Twin Paradox

- Frank and Mary are twins. Mary travels on a spacecraft at high speed (0.8c) to a distant star (8 light-years away) and returns. Frank remains on earth.
- Frank sees Mary's clock running slow hence she is younger than Frank on return.
- Mary sees Frank's clock running slow hence he is younger than Mary on return.
- Who is correct?

Twin Paradox

- In this example $\gamma = 5/3$ and $L_0 = 0.8$ light-years (proper length)
- Consider three frames
  - S, fixed to earth
  - S′, moving (coasting) to star
  - S″, moving (coasting) to earth

Twin Paradox

- Frank in S observes
  - On his clock, it takes Mary $L_0/V=8/0.8=10$ years to reach the star and 10 years to return. Frank has aged 20 years.
  - He observes Mary's clock to be running slow by $1/\gamma$ so Frank observes a time interval in S' for Mary of $10x3/5=6$ years and 6 years in S''. Mary has aged 12 years.

Twin Paradox

- Mary in S′ and S″ observes
  - Her clock is recording proper time and distance she travels is contracted by $1/\gamma$. Thus she observes a time interval of 3/5x8/0.8=6 years on the way to the star and 6 years on return. Mary has aged 12 years.
  - She also observes Frank's clock running slow (since he is moving relative to her). Mary observes a time interval in S to be 3/5x6=3.6 years on the way to the star and 3.6 years on return. Frank has aged 7.2 years.

Twin Paradox

- The answer is Frank ages 20 years and Mary ages 12 years.
- The problem is not symmetric.
  - Frank is in one inertial frame
  - Mary is in two inertial frames
  - Mary is also in an non-inertial (accelerated) frame at the turn-around
  - Mary must do her calculations differently

Twin Paradox

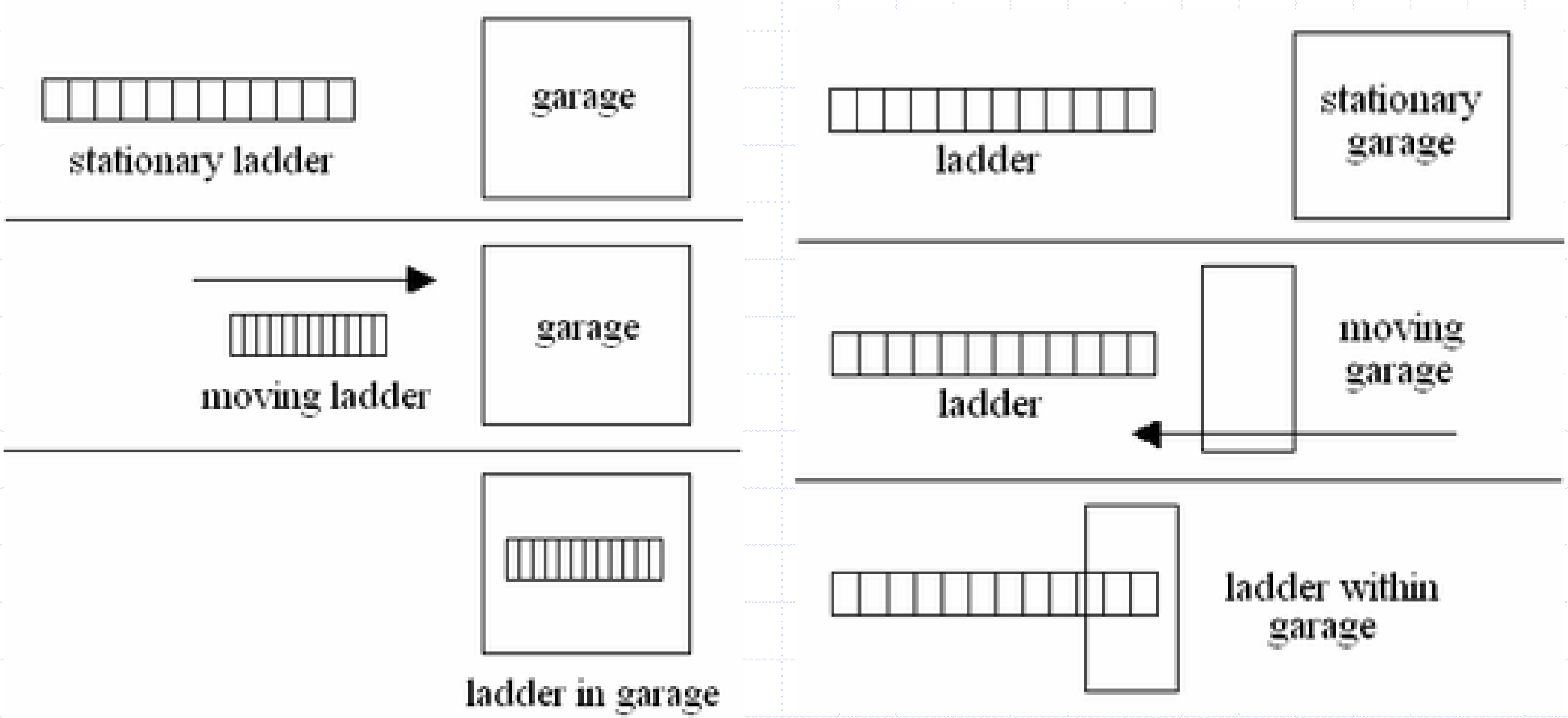
- A full analysis based on the Doppler effect will not be given now. But consider
  - Suppose that there are synchronous clocks in S. One on earth. One on the star.
  - In S′, these clocks are unsynchronized by an amount L_0V/c^2.
  - Mary is originally in S′ but when she stops at the star she is in S where all observers must agree the clocks are synchronous.
  - Thus in the negligible time it takes to stop, Frank must age considerably.

Twin Paradox

- Numerically
  - When in S′, Mary sees the clock in S at the star leading the one on earth by $L_0V/c^2=8x0.8=6.4$ years
  - After deceleration at the star she is in the S frame and must see the clock in S at the star synchronous with the one on earth.
  - Thus she sees Frank age 3.6+6.4=10 years.
    - She sees Frank's clock running slow on the trip out (3.6) plus the time the clock advances when she changes from frame S′ to frame S (6.4)

Barn and Pole Paradox

- Can we use special relativity to fit a 10m pole/ladder in a 5m barn/garage?



Barn and Pole Paradox

- In frame K (garage frame)
  - Front of garage is at x=0
  - Back of garage is at x=G=5
- In frame K′ (ladder frame)
  - Front of ladder is at x′=0
  - Back of ladder is at x′=-L=-10

Barn and Pole Paradox

- Event 1-Ladder front reaches garage front
  - In K and K′ let t=t′=0
- Event 2-Ladder front reaches garage back
  - In K,

$$t = \frac{G}{v}$$

  - In K′,

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \gamma\left(\frac{G}{v} - \frac{v}{c^2}G\right) = \sqrt{1-\frac{v^2}{c^2}}\,\frac{G}{v}$$

- Event 3-Ladder back reaches garage front
  - In K′, $x' = -L = \gamma(x - vt) = -\gamma vt$
  - In K, $t = \sqrt{1-\frac{v^2}{c^2}}\,\frac{L}{v}$

Barn and Pole Paradox

- In frame K, if time of event 2 = time of event 3 then the ladder will fit into the garage
  - An observer in K sees both ends of the ladder are in the garage at the same time

$$\frac{G}{v} = \sqrt{1 - \frac{v^2}{c^2}}\,\frac{L}{v}$$

$$\frac{v^2}{c^2} = 1 - \frac{G^2}{L^2} = 1 - \frac{5^2}{10^2} = 0.75$$

$$\beta = 0.866 \text{ and } \gamma = 2$$

Barn and Pole Paradox

- In frame K', event 2 (ladder front at garage back) occurs at (see p10)

$$t' = \sqrt{1-\frac{v^2}{c^2}}\frac{G}{v} = \sqrt{1-0.75}\frac{5}{0.866c} = 9.6ns$$

- In frame K', event 3 (ladder back at garage front) occurs at

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \gamma t = \gamma\frac{G}{v} = 2\frac{5}{0.866c} = 38.5ns$$

- Events 2 and 3 are not simultaneous in K'

Barn and Pole Paradox

- Does the ladder fit in K′ or not?
- Assume in frame K we quickly close the front and back doors when the ladder is inside
  - This happens at t=G/v=5/0.866c=19.25ns
- What time does front door close in K′?

$$t' = \gamma(t - \frac{vx}{c^2}) = 2(19.25ns - 0) = 38.5ns$$

- What time does back door close in K′?

$$t' = \gamma(t - \frac{vx}{c^2}) = 2(19.25ns - \frac{0.866c \times 5}{c^2}) = 9.6ns$$

Barn and Pole Paradox

- Summarizing, in K′
  - Ladder front at garage back at 9.6ns
  - Back door closes at 9.6ns
  - Ladder back at garage front at 38.5ns
  - Front door closes at 38.5
- Back door closes before the front door

Man and Manhole Paradox

- A 1 foot long ruler slides with $\gamma=12$ towards a 1 foot wide open hole in the ground (grid)
- In frame S′, the ruler's rest frame, the hole is only 1″ wide, so the rod easily passes over the hole
- In frame S, the hole's rest frame, the ruler appears only 1″ long so it is bound to fall into the hole under the influence of gravity
- Does the ruler fall in the hole or not?

Man and Manhole Paradox

- From Ap.J.Phys. V29 (1961) 365.

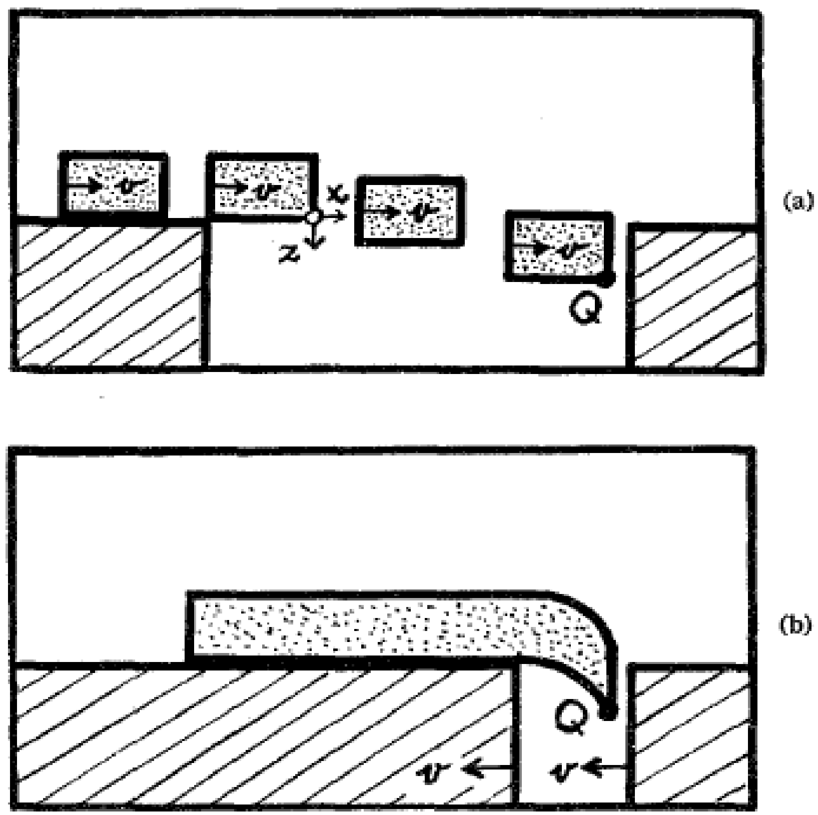


FIG. 1. (a) Sequence of four observations made by A at equal intervals of time t. (b) Observation made by B at one particular instant t'. (For convenience these diagrams are drawn for the case $\gamma=4$, not $\gamma=10$ as in the text.)

Man and Manhole Paradox

- Recall the Lorentz transformation equations

$$z = z' \text{ and } t = \gamma\left(t' + \frac{Vx'}{c^2}\right)$$

- In S (hole rest frame)

$$z = 0 \text{ for } t < 0$$

$$z = \frac{1}{2}at^2 \text{ for } t \geq 0$$

- In S′ (ruler rest frame)

$$z' = 0 \text{ for } x' < -\frac{c^2 t'}{V}$$

$$z' = \frac{1}{2}a\gamma^2\left(t' + \frac{Vx'}{c^2}\right)^2 \text{ for } x' \geq -\frac{c^2 t'}{V}$$

Man and Manhole Paradox

- Summary of the argument
- In the rest frame of the hole the rod will fall and be stopped by the far edge of the wall
- Then it must be that in the rest frame of the rod, the rod loses its rigidity and bends into the hole
- Furthermore the rod must sizably compress in S′ since the back end of the rod passes well into the hole

Man and Manhole Paradox

- Recently (2005) this explanation was challenged
- Referring to the following figures
  - V is the velocity of the car
  - d_0 is the proper length of the spoiler
  - w_0 is the proper length of the hole
- When the car's wheels leave the road, a horizontal stress propagates through the car at proper speed u_0

Man and Manhole Paradox

- From Eur.J.Phys. 26(2005) 19

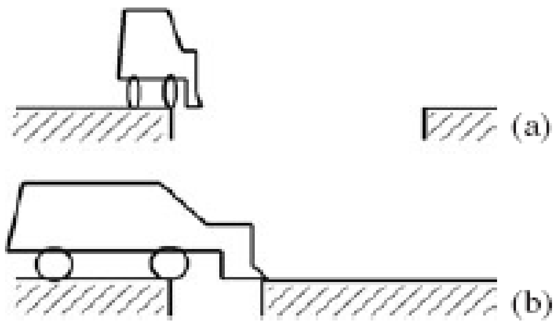


Figure 1. Situation as seen from stationary frame (a) and from moving frame (b).

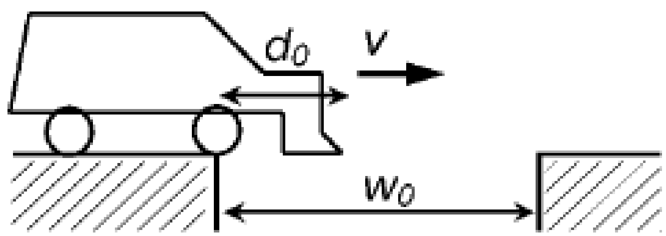


Figure 2. Schematic side view with proper lengths (when in rest or Newtonian).

Man and Manhole Paradox

- In the car's frame
  - The hole appears shorter by $w' = w_0/\gamma$
  - The stress effect of the hole's edge on the wheels will reach the spoiler at time $\Delta t' = d_0/u_0$
  - Thus the spoiler tip will not fall into the hole if

$$\frac{d_0}{u_0} > \left(\frac{w_0}{\gamma} - d_0 \right) / V$$

$$w_0 < \gamma \left(1 + \frac{V}{u_0} \right) d_0$$

  - We assumed the stiffness of the material is not affected by the speed of the car

Man and Manhole Paradox

- In the road's frame
  - Velocity transformation

$$u = \frac{u_0 + V}{1 + \frac{u_0 V}{c^2}}$$

  - The time Δt it takes before the front of the spoiler's motion is affected

$$\Delta t = \frac{d}{u - V} \text{ with } d = \frac{d_0}{\gamma}$$

  - The car moves $\Delta x = v\Delta t$ so the spoiler stays above the hole if $V\Delta t > (w_0 - d)$

Man and Manhole Paradox

- Still in the road's frame
  - Substituting expressions for d and Δt

$$w_0 < \left(\frac{u}{u-V}\right)\frac{d_0}{\gamma}$$

  - And substituting the expression for u, the spoiler tip will not fall into the hole if

$$w_0 < \gamma\left(1+\frac{V}{u_0}\right)d_0$$

  - Thus we find the same result for both frames

Man and Manhole Paradox

- These authors comment on the ruler problem that the rod’s proper material properties (like stiffness) does not change due to its speed relative to another system
- They define falling or bending to occur when the upper corner begins downward motion
- They further comment that the stress change propagation from lower to upper corner was overlooked by the first author
- Hence the ruler will pass unhindered over the hole (except if the ruler is exceptionally thin)