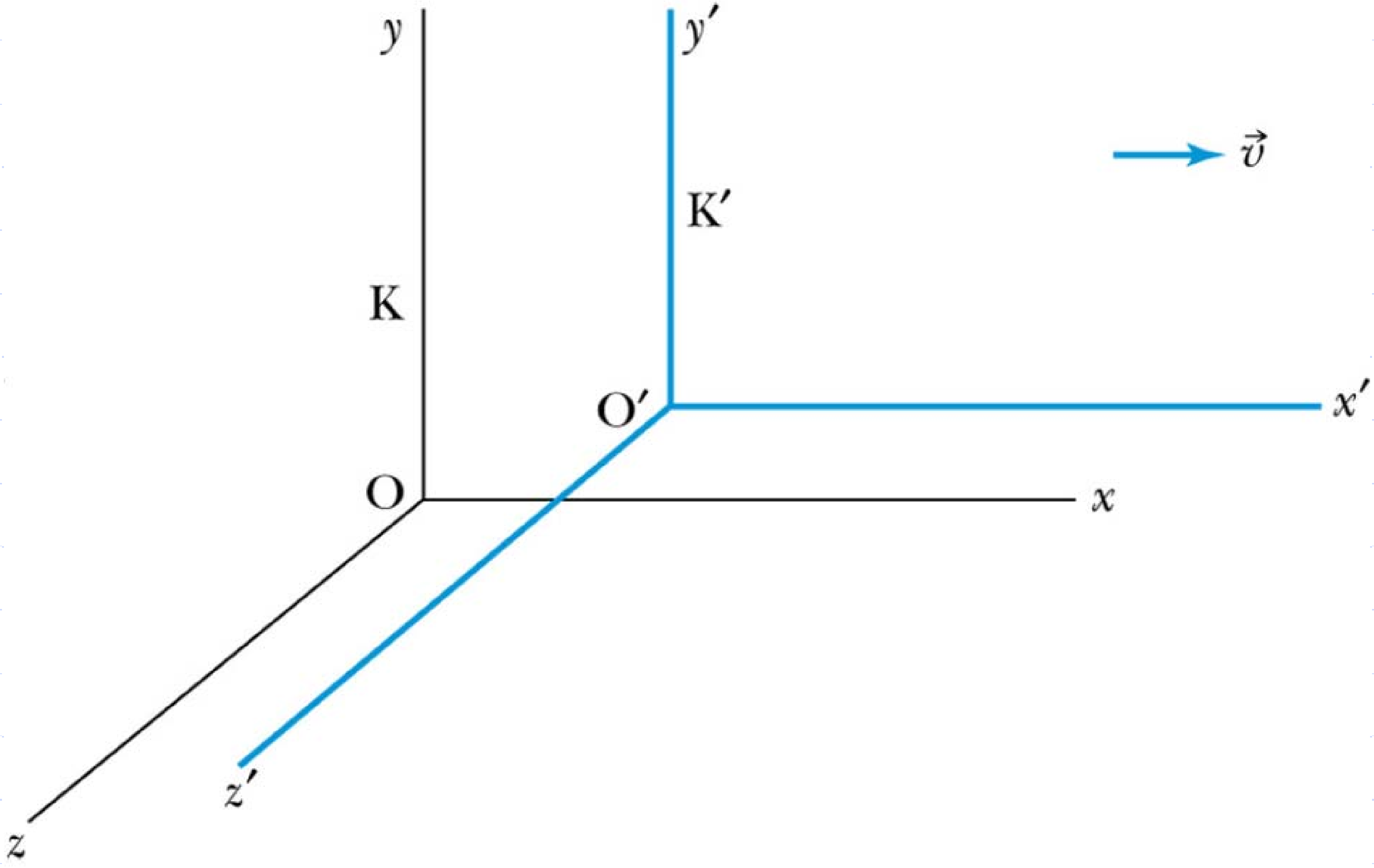
Newtonian Relativity

- A reference frame in which Newton's laws are valid is called an inertial frame
- Newtonian principle of relativity or Galilean invariance
  - If Newton's laws are valid in one reference frame, then they are also valid in a reference frame moving at uniform velocity relative to the first system
  - Thus this moving frame is also an inertial frame

Newtonian Relativity

- Consider



Newtonian Relativity

- Galilean transformation

$$x' = x - \vec{v}t \qquad x = x' + \vec{v}t$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = t \qquad t = t'$$

- Note time is the same in both systems

Newtonian Relativity

- Note Newton's laws are valid in both frames
  - The force and acceleration are the same in both frames
  - There is no way to detect which frame is moving and which is at rest

$$F'_y = ma'_y = m\frac{d^2y'}{dt'^2} = m\frac{d^2y}{dt^2} = ma_y = F_y$$

$$F'_x = ma'_x = m\frac{d^2x'}{dt'^2} = m\frac{d^2(x-vt)}{dt^2} = m\frac{d^2x}{dt^2} = F_x$$

Lorentz Transformation

- We immediately see the Galilean transformation is inconsistent with Einstein's postulates
  - If the velocity of light = c in frame K, the velocity of light = $c - V$ in frame K′
- The Lorentz transformation satisfies Einstein's postulates and also reduces to the Galilean transformation at low velocities
- A derivation is given in Thornton and Rex p30-31

Lorentz Transformation

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \gamma(t - vx / c^2) \qquad t = \gamma(t' + vx' / c^2)$$

$$\text{where } \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Lorentz Transformation

- Time dilation revisited
  - Let $\Delta t' = t_2' - t_1'$ be the proper time interval measured by a clock fixed at x_0' in K′

$$t_2 = \gamma(t_2' + \frac{x_2'V}{c^2}) = \gamma(t_2' + \frac{x_0'V}{c^2})$$

$$t_1 = \gamma(t_1' + \frac{x_1'V}{c^2}) = \gamma(t_1' + \frac{x_0'V}{c^2})$$

$$\Delta t = t_2 - t_1 = \gamma \Delta t'$$

- The clocks in S read a time longer than the proper time. The moving clock in S′ runs slow.

Lorentz Transformation

- Length contraction revisited
  - Consider a measuring rod with proper length $\Delta x' = x_2' - x_1'$. The interval Δx as viewed in S must have the positions measured at the same time t_0 in S.

$$x_2' = \gamma(x_2 - Vt_2) = \gamma(x_2 - Vt_0)$$

$$x_1' = \gamma(x_1 - Vt_1) = \gamma(x_1 - Vt_0)$$

$$\Delta x = \Delta x' / \gamma$$

- The length of the moving object as measured in S is shorter than the proper length

Lorentz Transformation

- Clock synchronization revisited
  - Consider two clocks synchronized in S′. Clock B′ at x_2' and clock A′ at x_1'. What times do they read at time t_0 in S?

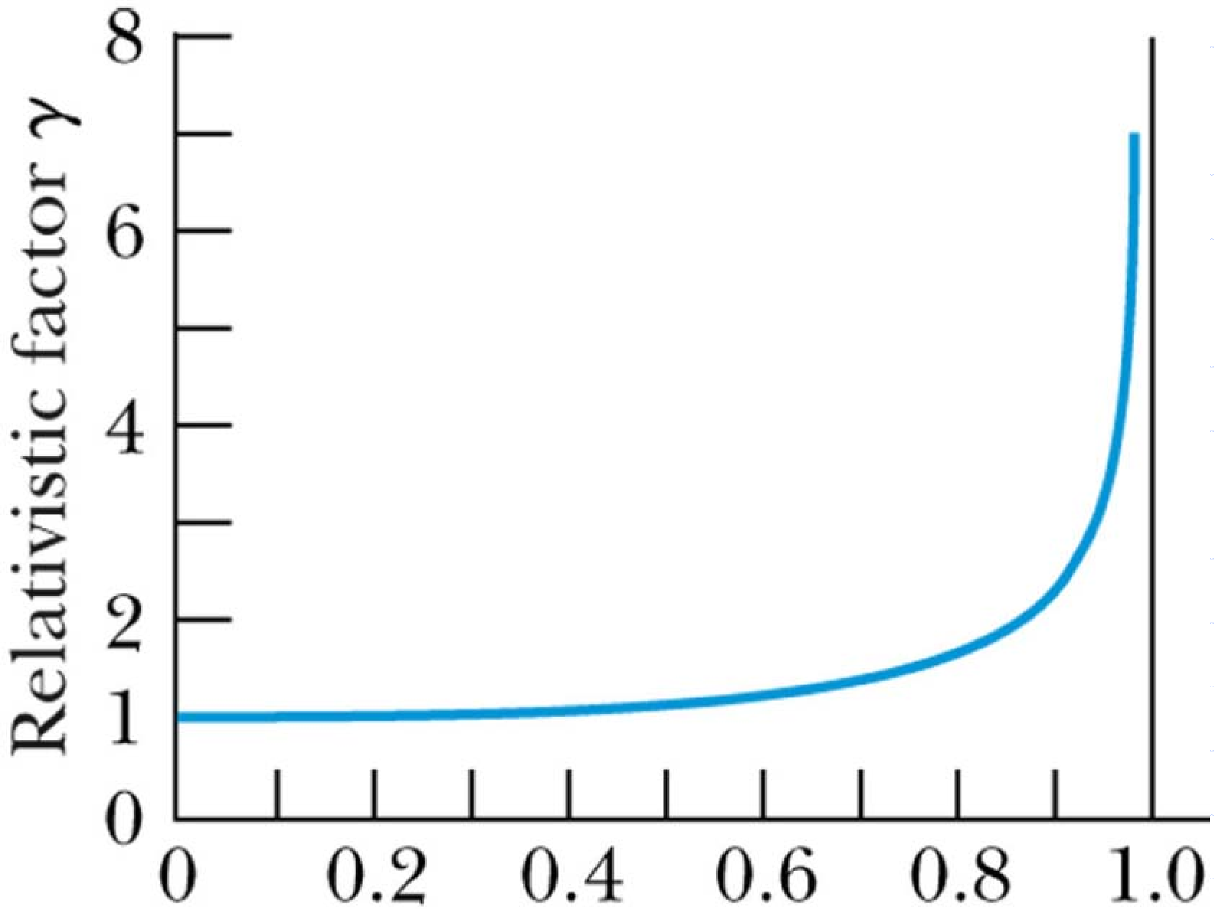
$$t_B' = \gamma\left(t_0 - \frac{x_2 V}{c^2}\right)$$

$$t_A' = \gamma\left(t_0 - \frac{x_1 V}{c^2}\right)$$

$$t_B' - t_A' = -\gamma\left(x_2 - x_1\right)\frac{V}{c^2} = -\left(x_2' - x_1'\right)\frac{V}{c^2} = -\frac{L_0 V}{c^2}$$

  - Agrees with results from the homework

Beta and Gamma



$\beta = \dfrac{v}{c}$

Invariants

- Invariant quantities have the same value in all inertial frames
  - In the next homework, you'll show

$$s^2 = x^2 + y^2 + z^2 - (ct)^2$$

$$s'^2 = x'^2 + y'^2 + z'^2 - (ct')^2$$

$$s^2 = s'^2$$

  - s^2 is the same for all inertial frames

Invariants

- Consider two events 1 and 2
- We define the spacetime interval as
  - $\Delta s^2 = \Delta x^2 - (c\Delta t)^2$
- Three cases
  - Lightlike ($\Delta s^2=0$)
    - The two events can be connected only by a light signal
  - Spacelike ($\Delta s^2>0$)
    - The two events are not causally connected. We can find an inertial frame where the events occur at the same time but at different positions in space
  - Timelike ($\Delta s^2<0$)
    - The two events are causally connected. We can find an inertial frame where the events occur at the same position in space but at different times

Addition of Velocities

- Recall the Galilean transformation between two frames K and K′ where K′ moves with velocity v with respect to K
  - Consider an object moving with velocity u in K and u' in K′

$$x' = x - vt$$

$$\frac{dx'}{dt'} = \frac{d(x - vt)}{dt} = \frac{dx}{dt} - v$$

$$u' = u - v$$

Galilean Transformation

$$x' = x - vt \qquad x = x' + vt$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = t \qquad t = t'$$

- Note time is the same in both systems

Addition of Velocities

- We know the Lorentz transformation should be used instead so

$$u_x = \frac{dx}{dt} = \frac{\gamma(dx' + vdt')}{\gamma\left[dt' + \left(\frac{v}{c^2}\right)dx'\right]} = \frac{u'_x + v}{1 + \left(\frac{v}{c^2}\right)u'_x}$$

similarly

$$u_y = \frac{u'_y}{\gamma\left[1 + \left(\frac{v}{c^2}\right)u'_x\right]}$$

$$u_z = \frac{u'_z}{\gamma\left[1 + \left(\frac{v}{c^2}\right)u'_x\right]}$$

Lorentz Transformation

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \gamma(t - vx/c^2) \qquad t = \gamma(t' + vx'/c^2)$$

$$\text{where } \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Addition of Velocities

- Swapping primed and unprimed variables and letting v go to –v

$$u'_x = \frac{u_x - v}{1 - \left(\frac{v}{c^2}\right) u_x}$$

$$u'_y = \frac{u_y}{\gamma \left[1 - \left(\frac{v}{c^2}\right) u_x\right]}$$

$$u'_z = \frac{u_z}{\gamma \left[1 - \left(\frac{v}{c^2}\right) u_x\right]}$$

Addition of Velocities

- Example - let $u'_x=c,\ u'_y=0,\ u'_z=0$

$$u_x = \frac{(c+V)}{\left(1+\frac{cV}{c^2}\right)} = c, u_y = 0, u_z = 0$$

- Example - *let* $u'_x=0,\ u'_y=c,\ u'_z=0$

$$u_x = V,\ u_y = \frac{c}{\gamma},\ u_z = 0$$

$$\tan\theta = \frac{u_x}{u_y} = \frac{\gamma V}{c}$$

Addition of Velocities

- A rocket blasts off from the earth at v=0.90*c*
- A second rocket follows in the same direction at velocity 0.98*c*
- What is the relative velocity of the rockets using a Galilean transformation
- What is the relative velocity of the rockets using a Lorentz transformation?

Lorentz Transformation

- Last time we argued that

$$y = y' \text{ and } z = z'$$

- The most general linear transformation for x=f(x′,t′) is

$$x = \gamma x' + \alpha t' = \gamma\left(x' + \frac{\alpha}{\gamma} t'\right)$$

- At low velocities, γ→1 and α/γ→V

$$x = \gamma\left(x' + Vt'\right)$$

- The inverse transformation is the same except for the sign of relative motion

$$x' = \gamma\left(x - Vt\right)$$

Lorentz Transformation

- For a light pulse in S we have *x=ct*
- For a light pulse in S′ we have *x′=ct′*
- Then

$$x = ct = \gamma\left(ct' + Vt'\right) = \gamma\left(c + V\right)t'$$

$$x' = ct' = \gamma\left(ct - Vt\right) = \gamma\left(c - V\right)t$$

$$ct' = \gamma\left(c - V\right)\frac{\gamma}{c}\left(c + V\right)t'$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$$

Lorentz Transformation

- For the *t* transformation

$$x' = \gamma(x - Vt)$$

$$x' = \gamma[\gamma(x' + Vt') - Vt]$$

$$t = \gamma t' + \left(\gamma - \frac{1}{\gamma}\right)\frac{x'}{V}$$

$$t = \gamma\left(t' + \frac{Vx'}{c^2}\right)$$