Units

- The work done in accelerating an electron across a potential difference of 1V is
  - W = charge x potential
  - $W = (1.602\times10^{-19} C)(1V) = 1.602\times10^{-19}$ J
  - $W = (1e)(1V) = 1$ eV
- $1 \text{ eV} = 1.602\times10^{-19}$ J
  - In particle and nuclear physics, MeV is more common
    - $\text{MeV} = 10^6$ eV
    - $\text{GeV} = 10^9$ eV
    - $\text{TeV} = 10^{12}$ eV

Units

- $E^2=p^2c^2 + m^2c^4$
  - Units of energy = MeV
  - Units of momentum = MeV/c
  - Units of mass = MeV/c^2
- Rest energy of a proton = mc^2 = 938 MeV
  - $E=(1.67\times10^{-27}kg)(3\times10^{8}m/s)^2=1.5\times10^{-10}J$
  - $E=(1.5\times10^{-10}J)(1eV/1.602\times10^{-19}J)=938$ MeV
- Mass of a proton = 938 MeV/c^2
- Mass of an electron = 0.511 MeV/c^2

Relativistic Energy

- Photons
  - $m=0$ so $E = pc$
  - $E = \gamma mc^2 = pc = \gamma muc$ so $u=c$ for a massless particle

Binding Energy

- Of course, any potential energy must be added to the kinetic energy and rest mass energy
- One common potential energy in atomic and nuclear physics is called the binding energy
  - Binding energy is the potential energy associated with the force that holds a system together
  - Binding energy is the energy needed to separate the system into its constituent parts

Binding Energy

- Binding energy B

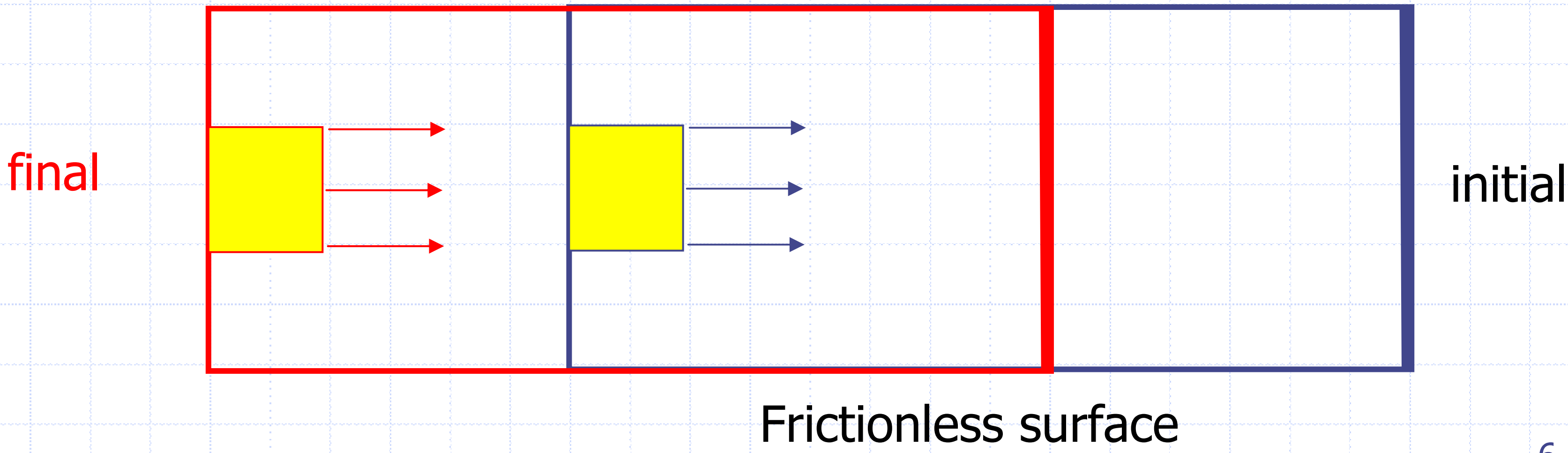
$$B = \sum_i m_i c^2 - M_{bound} c^2$$

- Hydrogen atom
  - proton + electron = hydrogen atom
  - 938.272 MeV + 0.511 MeV - 938.791 MeV = 13.6eV

- Deuteron
  - proton + neutron = deuteron
  - 938.27MeV + 939.57MeV - 1875.61MeV = 2.23MeV

Relativistic Energy

- $E = \gamma mc^2$ means mass and energy are equivalent
- Commonplace in nuclear and particle physics
- Einstein's box



Relativistic Energy

- The laser fires a burst of photons
  - The photons carry momentum p
  - The box of mass M recoils (nonrelativistically)
    - $Mv = p = E/c$
  - When the photons are absorbed on the far side of the box the motion stops
- But there are no external forces on the box hence we would expect the center-of-mass of the box to remain unchanged

Relativistic Energy

- For the center-of-mass to remain unchanged we must have
  - $Mx_1 = mx_2 = mL$
- The box will move to the left x_1
  - $x_1 = vt = vL/c = EL/Mc^2$
- Thus
  - $MEL/Mc^2 = mL$ or $m = E^2/c$
- The energy of the photons is equivalent to a mass m, $E = mc^2$

Four-Vectors

- Given the intimate connection between space and time it is natural to group *ct* and ***x*** into a vector
- We define any four quantities that transform like *(ct,**x**)* under a Lorentz transformation to be a four-vector
- We use the notation

$$\left(ct,\vec{x}\right)=\left(x^0,x^1,x^2,x^3\right)=x^\mu$$

$$\left(\frac{E}{c},\vec{p}\right)=\left(p^0,p^1,p^2,p^3\right)=p^\mu$$

Four-Vectors

- Four-vectors follow the usual rules for vector addition, multiplication, …
- We also define

$$\left(x_0, -x_1, -x_2, -x_3\right) = x_\mu$$

- Upper index vectors == contravariant vectors
- Lower index vectors == covariant vectors
- If there is an upper and lower index of the same type (e.g. μ) a sum over repeated indices is implied

Four-Vectors

- We define the usual scalar product of two four-vectors

$$A \cdot B = A_\mu B^\mu = A^\mu B_\mu = A^0 B^0 - \vec{A} \cdot \vec{B}$$

- Scalar products are Lorentz invariant

$$x^\mu x_\mu = c^2 t^2 - x^2 - y^2 - z^2 = -s^2$$

$$p^\mu p_\mu = \frac{E^2}{c^2} - \vec{p}^{\,2} = m^2 c^2$$

Lorentz Transformation

$$p'_x = \gamma(p_x - VE/c^2) \qquad p_x = \gamma(p'_x + VE'/c^2)$$

$$p'_y = p_y \qquad p_y = p'_y$$

$$p'_z = p_z \qquad p_z = p'_z$$

$$E' = \gamma(E - Vp_x) \qquad E = \gamma(E' + Vp'_x)$$

$$\text{where } \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Lorentz Transformation

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \gamma(t - vx/c^2) \qquad t = \gamma(t' + vx'/c^2)$$

$$\text{where } \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Particle Production

- In particle physics we transform kinetic energy into mass on a daily basis
  - $E=mc^2$ (when $p=0$) so the higher the energy the larger the mass
- Consider the reaction $pp \rightarrow pp\pi^0$
- We have for masses
  - M_p=938 MeV/c^2
  - M_π=135 MeV/c^2
- Can you create a π^0 by colliding a 135 MeV proton with a proton at rest?

Particle Production

- Though one can determine the threshold kinetic energy for π^0 production from energy and momentum conservation considerations, it is easier to use the invariant s
  - s^2 = (center-of-mass energy)^2

$$s^2 = (P_1 + P_2)^2 \quad = (P_3 + P_4)^2 \text{any frame}$$

$$s^2 = \left(E_1 + E_2 + \ldots\right)^2 - c^2\left(\vec{p}_1 + \vec{p}_2 + \ldots\right)^2$$

$$s^2 = \left(E_3 + E_4 + \ldots\right)^2 - c^2\left(\vec{p}_3 + \vec{p}_4 + \ldots\right)^2$$

Particle Production

- In the center-of-mass frame, the $pp\pi^0$ can be produced at rest

$$s^2 = (E_3 + E_4 + E_5)^2 = (m_p c^2 + m_p c^2 + m_{\pi^0} c^2)^2$$

$$s^2 = (938 + 938 + 135)^2 MeV^2 = 2011^2 MeV^2$$

- In the lab frame (where one of the protons is at rest)

$$s^2 = (E_1 + E_2)^2 - c^2 (p_1 + p_2)^2$$

$$s^2 = (E_1 + m_p c^2)^2 - c^2 p_1^2 = (E_1 + m_p c^2)^2 - (E_1^2 - m_p^2 c^4)$$

$$s^2 = 2E_1 m_p c^2 + 2m_p^2 c^4 = (m_p c^2 + m_p c^2 + m_{\pi^0} c^2)^2$$

$$(2 \times 938)E_1 + 2(938)^2 = (938 + 938 + 135)^2$$

$$E_1 = 1217.7 MeV$$

$$T_1 = E_1 - m_p c^2 = 279 MeV$$

Particle Production

- Thus it takes a proton with T=280 MeV to collide with a proton at rest to produce a π^0
- Taking a closer look at

$$s^2 = 2E_1 m_p c^2 + 2m_p^2 c^4 = \left(m_p c^2 + m_p c^2 + m_{\pi^0} c^2\right)^2$$

$$m^* = \sqrt{2E_1 m_p c^2 + 2m_p^2 c^4}$$

- The energy available for particle production goes as the square root of the beam energy (in the lab frame)
- This is because energy goes into the motion of the center-of-mass

Particle Production

- In the center-of-mass frame there is no "wasted" energy

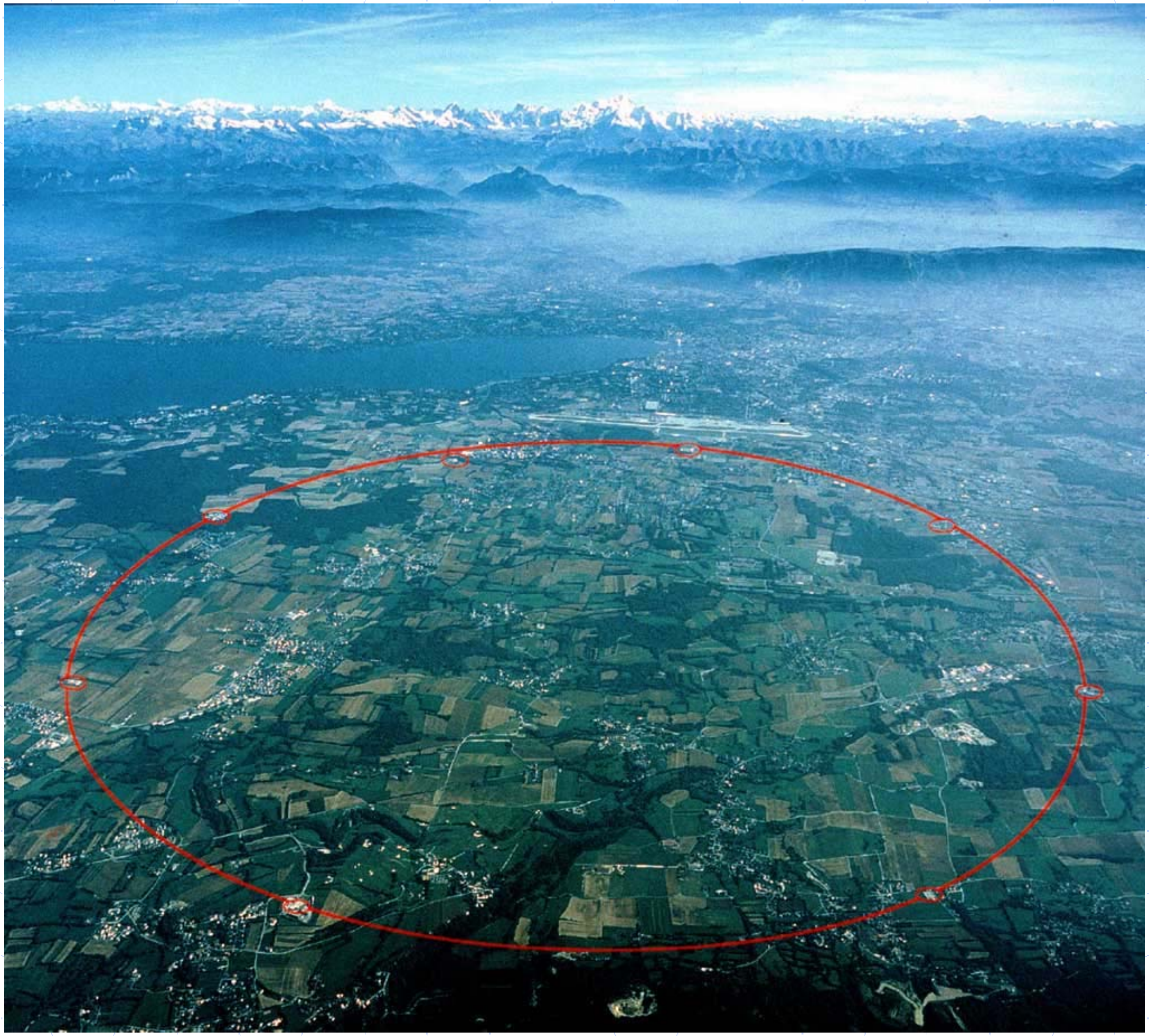
$$s^2 = (P_1 + P_2)^2 = m_1^2 c^4 + m_2^2 c^4 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2)$$

$$m^* \approx \sqrt{4E_1 E_2} = 2E_1$$

- This is why particle physics experiments are performed in the center-of-mass frame (i.e. using colliding beams of particles)

Large Hadron Collider at CERN

- 7 TeV protons colliding with 7 TeV protons



Electromagnetism

- What led Einstein to special relativity was the conviction that Maxwell's equations were invariant in all inertial frames
  - This is called Lorentz covariance
  - It means that the fundamental equations have the same form in all inertial frames

Maxwell's Equations

- In differential form

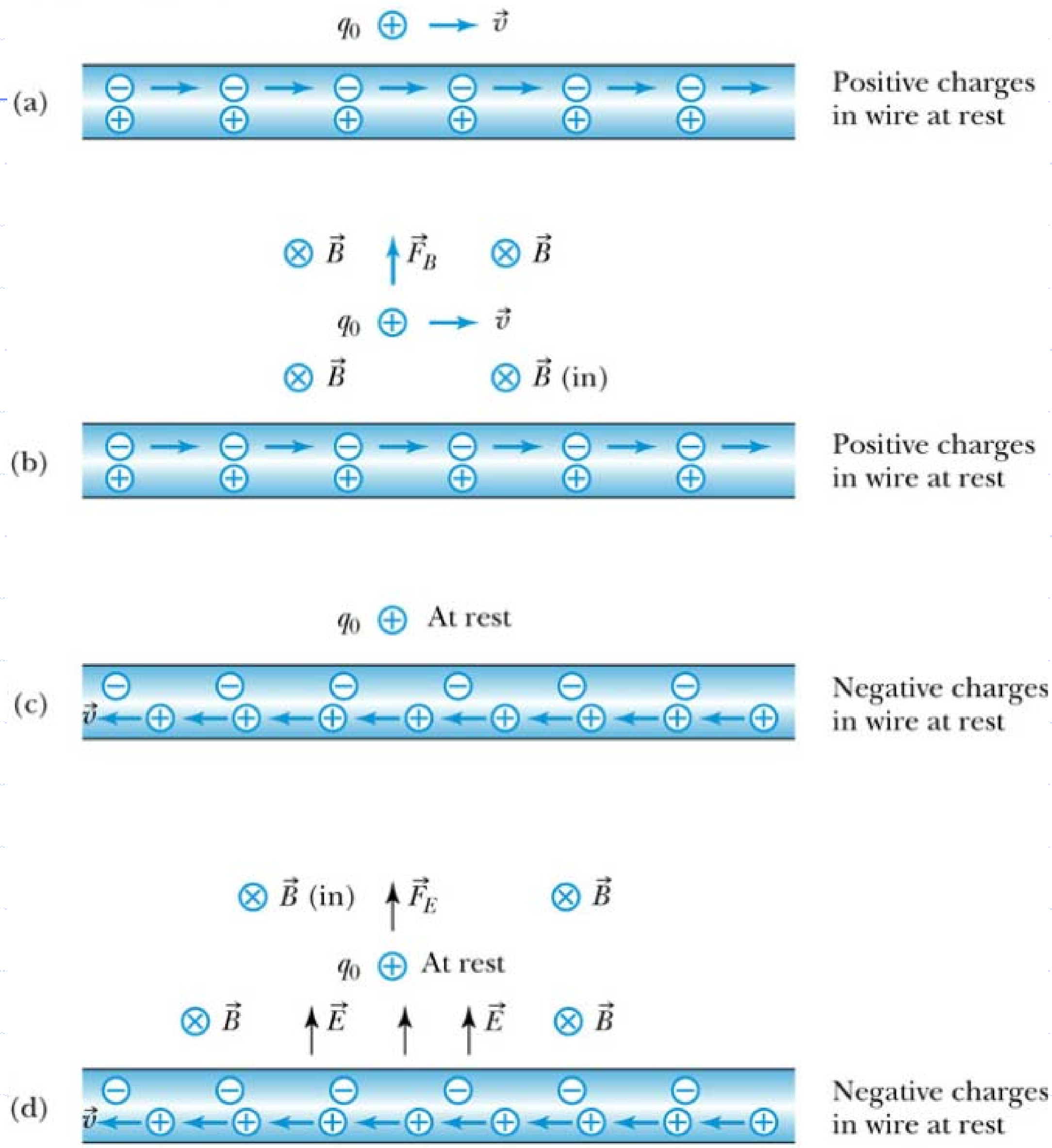
- Gauss's law (E field): $\vec{\nabla} \cdot \vec{E} = \rho / \varepsilon_0$

- Gauss's law (B field): $\vec{\nabla} \cdot \vec{B} = 0$

- Faraday's law: $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

- Ampère's law: $\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$

Electromagnetism



Electromagnetism

- In frame K (Figure (a)), what is the force on q_0?
  - Pay attention to the orientation of the positive and negative charges. The wire is neutral.
  - The answer is

$$\vec{F} = \frac{q\mu_0\lambda v^2}{2\pi r}\hat{i}$$

Electromagnetism

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{E} = 0$$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 \lambda v}{2\pi r} \text{ (Ampere's law)}$$

$$\text{thus } \vec{F} = \frac{q\mu_0 \lambda v^2}{2\pi r}\hat{i}$$

Electromagnetism

- In frame K′ (Figure (c)), what is the force on q_0?
  - Pay attention to the orientation of the positive and negative charges. The wire is neutral.
  - The answer is

$$\vec{F}' = \gamma \frac{q\mu_0 \lambda v^2}{2\pi r}\hat{i}$$

Electromagnetism

$$\vec{F}' = q(\vec{E}' + \vec{v}' \times \vec{B}')$$

$$\vec{v}' \times \vec{B}' = 0$$

$$\lambda'_+ = \lambda\gamma \text{ and } \lambda'_- = -\lambda/\gamma$$

$$\lambda' = \lambda'_+ + \lambda'_- = \frac{\lambda}{\sqrt{1-\frac{v^2}{c^2}}} - \lambda\sqrt{1-\frac{v^2}{c^2}} = \gamma\lambda\frac{v^2}{c^2}$$

$$E' = \gamma\lambda\frac{v^2}{c^2}\frac{1}{2\pi r}\frac{1}{\varepsilon_0} \text{ (Gauss' law)}$$

$$\vec{F}' = \gamma\frac{q\mu_0\lambda v^2}{2\pi r}\hat{i} \text{ (since } \frac{1}{\mathrm{c}^2} = \mu_0\varepsilon_0)$$

Electromagnetism

- It turns out that in this example (only) where the force is perpendicular to the velocity of the frame AND the test particle is at rest in the K′ frame then

$$F_T = F'_T / \gamma$$

- The electrostatic force in the K′ frame is identical in magnitude (after transformation) to the magnetic force in the K frame
- Whether you observe a magnetic or electric force is frame dependent
- Electrostatics plus relativity implies a magnetic field must exist