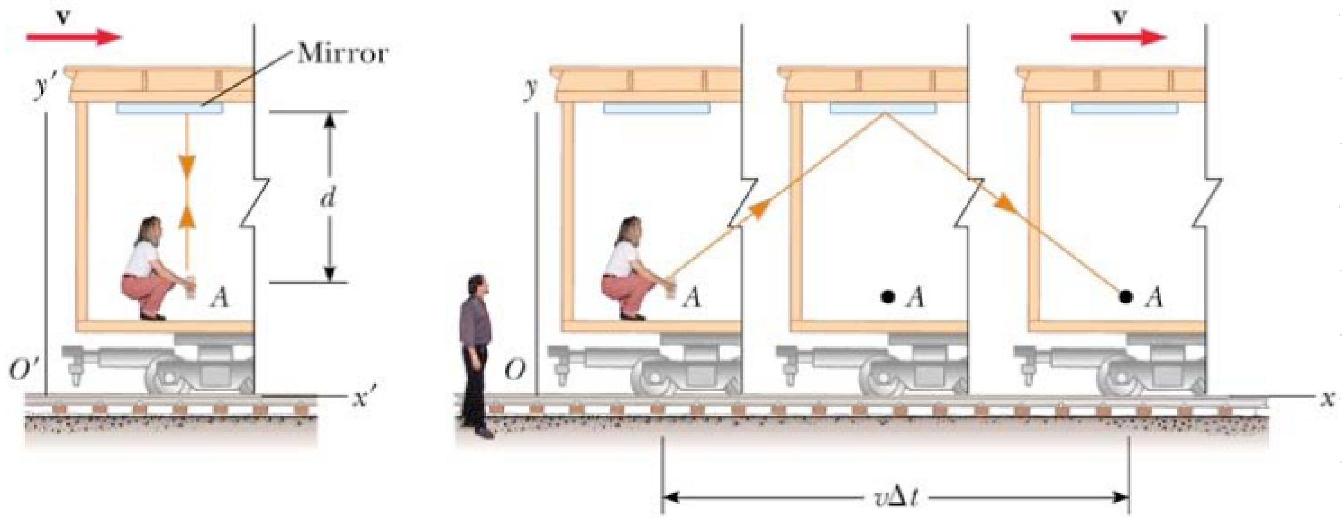
Time Dilation

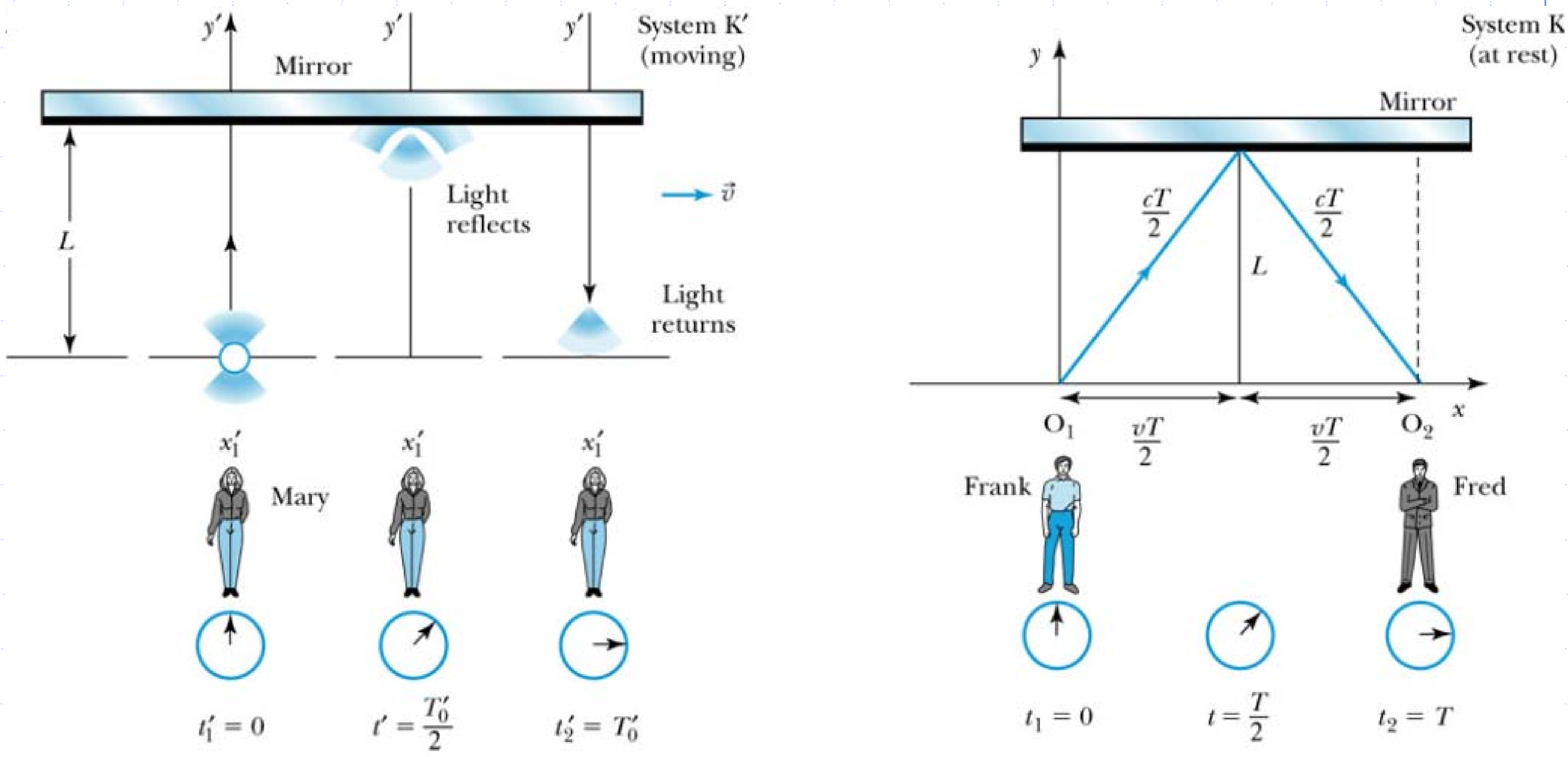
Reference frame of
observer *O'* on train

Reference frame of
observer *O* on ground



- Observer O is on the ground
- Observer O' is on the train moving with velocity v relative to the ground
- The time interval between the two events will be different for the different observers

Time Dilation



Time Dilation

- The observer in frame O′ measures the time interval between light flash and light return to be

$$\Delta t' = \frac{\text{distance}}{\text{velocity}} = \frac{2d}{c}$$

- The time between events that occur at the same position in a reference frame is called the proper time
  - Thus the proper time is measured in a reference frame where the clock is "at rest"

Time Dilation

- The observer in frame O measures the time interval between light flash and light return to be

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Thus we have

$$\Delta t = \gamma \Delta t' \text{ where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Time Dilation

- We interpret $\Delta t = \gamma \Delta t'$ as "moving clocks run slow"
  - The "tick" of a clock moving with uniform velocity with respect to an observer runs more slowly
  - The proper time interval is always the shortest time interval
  - It's useful to keep in mind that Δt or $\Delta t'$ is the time interval between two events and gamma is always > 1

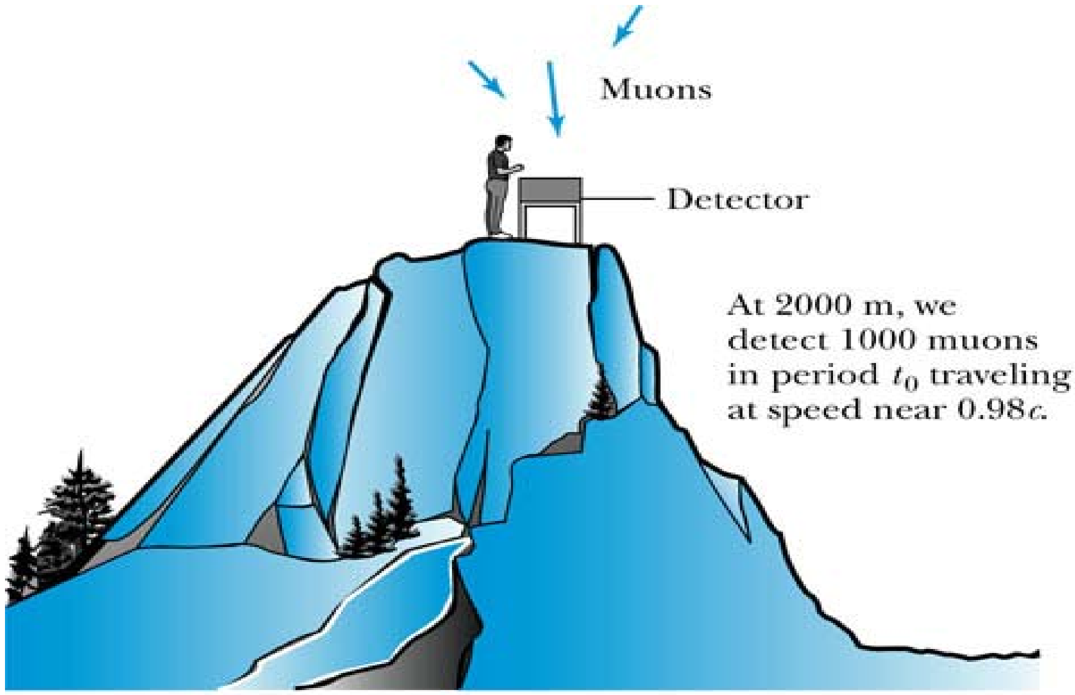
Time Dilation

- Consider a train and clock moving with $\gamma=2$ and let's take our two events to be minute hand at 1 and minute hand at 2
  - What time interval does an observer on the train record?
  - What time interval does an observer on the ground record?
- But an observer on the train sees the ground moving with $\gamma=2$
  - What time interval does an observer on the train record (for a ground clock)
  - What time interval does an observer on the ground record (for a ground clock)
- Which point of view is correct?

Time Dilation

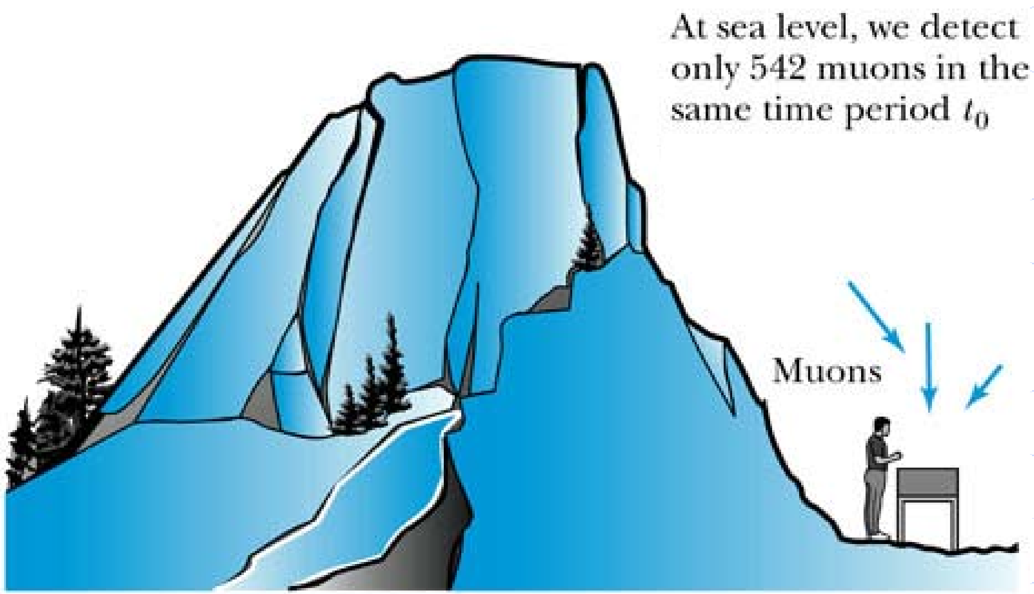
- Time dilation is real and measurable
- Consider cosmic ray muons
  - Muons are the most numerous charged particle at sea level
  - You can think of muons as heavy electrons
  - Intensity ~ 1/cm^2/min
  - $\beta = v/c \sim 0.98$ and $\gamma \sim 5$
  - Muons are unstable and decay with a lifetime of $\tau = 2.2\mu s$
    - $N = N_0 \exp(-t/\tau)$

Time Dilation



- How many muons will be detected at sea level?
- Expect N = 1000 exp(-6.8/2.2) = 45

Time Dilation

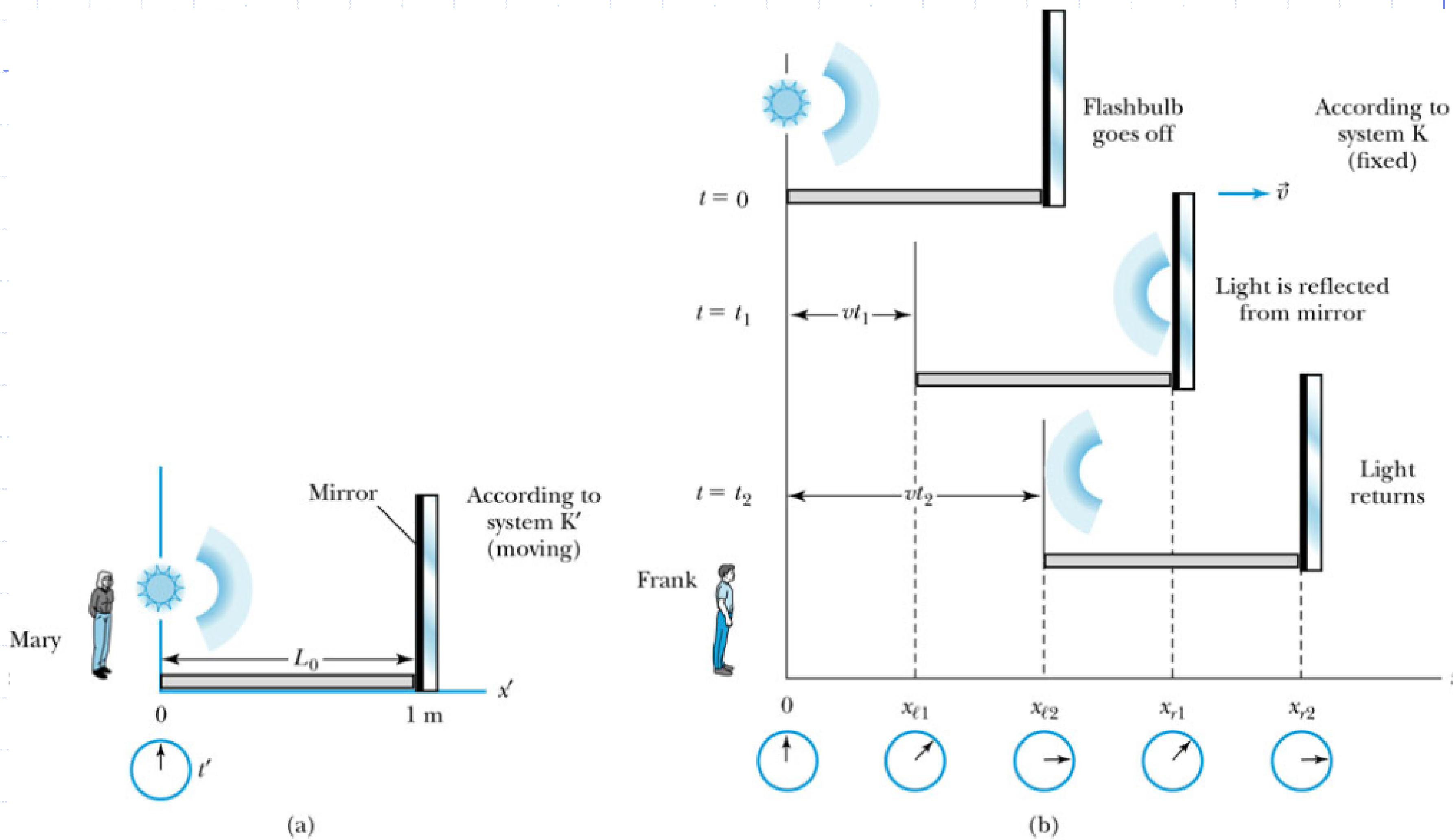


- Answer: 539 = 1000 exp (-6.8/11) since the muon clock (lifetime) is running slow by 1/γ

Length Contraction

- Consider again a train moving with velocity v relative to the ground
  - Observer O is on the ground
  - Observer O' is on the train moving with velocity v relative to the ground
- Measure the length of the train car by placing a mirror at the back end of the car and sending a light pulse down and back
- The length of the train car will be different for the different observers

Length Contraction



Length Contraction

- The observer in frame O′ measures the length of the train car to be

$$\Delta x' = \frac{c\Delta t'}{2}$$

- The length of an object measured in a frame where the object is at rest is called the proper length
  - $\Delta x' == L_0$

Length Contraction

- The observer in frame O measures the length of the train car to be

$$\Delta x = \frac{c\Delta t}{2}\left(1 - \frac{v^2}{c^2}\right)$$

- Which uses the measured time down and time back

$$\Delta t_1 = \frac{\Delta x + v\Delta t_1}{c} \text{ and } \Delta t_2 = \frac{\Delta x - v\Delta t_2}{c}$$

Length Contraction

- The time intervals measured by O and O′ are related by

$$\Delta t = \Delta t' \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \text{ and } \Delta x' = \frac{c\Delta t'}{2}$$

- Thus we find

$$\Delta x = \Delta x' \sqrt{1-\frac{v^2}{c^2}} = \frac{\Delta x'}{\gamma}$$

Length Contraction

- We interpret $\Delta x = \Delta x' / \gamma$ as "moving objects are shortened"
  - The length of an moving object is measured to be smaller by a stationary observer
  - The proper length L_0 is always the longest one ($L = L_0/\gamma$)
  - It is important to note that Δx
    - Is the length of the moving object as measured in O where the ends are measured at the same instant of time

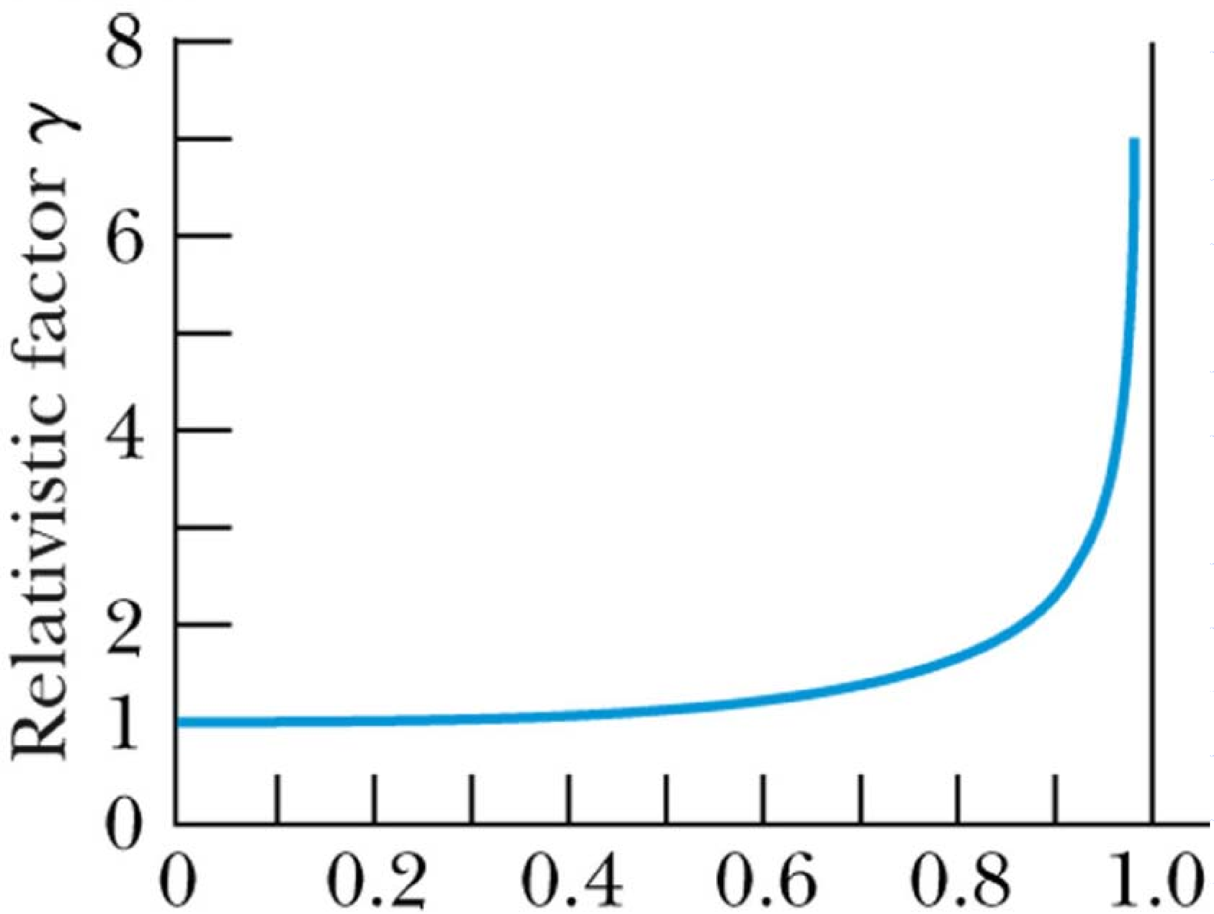
Length Contraction

- Cosmic rays revisited
  - The muon sees itself at rest and the mountain moving by
  - The muon measures the proper time for its lifetime = 2.2μs
  - The muon measures a contracted length for the distance of mountain top to sea level = 2000m/5=400m
    - The time to travel this distance is 400/0.98c=1.36μs
- Answer: 539 = 1000 exp (-1.36/2.2)

Length Contraction

- What about lengths transverse to the velocity V?
  - Consider a rod of proper length 1m with a paintbrush on one end. Align the rod in the y-direction. Move the rod very slowly in the x-direction making a red paint line on a wall at y=1m.
  - Now move the rod at relativistic speeds. An observer in the K frame (at rest relative to the wall) could see the length of (vertical) rod shortened. The observer would see a blue paint line on the wall below the red paint line.
  - An observer in the K′ frame (at rest relative to the rod) would see the red paint line shortened. Hence this observer would see a blue paint line above the red paint line.
  - Both observations cannot be true! This implies an asymmetry with respect to the direction of motion and violates postulate 1 of special relativity.
- Transverse lengths (y or z) must be the same for all inertial frames in relative motion along x

Beta and Gamma



$\beta = \frac{v}{c}$