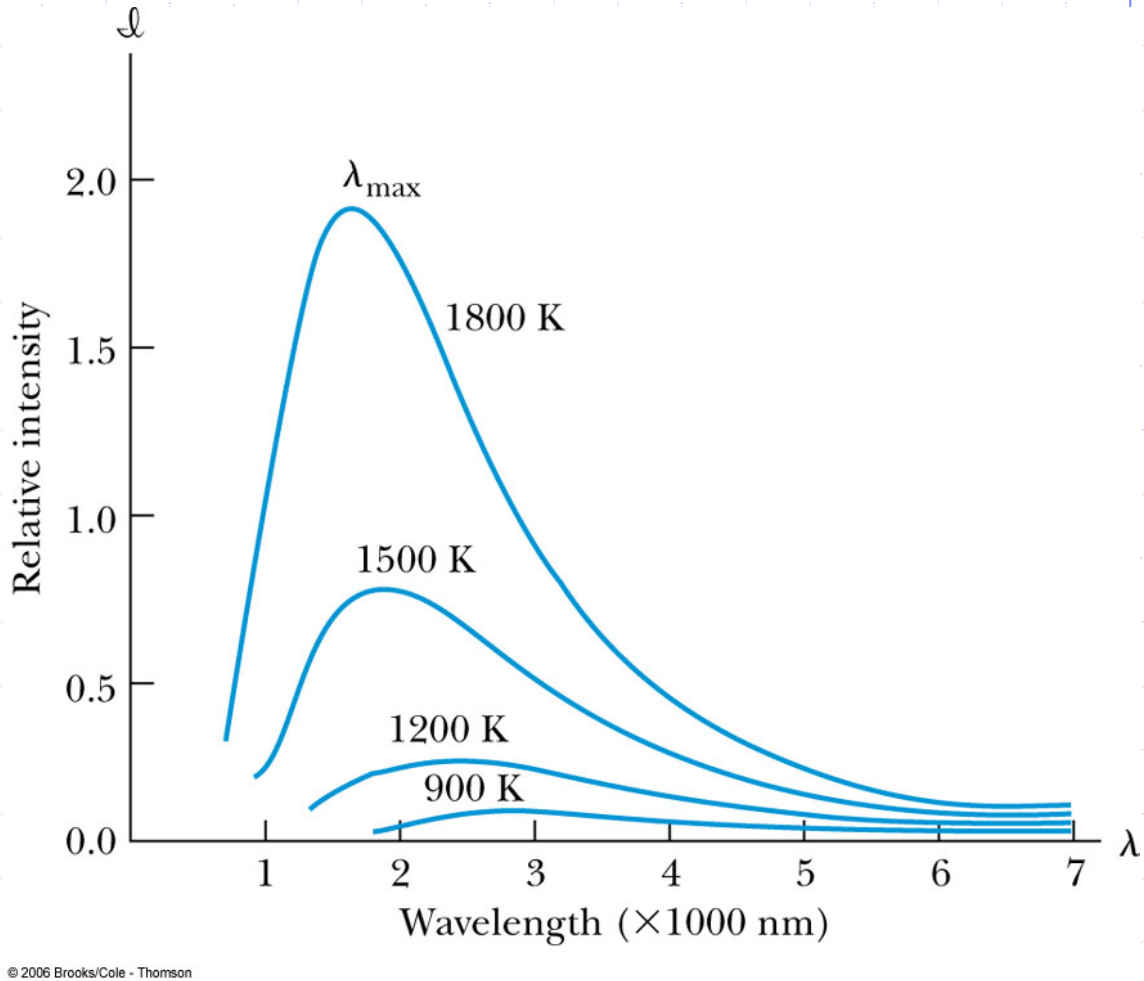
Blackbody Radiation

- A blackbody is a perfect absorber of radiation
- A simple blackbody is given by a hole in a wall of some enclosure
- Both absorption and emission can occur
- The radiation properties of the cavity are independent of the enclosure material

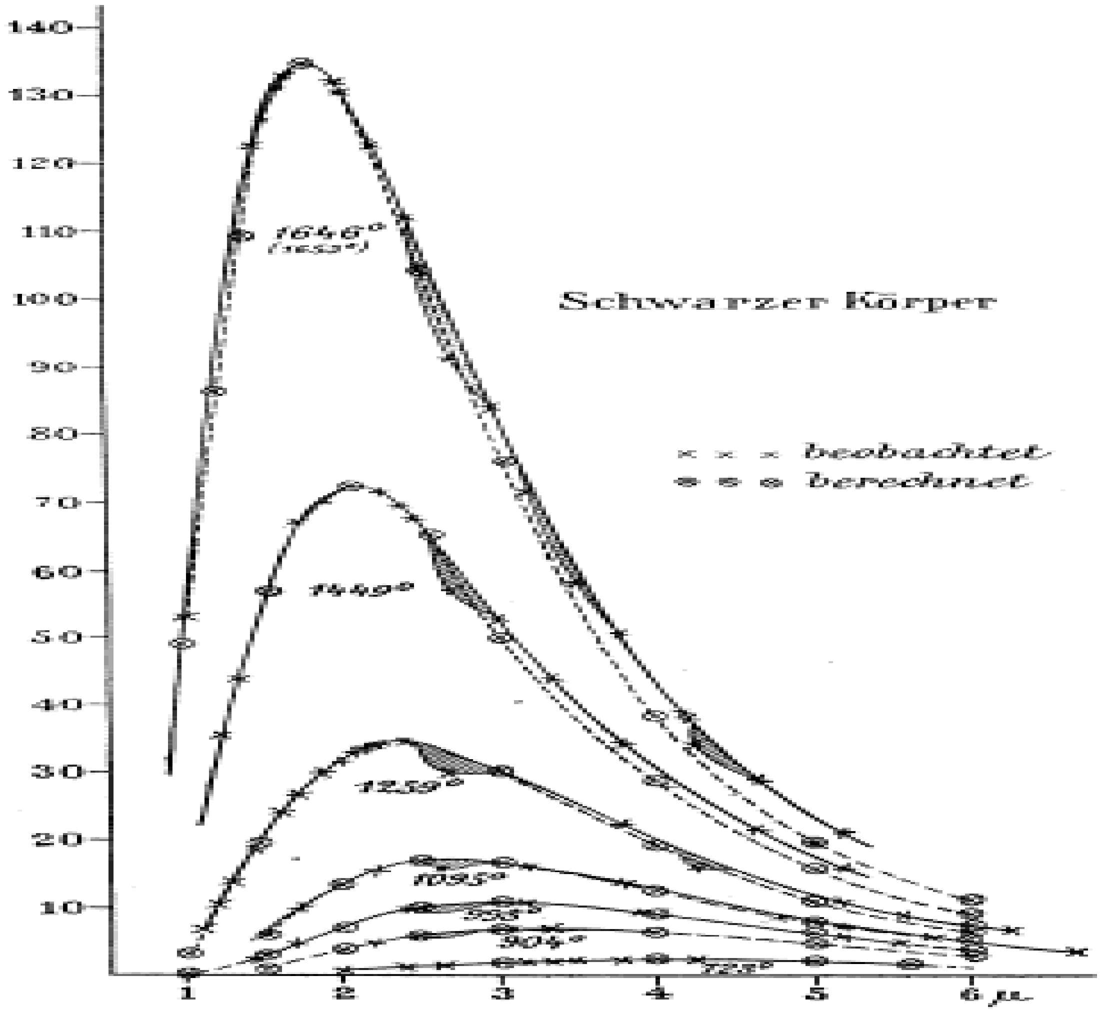


Blackbody Radiation

- The y axis is energy / time / area / wavelength



Blackbody Radiation



Blackbody Radiation

- Wien's displacement law

$$\lambda_{\max} T = 2.898 \times 10^{-3} mK$$

  - Wavelength decreases as T increases

- Stefan-Boltzmann law

$$R(T) = \varepsilon \sigma T^4$$

$$\sigma = 5.6705 \times 10^{-8} W / \left(m^2 K^4\right)$$

  - Total power / area radiated increases as T^4

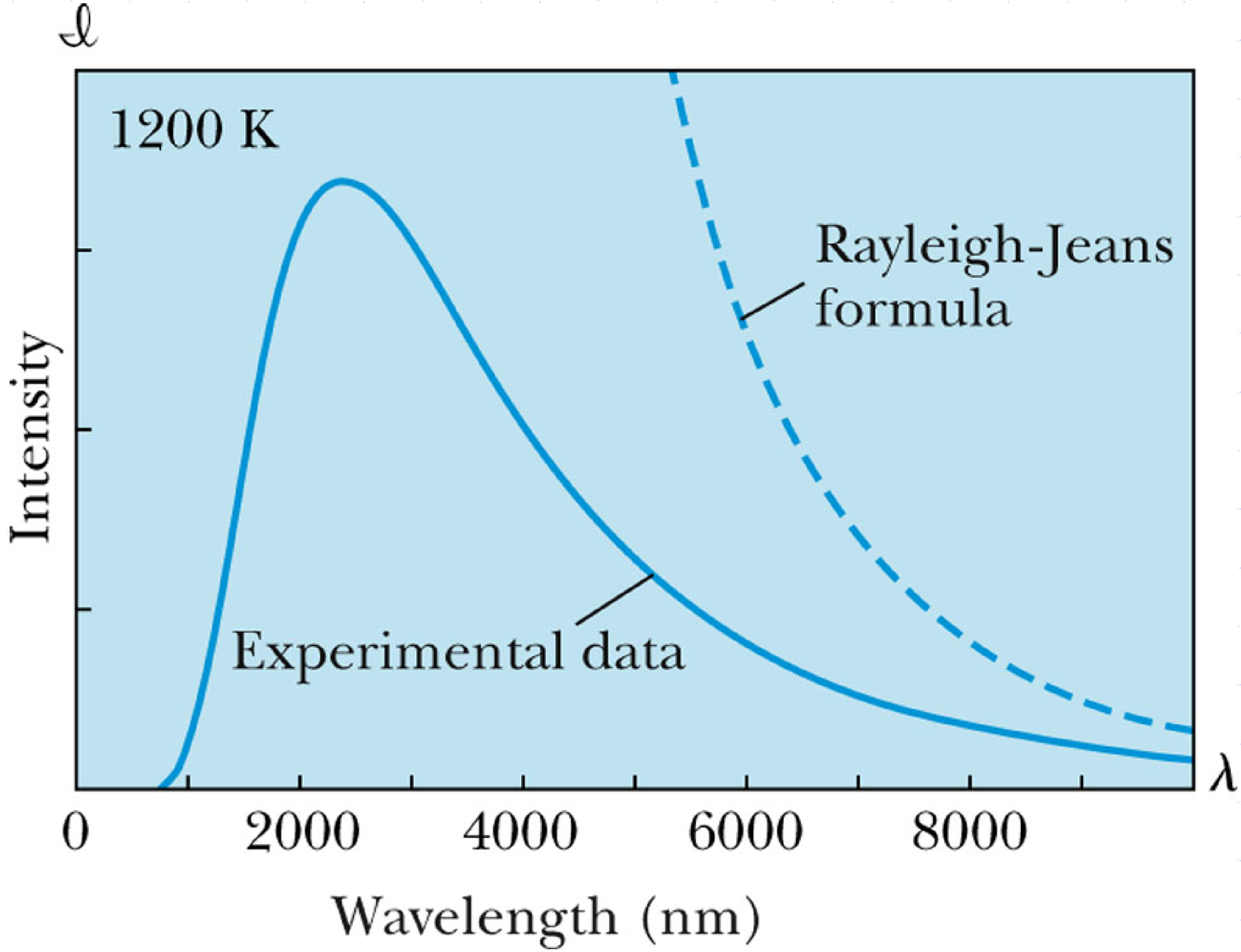
Blackbody Radiation

- Attempts to calculate the spectral distribution of blackbody radiation from first principles failed
- The best description was given by the Rayleigh-Jeans formula

$$I(\lambda, T) = \frac{2\pi ckT}{\lambda^4}$$

- This described the distribution at long wavelengths but increased without limit as $\lambda \rightarrow 0$
  - Ultraviolet catastrophe

Blackbody Radiation



Blackbody Radiation

- Planck was able to calculate the correct distribution by assuming energy was quantized (he was desperate)
  - Microscopic (atomic) oscillators can only have certain discrete energies

$$E_n = nhf$$

$$h = 6.6261 \times 10^{-34}\,Js$$

  - The oscillators can only absorb or emit energy in multiples of

$$\Delta E = hf$$

Blackbody Radiation

- Planck's radiation law agreed with data

$$I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

- It leads directly to Wien's displacement law and the Stefan-Boltzmann law
- It agrees with Rayleigh-Jeans formula for large wavelengths
  - See derivations for both in Thornton and Rex

Blackbody Radiation

- We represent a blackbody by a cavity heated to temperature T and connected to the outside by a small hole
- We'll assume a metal cavity in the form of a cube (an oven with a pinhole)
- Thermal agitation causes the electrons in the wall to oscillate (accelerate) thus producing electromagnetic radiation
- The electromagnetic radiation forms standing waves inside the cavity with nodes at the metallic surfaces

Blackbody Radiation

- The calculation of Planck's law has five parts
  - $N(f)df$ = Number of standing waves with frequencies between f and $f+df$
    - We'll do the one-dimensional case and just write down the result for the three-dimensional case
  - ε = average energy per standing wave
    - We'll do the calculation
  - Divide by the volume
  - Change variables from frequency to wavelength
  - Multiply by $c/4$ to change from energy/volume/wavelength to energy/time/area/wavelength

Blackbody Radiation

- We first calculate the number of standing waves in the frequency interval from *f* to *f+df*
- Consider a one-dimensional "cavity" of length a (think of possible waves on a string with fixed endpoints)
- The electric field is given by

$$E(x,t) = E_0 \sin\left(\frac{2\pi x}{\lambda}\right)\sin(2\pi f t)$$

where $f\lambda = c$

  - So the amplitude has a sinusoidal space variation which is oscillating in time sinusoidally

Blackbody Radiation

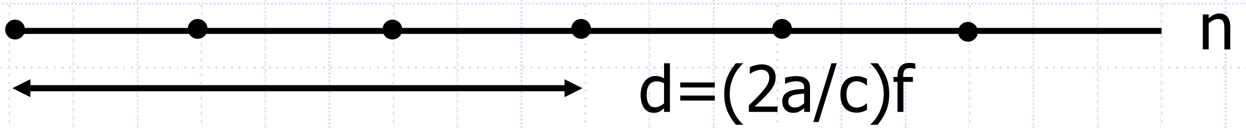
- We want the amplitude of the electric field to vanish at x=0 and x=a
  - At x=0, this is satisfied automatically
  - At x=a, we must have $\frac{2a}{\lambda} = n, \ \mathrm{n} = 1,2,3,\ldots$
- This determines the allowed values for the wavelength
  - What do possible standing waves look like?

Blackbody Radiation

- It's a bit easier to work in terms of frequency

$$f = \frac{cn}{2a}$$

- We represent the allowed values of frequency on a line where we plot a point at every integral value of *n*



- We'll use this line to find *N(f)df*, the number of allowed frequencies (standing waves) in the range *f* to *f+df*

Blackbody Radiation

- Looking at the line, the number of points between *f* and *f+df* is

$$N(f)df = 2 \times \frac{2a}{c} df$$

  - Where we multiplied x2 to account for the two possible polarizations of the electromagnetic wave

Blackbody Radiation

- The calculation for a three-dimensional cavity is similar but somewhat more complicated
- We'll just write down the result

$$N(f)df = \pi\left(\frac{2a}{c}\right)^3 f^2 df$$

- And for later use note

$$\frac{df}{d\lambda} = -\frac{c}{\lambda^2} \text{ since } f = \frac{c}{\lambda}$$

Blackbody Radiation

- Next we calculate the average energy per standing wave
- Classically this is just

$$\bar{\varepsilon} = \frac{\int_0^\infty \varepsilon P(\varepsilon)\, d\varepsilon}{\int_0^\infty P(\varepsilon)\, d\varepsilon} = \frac{\int_0^\infty \frac{\varepsilon}{kT} e^{-\frac{\varepsilon}{kT}}\, d\varepsilon}{\int_0^\infty \frac{1}{kT} e^{-\frac{\varepsilon}{kT}}\, d\varepsilon} = kT$$

where $P(\varepsilon)$ is the Boltzmann factor $\frac{1}{kT} e^{-\frac{\varepsilon}{kT}}$

and we used $\int \mathrm{x}e^{\mathrm{cx}} = \frac{e^{cx}}{c^2}(cx-1)$

- This is the same result the equipartition theorem gives for two degrees of freedom

Kinetic Theory of Gases

- Based on "atomic" theory of matter
- Results include
  - Speed of a molecule in a gas
    - $v_{rms} = (<v^2>)^{1/2} = (3kT/m)^{1/2}$
  - Equipartition theorem
    - Internal energy $U = f/2\ NkT = f/2\ nRT$
  - Heat capacity
    - $C_V = (dU/dT)_V = f/2\ R$
  - Maxwell speed distribution

$$f(v) = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

Blackbody Radiation

- Continuing with classical calculation we have

$$u(f)df = \pi\left(\frac{2a}{c}\right)^3 f^2 df \times kT \times \frac{1}{a^3} = \frac{8\pi f^2 kT}{c^3} df$$

- And changing variables from frequency to wavelength

$$u(\lambda)d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

- Multiply by *c/4* to find the Rayleigh-Jeans formula

$$I(\lambda, T) = \frac{2\pi ckT}{\lambda^4}$$

Blackbody Radiation

- Now redo the calculation ala Planck
- Let *E=nhf*, then the average energy is

$$\bar{\varepsilon} = \frac{\sum_{n=0}^{\infty} \varepsilon P(\varepsilon)}{\sum_{n=0}^{\infty} P(\varepsilon)} = \frac{\sum_{n=0}^{\infty} \frac{nhf}{kT} e^{-\frac{nhf}{kT}}}{\sum_{n=0}^{\infty} \frac{1}{kT} e^{-\frac{nhf}{kT}}} = \frac{kT \sum_{n=0}^{\infty} n\alpha e^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}}$$

where $\alpha = \frac{hf}{kT}$

and $\frac{1}{kT} e^{-\frac{\varepsilon}{kT}}$ is the Boltzmann factor P(ε)

Blackbody Radiation

- To evaluate this we use standard "tricks" from statistical mechanics

$$-\alpha\frac{d}{d\alpha}\ln\sum_{n=0}^{\infty}e^{-n\alpha}=\frac{-\alpha\frac{d}{d\alpha}\sum_{n=0}^{\infty}e^{-n\alpha}}{\sum_{n=0}^{\infty}e^{-n\alpha}}=\frac{-\sum_{n=0}^{\infty}\alpha\frac{d}{d\alpha}e^{-n\alpha}}{\sum_{n=0}^{\infty}e^{-n\alpha}}$$

$$=\frac{\sum_{n=0}^{\infty}n\alpha e^{-n\alpha}}{\sum_{n=0}^{\infty}e^{-n\alpha}}$$

Blackbody Radiation

- And note

$$\sum_{n=0}^{\infty} e^{-n\alpha} = 1 + e^{-\alpha} + e^{-2\alpha} + e^{-3\alpha} + \ldots$$

$$= 1 + X + X^{2} + X^{3} + \ldots$$

$$= (1 - X)^{-1}$$

Blackbody Radiation

- Putting these together we have

$$\bar{\varepsilon} = kT\left(-\alpha \frac{d}{d\alpha} \ln \sum_{n=0}^{\infty} e^{-n\alpha}\right) = -hf \frac{d}{d\alpha} \ln \sum_{n=0}^{\infty} e^{-n\alpha}$$

$$\bar{\varepsilon} = -hf \frac{d}{d\alpha} \ln\left(1 - e^{-\alpha}\right)^{-1} = \frac{-hf}{\left(1 - e^{-\alpha}\right)^{-1}}(-1)\left(1 - e^{-\alpha}\right)^{-2} e^{-\alpha}$$

$$\bar{\varepsilon} = \frac{hfe^{-\alpha}}{1 - e^{-\alpha}} = \frac{hf}{e^{\alpha} - 1} = \frac{hf}{e^{\frac{hf}{kT}} - 1} = \frac{hc}{\lambda} \frac{1}{e^{\frac{hc}{\lambda kT}}}$$

Blackbody Radiation

- We already know the number of standing waves per volume

$$N(\lambda)d\lambda = \frac{8\pi}{\lambda^4}d\lambda$$

- So the energy per volume is

$$u(\lambda)d\lambda = \frac{hc}{\lambda}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\frac{8\pi}{\lambda^4}d\lambda$$

- And changing units to spectral intensity (multiply by c/4) gives Planck's formula

$$I(\lambda,T) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$$

Blackbody Radiation

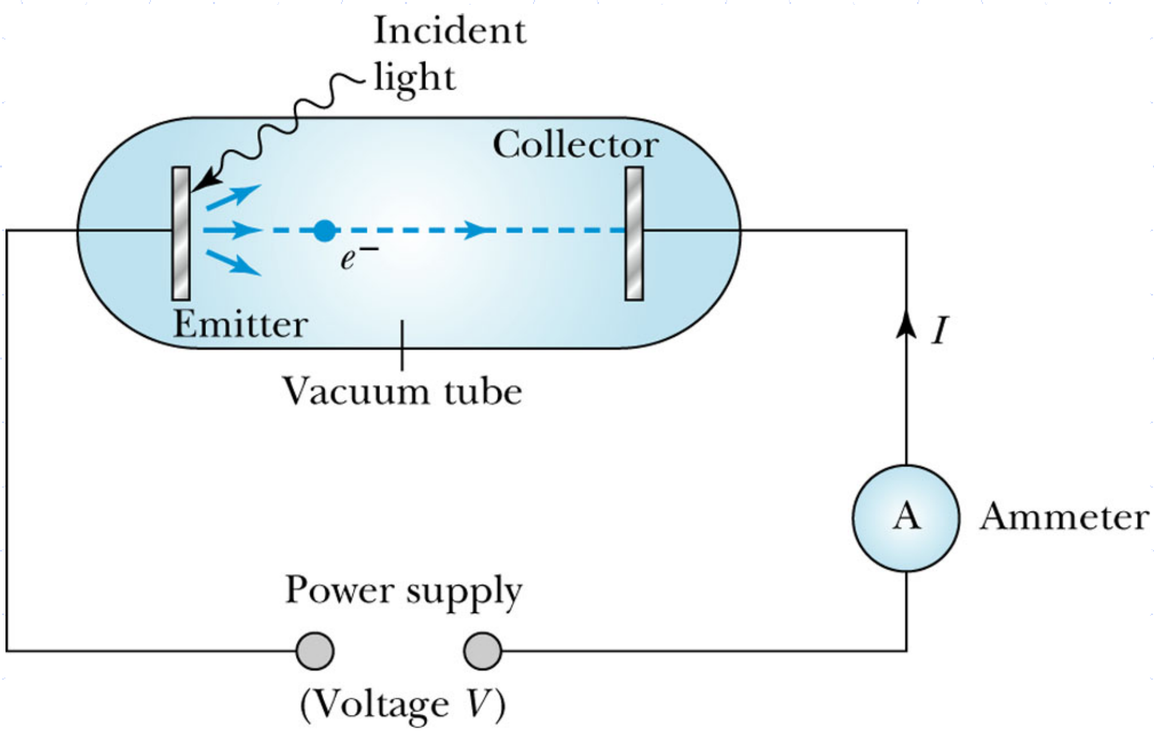
- You can see how Planck avoided the ultraviolet catastrophe
  - Because the energy is proportional to the frequency
    - The average energy is kT when the possible energies are small compared to kT
    - The average energy is extremely small when the possible energies are large compared to kT (because P(ε) is extremely small)

Blackbody Radiation

- Planck's paper is generally considered to be the birthplace of quantum mechanics
  - Revisionist history?
  - Planck did not pay too much attention to energy quantization
  - Neither did anyone else
  - There is controversy of whether he even intended the energy of an oscillator to be nhf

Photoelectric Effect

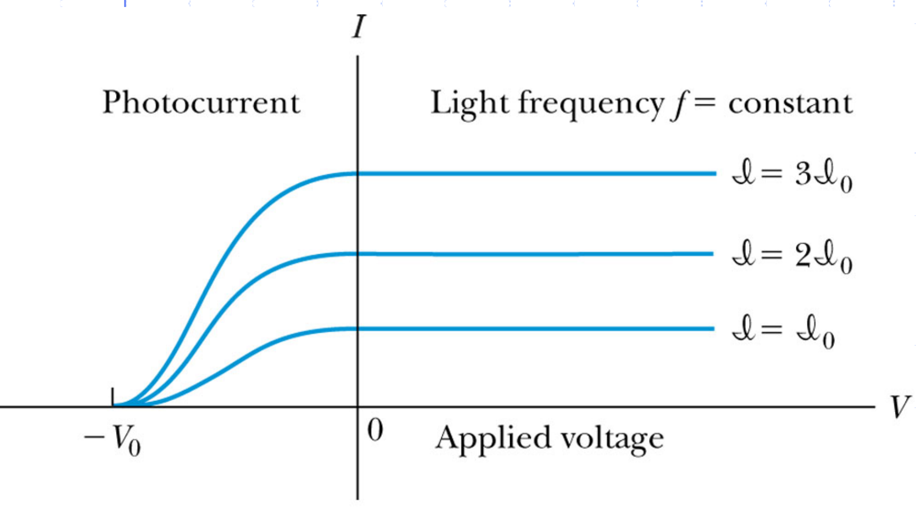
- Light incident on a metal will eject electrons
  - Aside, other means of doing this are with temperature, electric fields, and particle bombardment

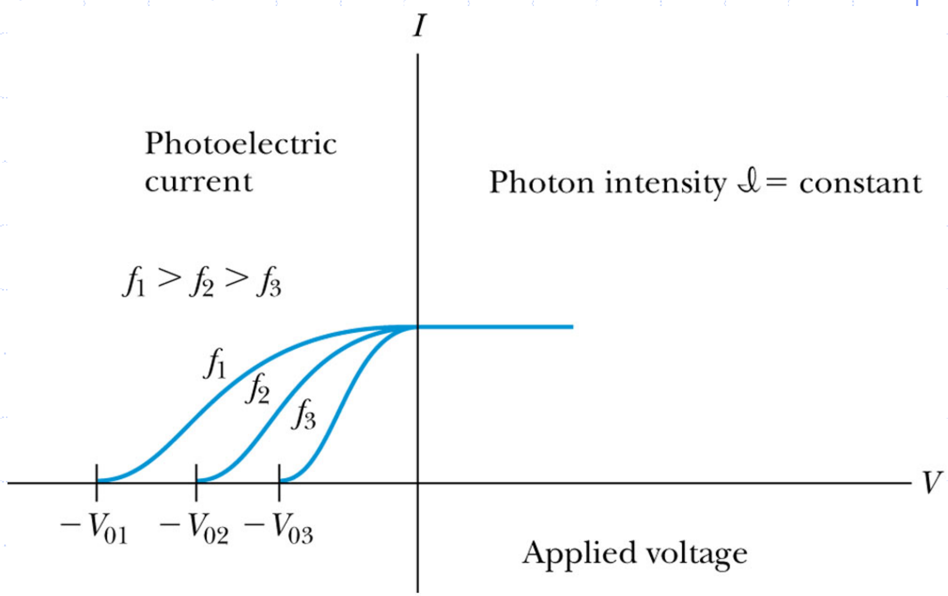


Photoelectric Effect

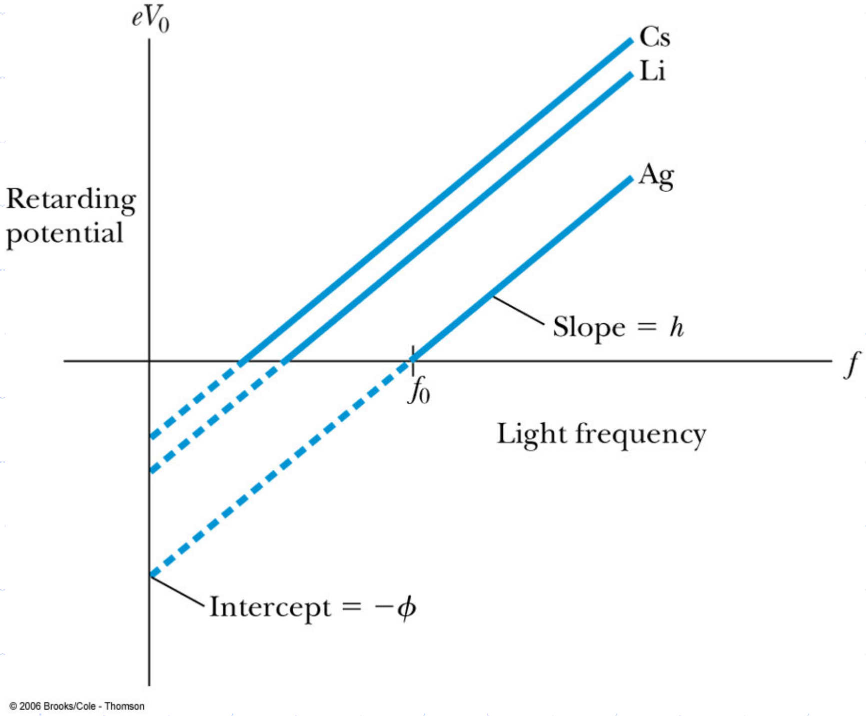
- Experiments showed
  - The kinetic energy of the photoelectrons are independent of the light intensity
  - The maximum kinetic energy of the photoelectrons depends on the light frequency
  - The smaller the work function φ the smaller the threshold frequency to produce photoelectrons
  - The number of photoelectrons is proportional to light intensity
  - The photoelectrons are emitted instantaneously (on the order of nanoseconds)

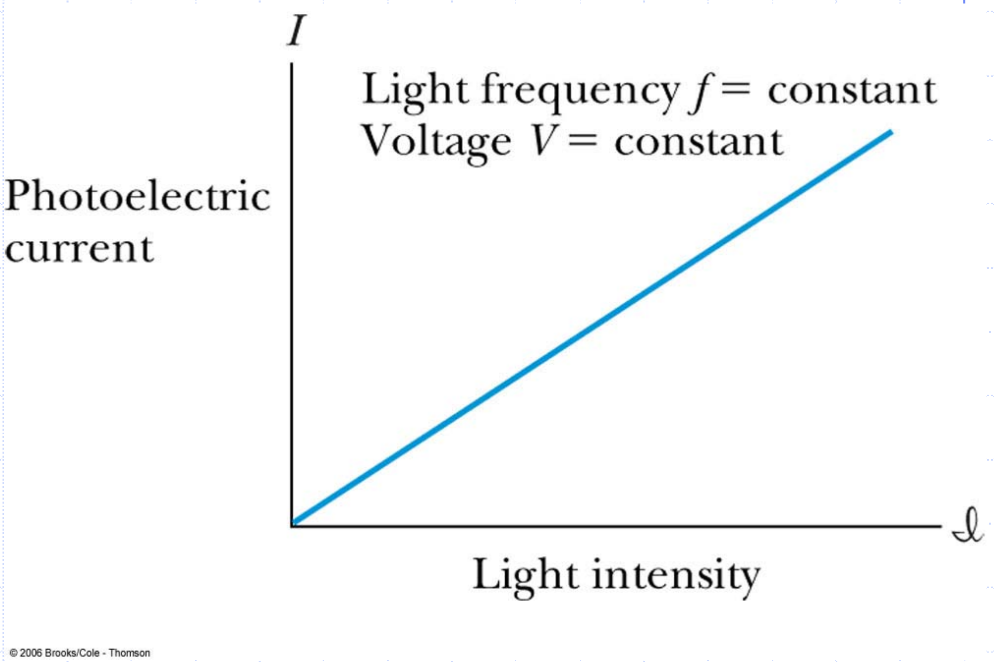
Photoelectric Effect





Photoelectric Effect





Photoelectric Effect

- Explained by Einstein in one of his annus mirabilis papers
- In his paper he assumed
  - Electromagnetic field was quantized
  - Light quanta were localized in space (like particles) == photons
  - Energy $E = hf$
  - In the photoelectric process, the energy quanta (photons) are completely absorbed

Photoelectric Effect

- Thus photons penetrate the surface of the metal and are absorbed by electrons
- The electrons overcome attractive forces that normally hold them in the material and escape
- Conservation of energy gives
  - $hf = 1/2mv^2_{max} + \varphi$
- And consequently he predicted
  - $1/2mv^2_{max} = eV_0 = hf - \varphi$
  - Note h/e can be measured from the slope

Photoelectric Effect

- This was strange since it involved Planck's constant h
- This was a difficult experiment to carry out
  - It took almost a decade to verify
  - Millikan was the principle experimenter (who was trying to prove Einstein's theory wrong)
- The end result was proof that light energy is quantized and E=hf

Photoelectric Effect