Doppler Effect

- The Doppler effect is the shift in frequency and wavelength of waves that results from a moving source and/or moving observer relative to the wave medium
- Consider a point source of traveling waves (a siren) at some frequency f_0
  - We have $\lambda = V/f_0$ where V is the wave velocity and V depends on the characteristics of the medium alone

Doppler Effect

- A source moving towards an observer with velocity v_s
  - The observer sees wave fronts that are "squeezed" in the direction of v_s but the wave velocity remains unchanged

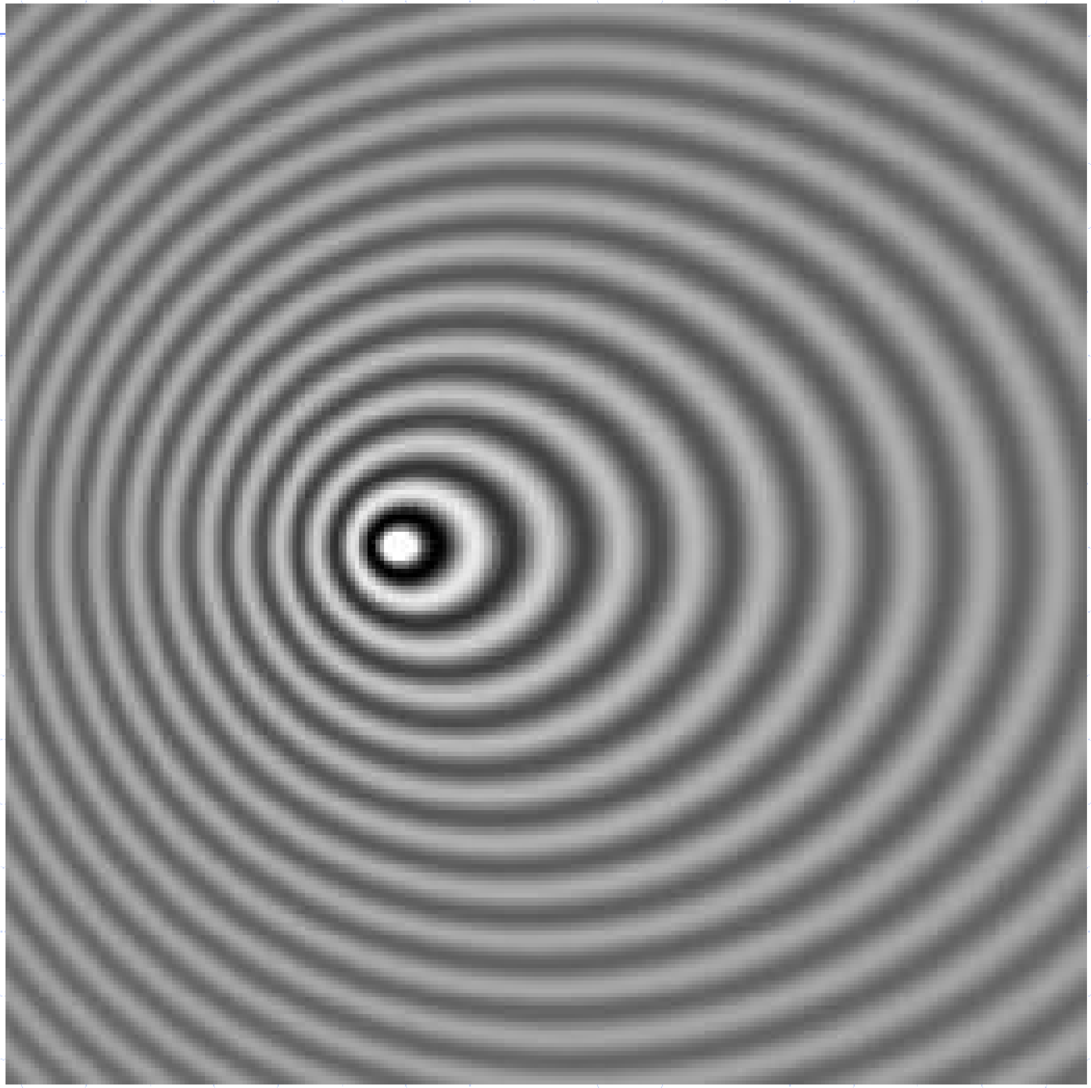
$$\lambda' = \lambda - \frac{v_s}{f_0} = \frac{V - v_s}{f_0}$$

$$f = \frac{V}{\lambda'} = f_0\left(\frac{V}{V - v_s}\right) = \frac{f_0}{1 - \frac{v_s}{V}}$$

- A source moving away from an observer

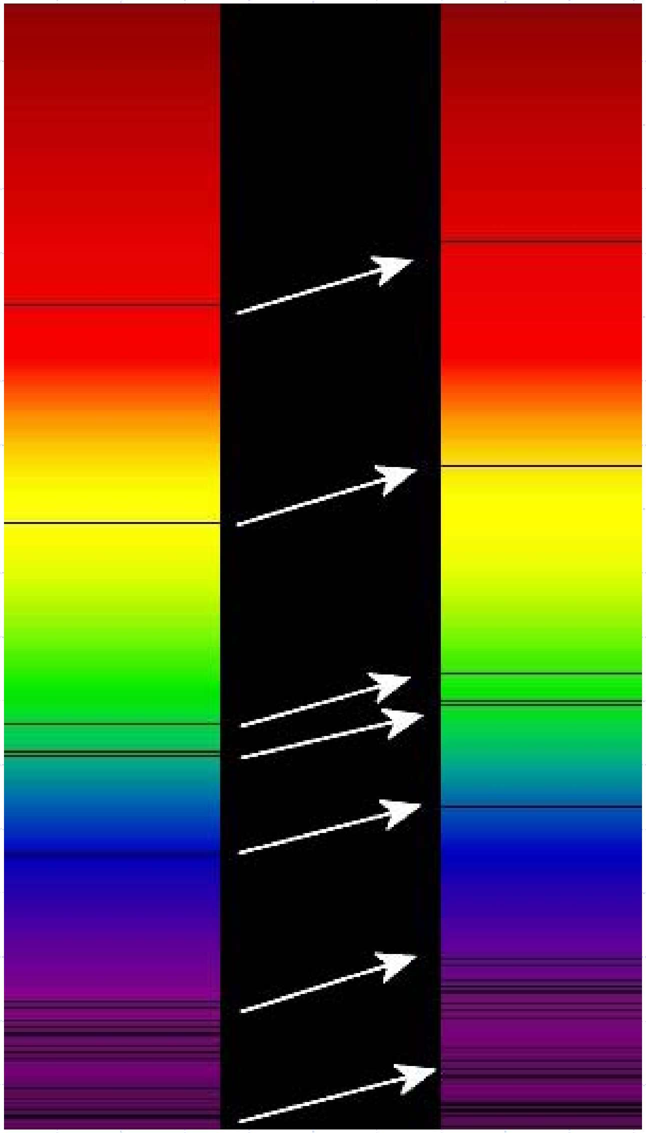
$$f = \frac{f_0}{1 + \frac{v_s}{V}}$$

Doppler Effect



Doppler Effect

Optical spectrum of sun



Optical spectrum of cluster of distant galaxies

Doppler Effect

- An observer moving towards a source with velocity v_o
  - Here an observer will see the same wavelength λ but with a higher wave velocity (e.g. wading into an ocean)

$$v' = V + |v_o|$$

$$f = \frac{v'}{\lambda} = \frac{V + |v_o|}{\lambda} = f_0 + \frac{|v_o|}{\lambda} = f_0\left(1 + \frac{|v_o|}{V}\right)$$

- An observer moving away from a source

$$f = f_0\left(1 - \frac{|v_o|}{v}\right)$$

Doppler Effect

- In the case of light waves
  - Classical, moving source

$$f = \frac{f_0}{1 \mp \frac{V}{c}}$$

  - Classical, moving observer

$$f = \left(1 \pm \frac{V}{c}\right) f_0$$

- Makes sense, the frequency is higher if observer and source approach each other and lower if they are receding

Doppler Effect

- But
  - This is a classical calculation
  - There is an asymmetry between moving source and moving observer
    - Moving source

$$f = \frac{f_0}{1 \mp \frac{V}{c}} \approx \left(1 \pm \frac{V}{c} \pm \frac{V^2}{c^2} \pm \ldots\right) f_0$$

    - Moving observer

$$f = \left(1 \pm \frac{V}{c}\right) f_0$$

Doppler Effect

- The relativistic Doppler effect calculation differs from the classical one because of time dilation
- Consider a source of frequency f_0 fixed in S′
  - As usual, S′ is moving with velocity V relative to S
  - $N = f_0 \Delta t'$ waves are emitted in $\Delta t'$
  - $\Delta t = \gamma \Delta t'$

Doppler Effect

- In S, with the source in S′ moving towards the observer in S
  - During Δt, the first wave moves $c\Delta t$ and the source moves $V\Delta t$
  - The wavelength in S is

$$\lambda = \frac{c\Delta t - V\Delta t}{N} \text{ and } f = \frac{c}{\lambda}$$

$$f = \frac{cN}{c\Delta t - V\Delta t}$$

  - Now

$$N = f_0 \Delta t' = f_0 \frac{\Delta t}{\gamma}$$

Doppler Effect

- Still in S, we find the relativistic Doppler effect for the source approaching

$$f = \frac{c f_0 \sqrt{1-\beta^2}}{c(1-\beta)} = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} f_0$$

- For the source receding, we have

$$f = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} f_0$$

Doppler Effect

- For completeness, consider the source of frequency f_0 be fixed in S and let Δt=λ/c
- In S', let the observer see the source receding

$$c\Delta t' = V\Delta t' + \frac{\lambda}{\gamma} = V\Delta t' + \frac{c\Delta t}{\gamma}$$

$$\Delta t' = \frac{\sqrt{1-\frac{V^2}{c^2}}\,c\Delta t}{c-V} = \frac{\sqrt{1+\frac{V}{c}}}{\sqrt{1-\frac{V}{c}}}\Delta t$$

$$f' = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}\, f_0$$

Doppler Effect

- In S', if the observer sees the source approaching we would find

$$f' = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} f_0$$

Doppler Effect

- In summary
  - For source and/or observer approaching

$$f = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} f_0$$

  - For source and/or observer receding

$$f = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} f_0$$

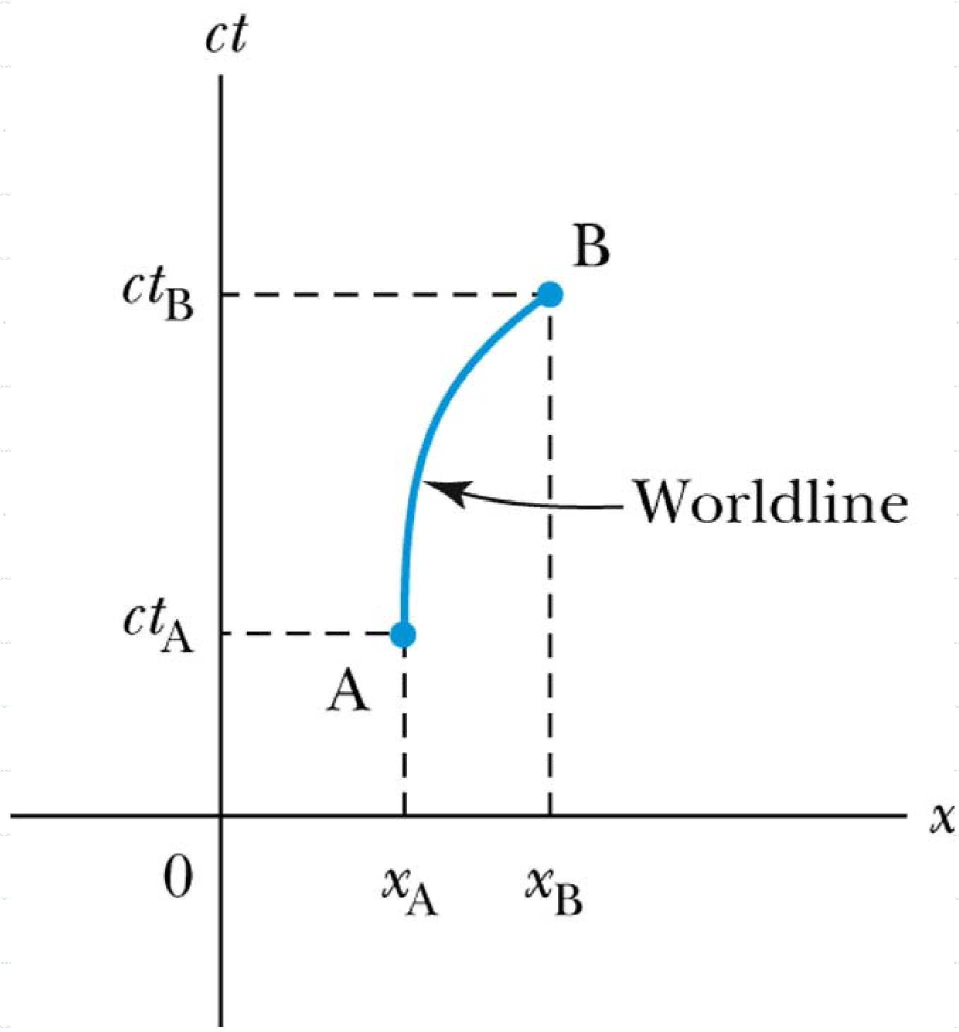
- In special relativity, the two situations (source moving or observer moving) give the same result
- There is also a transverse Doppler effect that does not occur classically

$$f = \frac{f_0}{\gamma}$$

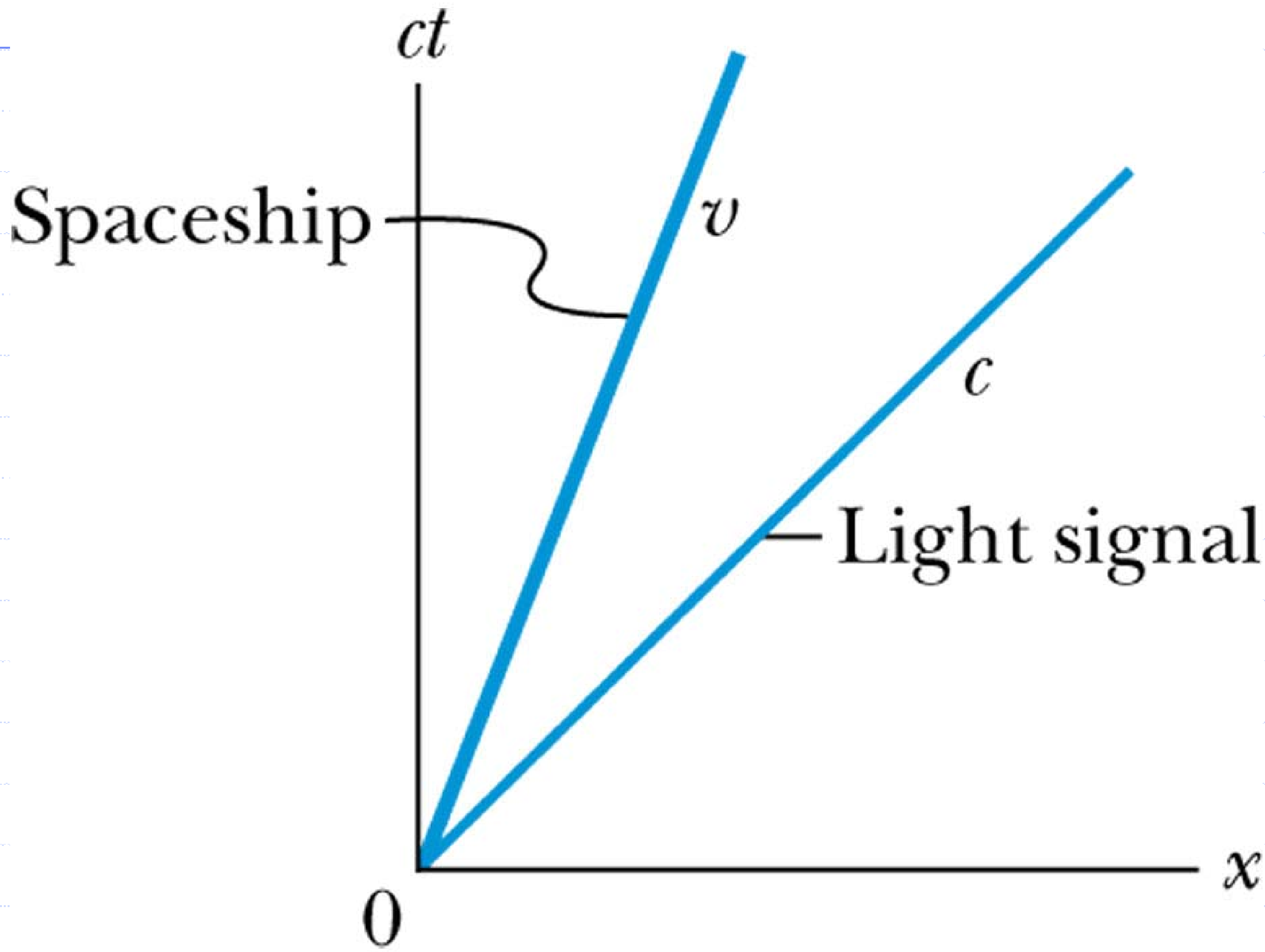
Doppler Effect

- An astronaut tries to get out of a traffic violation for running a red light (λ=670nm) by arguing the Doppler effect made the light appear green (λ=540nm) .
- What was the velocity of the astronaut?

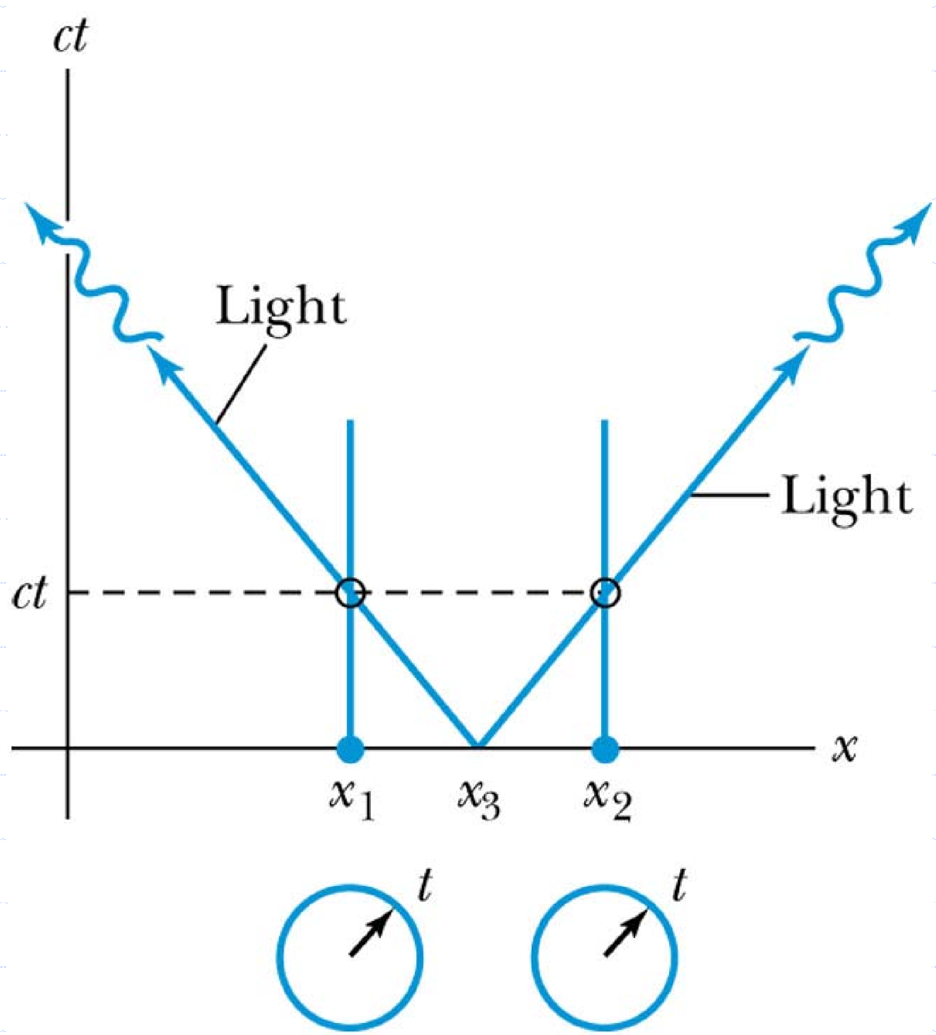
Spacetime Diagrams



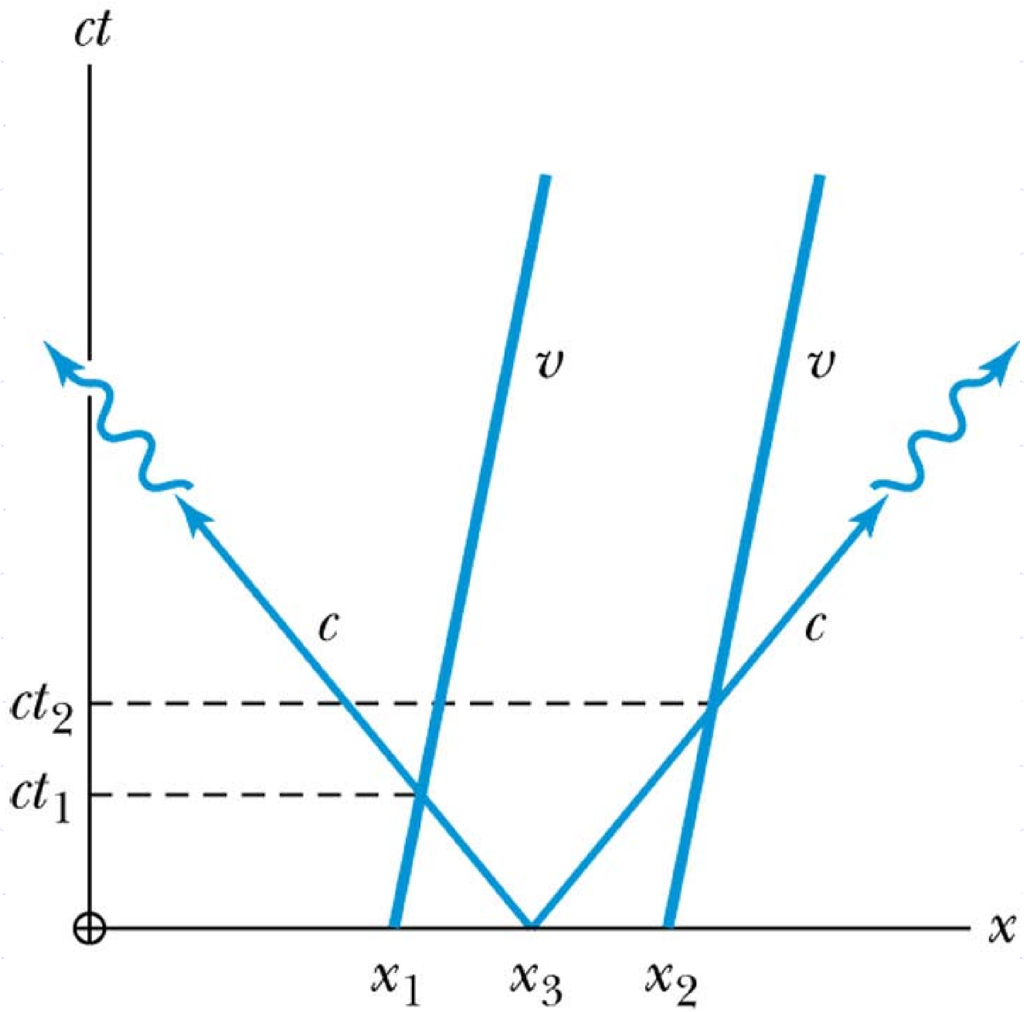
Spacetime Diagrams



Spacetime Diagrams

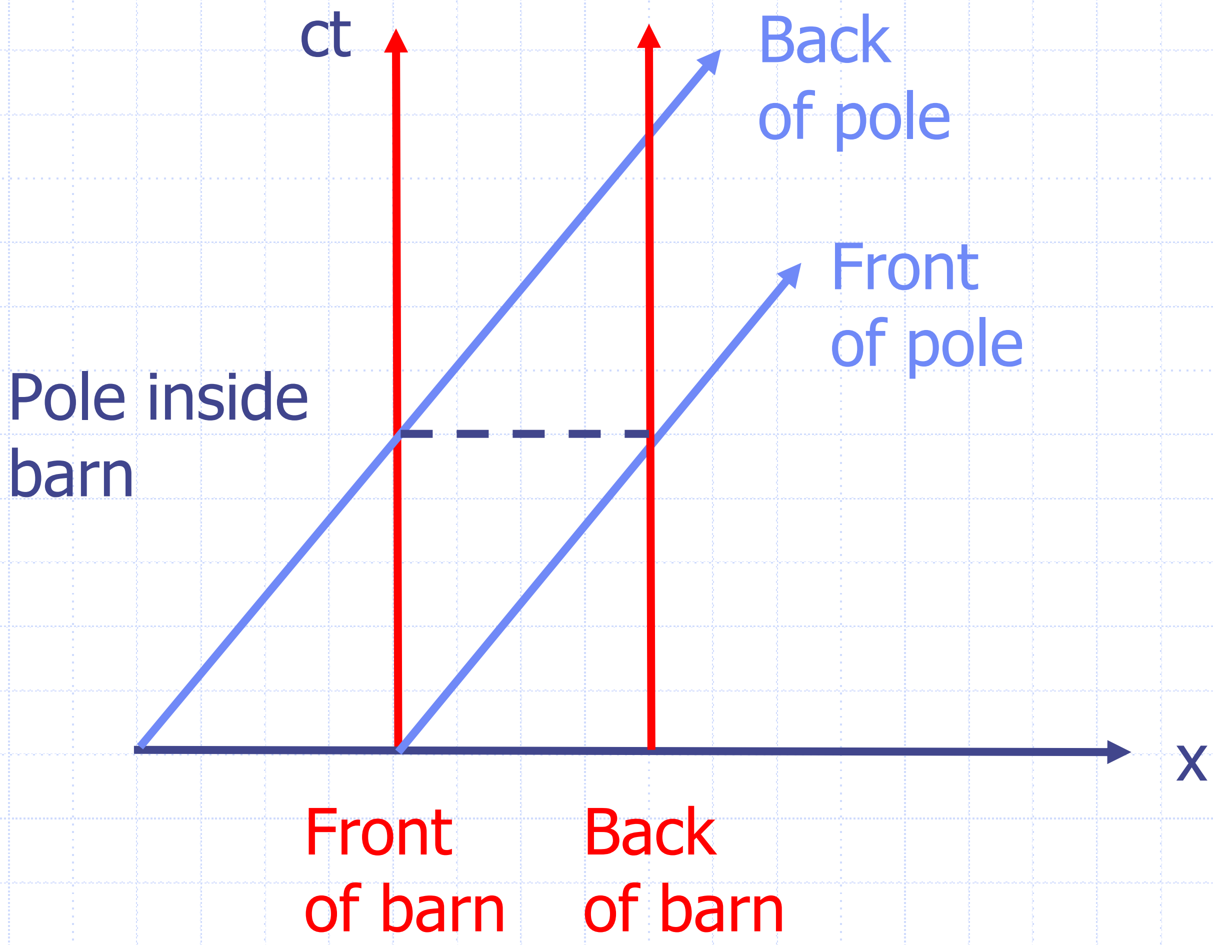


Spacetime Diagrams



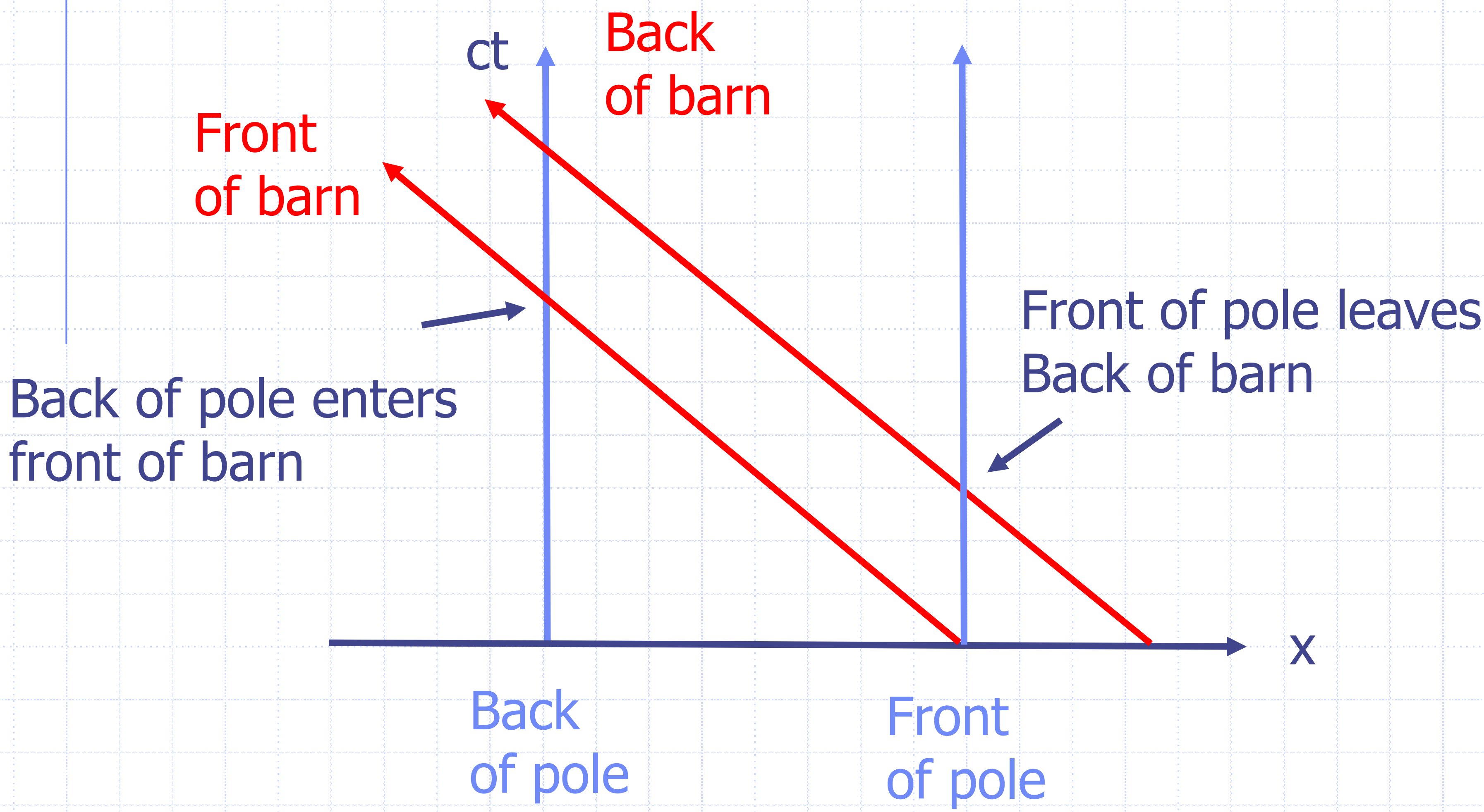
Spacetime Diagrams

- Barn and pole paradox – Barn frame

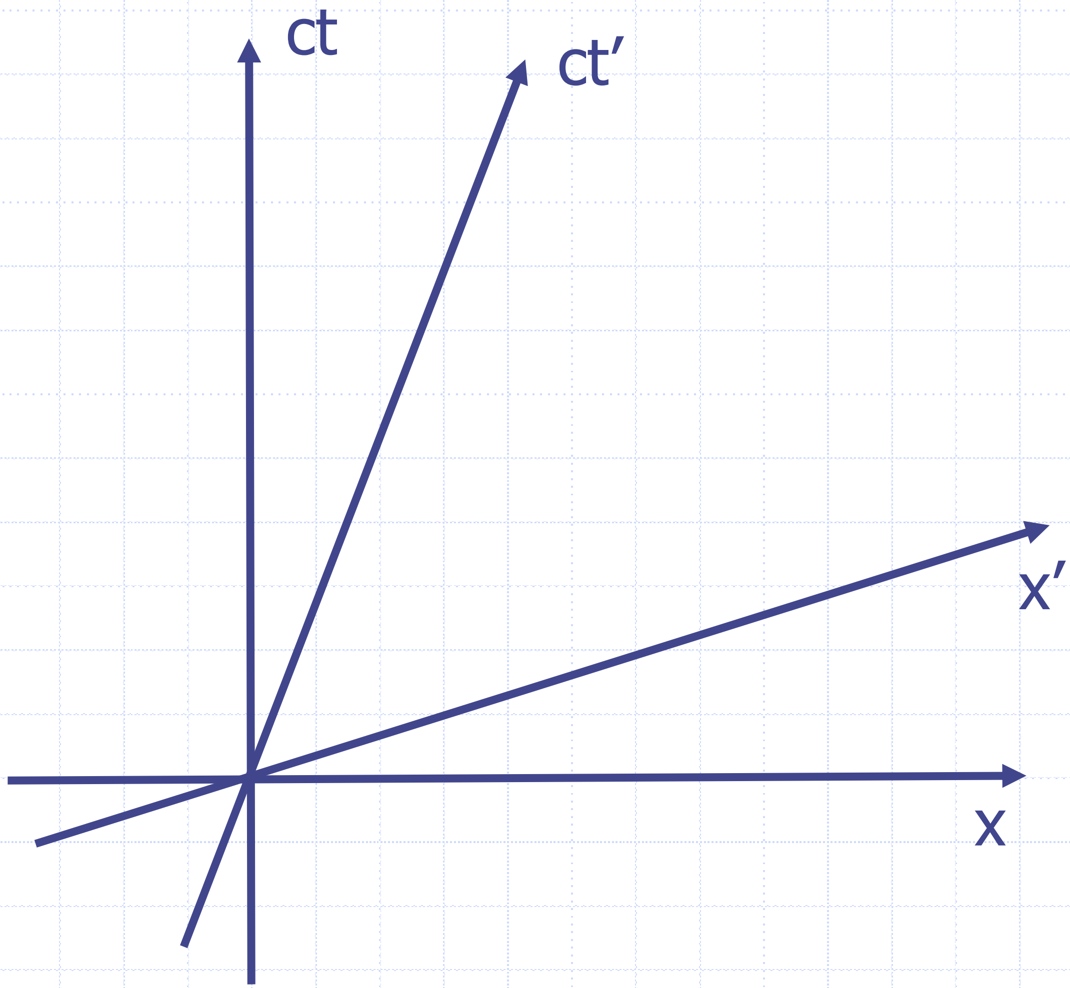


Spacetime Diagrams

- Barn and pole paradox – Pole frame



Spacetime Diagrams



Spacetime Diagrams