#9

7.6

$n=2$, $l=1$

$$R_{21} = \frac{r}{a_0} \frac{e^{-r/2a_0}}{\sqrt{3}\,(2a_0)^{3/2}}$$

$$= C\, r e^{-r/2a_0}$$

$$\frac{dR}{dr} = C\left(1 - \frac{r}{2a_0}\right) e^{-r/2a_0}$$

$$r^2 \frac{dR}{dr} = C\left(r^2 - \frac{r^3}{2a_0}\right) e^{-r/2a_0}$$

$$\frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) = C\left(2r - \frac{r^2}{2a_0} - \frac{3r^2}{2a_0} + \frac{r^3}{4a_0^2}\right) e^{-r/2a_0}$$

$$= C\left(2r - \frac{2r^2}{a_0} + \frac{r^3}{4a_0^2}\right) e^{-r/2a_0}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{2\mu}{\hbar^2}\left(E + \frac{e^2}{4\pi\epsilon_0 r} - \frac{\hbar^2}{2\mu}\frac{2}{r^2}\right)R = 0$$

$$c\left(\frac{2}{r} - \frac{2}{a_0} + \frac{r}{4a_0^2}\right)$$

$$+\left(\frac{2\mu E r}{\hbar^2} + \frac{2\mu e^2}{\hbar^2 4\pi\epsilon_0} - \frac{2}{r}\right)c = 0$$

✓

①

$$\left(\frac{2}{r} - \frac{2}{r}\right) + \left(\frac{1}{4a_0^2} + \frac{2\mu E}{\hbar^2}\right)r$$

$$+\left(-\frac{2}{a_0} + \frac{2\mu e^2}{\hbar^2 4\pi\epsilon_0}\right) = 0$$

②

① $$\frac{2\mu E}{\hbar^2} = -\frac{1}{4a_0^2}$$

$$E = -\frac{\hbar^2}{8\mu a_0^2} = -\frac{E_0}{4}$$ ✓

(2)

$$-\frac{\cancel{2}}{a_0} + \frac{\cancel{2}\mu e^2}{\hbar^2 4\pi\epsilon_0} = 0$$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{\mu e^2}$$ ✓

7.9

$n = 6$

$\Rightarrow$ $l = 5$ $\quad m_l = \pm 5, \dots, 0$

$l = 4$ $\quad m_l = \pm 4, \dots, 0$

$l = 3$ $\quad m_l = \pm 3, \dots, 0$

$l = 2$ $\quad m_l = \pm 2, \dots, 0$

$l = 1$ $\quad m_l = \pm 1, 0$

$l = 0$ $\quad m = 0$

there are $n^2 = 36$ terms

7.10

$3p \Rightarrow n=3,\ l=1 \Rightarrow m_l = \pm 1, 0$

$L = \sqrt{l(l+1)}\,\hbar = \sqrt{2}\,\hbar$

$L_z = 0, \hbar, -\hbar$

$L_x^2 + L_y^2 = L^2 - L_z^2$

7.25

$5f \Rightarrow m = \pm 3, \pm 2, \pm 1, 0$

$\Delta E = \mu_B B$

$= 5.788 \times 10^{-5} \frac{eV}{T} \cdot 3T = 1.74 \times 10^{-4} eV$

m_ℓ	ΔE
± 3	$5.21 \times 10^{-4} eV$
± 2	$3.47 \times 10^{-4} eV$
± 1	$1.74 \times 10^{-4} eV$
0	0

7.40

$$\langle r \rangle = \int_0^\infty r\,|R(r)|^2 r^2\,dr$$

$$= \frac{1}{(2a_0)^3}\int_0^\infty \left(4 - \frac{4r}{a_0} + \frac{r^2}{a_0^2}\right) r^3 e^{-r/a_0}\,dr$$

$$\int_0^\infty r^n e^{-r/a_0}\,dr = n!\,(a_0)^{n+1}$$

$$= \frac{1}{8a_0^3}\left[4\cdot 3!\,a_0^4 - \frac{4}{a_0}4!\,a_0^5 + \frac{1}{a_0^2}5!\,a_0^6\right]$$

$$= \frac{a_0}{8}(24 - 96 + 120) = \boxed{6a_0}$$

$$\langle r \rangle = \int_0^\infty r\,|R(r)|^2\, r^2\, dr$$

$$= \frac{1}{24 a_0^3} \int_0^\infty r^3 \left(\frac{r}{a_0}\right)^2 e^{-r/a_0}\, dr$$

$$= \frac{1}{24 a_0^5} \int_0^\infty r^5\, e^{-r/a_0}\, dr$$

$$= \frac{1}{24 a_0^5}\, 5!\, a_0^6 = \boxed{5 a_0}$$