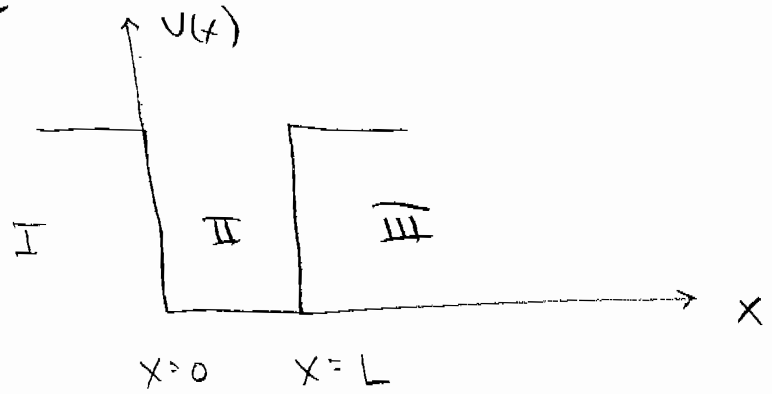
#1 #8

6.25



region III

$$E e^{k_{III}x} + F e^{-k_{III}x} \rightarrow F e^{-k_{III}x}$$

region II

$$C \sin kx + D \cos kx \quad \text{or}$$
$$C e^{ikx} + D e^{-ikx}$$

$$k_{III} = \frac{\sqrt{2M(V_0 - E)}}{\hbar} \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

at $x = L$ ψ must be continuous

$$C e^{ikL} + D e^{-ikL} = F e^{-k_{III} L}$$

at $x = L$ $\frac{d\psi}{dx}$ must be continuous

$$ikC e^{ikL} + -ikD e^{-ikL} = -k_{III} F e^{-k_{III} L}$$

$$\Rightarrow \quad \frac{C e^{ikL} + D e^{-ikL}}{\cancel{e^{-k_{III} L}}} = \frac{ikC e^{ikL} - ikD e^{-ikL}}{-k_{III} \cancel{e^{-k_{III} L}}}$$

$$\Rightarrow \quad \frac{C}{D} = \frac{ik - k_{III}}{ik + k_{III}} e^{-2ikL}$$

#2

6.32

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = A^2 \int_{-\infty}^{\infty} x^2 e^{-\alpha x^2} dx$$

look up the $\int$

$$= A^2 \frac{1}{2\alpha} \sqrt{\frac{\pi}{\alpha}}$$

$$= 1$$

$$\Rightarrow A^2 = 2 \frac{1}{\sqrt{\pi}} \alpha^{3/2}$$

$$\Rightarrow \boxed{A = \sqrt{2}\, \pi^{-1/4} \alpha^{3/4}}$$

$$\langle x \rangle = A^2 \int_{-\infty}^{\infty} x^3 e^{-\alpha x^2} dx$$

this is an odd fcn of x over $\int_{-\infty}^{\infty}$

$$\Rightarrow \boxed{\langle x \rangle = 0}$$

$$\langle x^2 \rangle = A^2 \int_{-\infty}^{\infty} x^4 e^{-\alpha x^2} dx$$

look up the $\int$

$$= A^2 \frac{3}{4\alpha^2} \sqrt{\frac{\pi}{\alpha}}$$

$$= \frac{2}{\sqrt{\pi}} \alpha^{3/2} \frac{3}{4} \alpha^{-5/2} \sqrt{\pi}$$

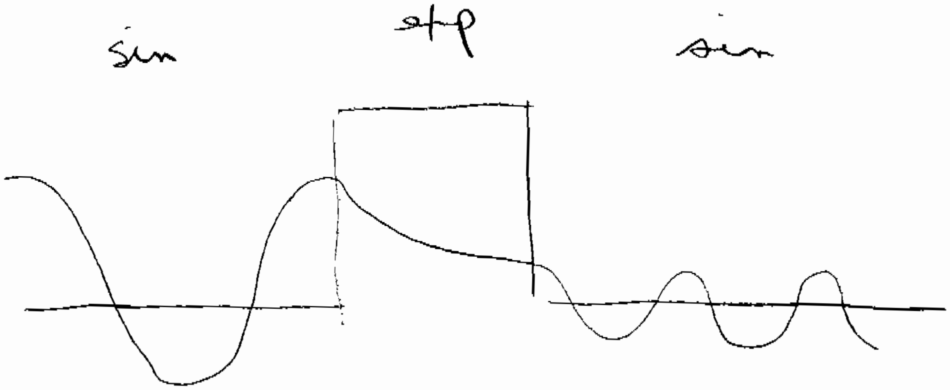
$$\boxed{\langle x^2 \rangle = \frac{3}{2} \alpha}$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{3}{2\alpha}}$$

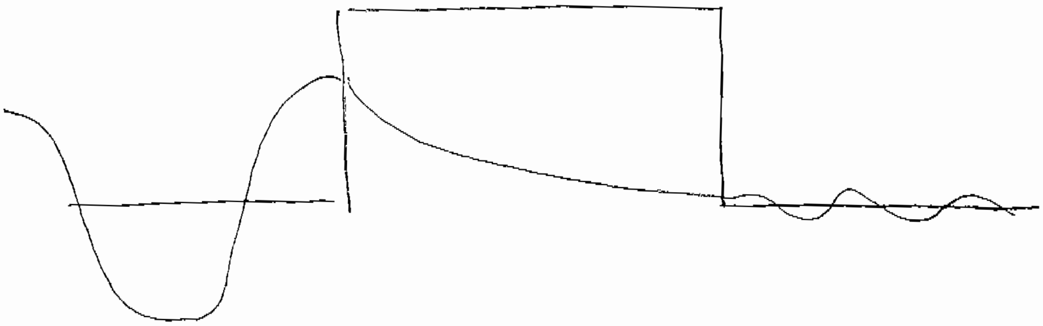
#3

6.45

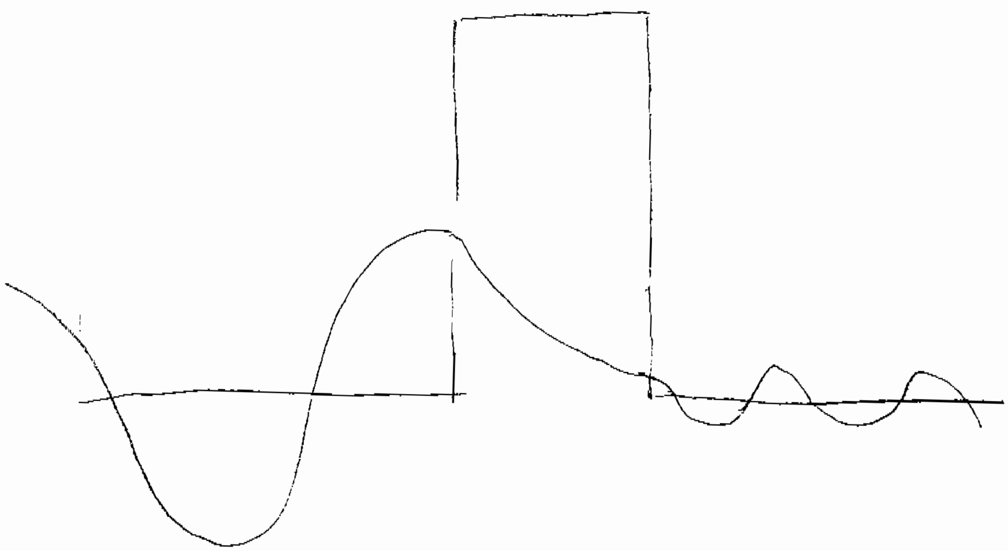
a)



b)



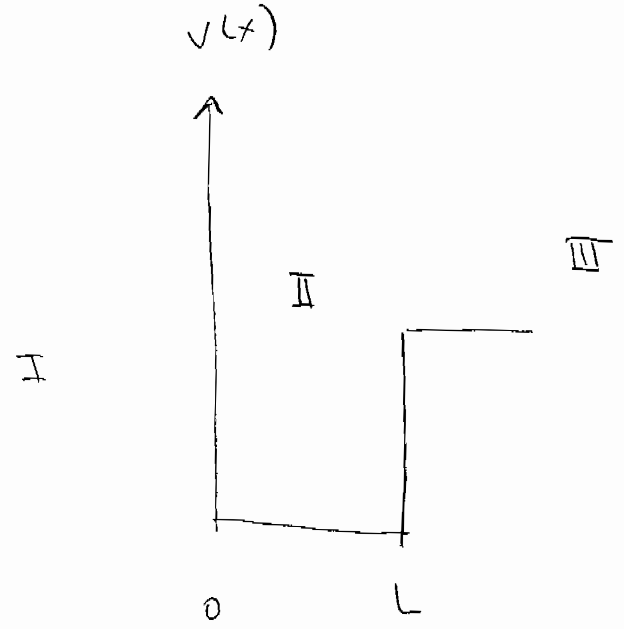
c)



a) > c) > b)

#4

6.49



region I $\quad \psi(x) = 0$

region II $\quad \psi(x) = A \sin kx + B \cos kx$

$\psi(0) = 0 \Rightarrow A \sin kx$

region III $\quad \psi(x) = Ce^{k_{III}x} + De^{-k_{III}x}$

$= De^{-k_{III}x}$ since $\psi \to 0$ as $x \to \infty$

$$K = \sqrt{\frac{2mE}{\hbar}} \qquad k_{III} = \sqrt{\frac{2m(V_0 - E)}{\hbar}}$$

apply boundary conditions

$$A \sin KL = D e^{-k_{III} L}$$

$$K A \cos KL = -k_{III} D e^{-k_{III} L}$$

$$\Rightarrow \quad \frac{\tan KL}{K} = -\frac{1}{K_{III}}$$

$$\Rightarrow \quad \boxed{K_{III} \tan KL = -K}$$

#5

6.50

now let $\alpha = kL$

$\beta = k_{III} L$

$V_0 = \frac{4\hbar^2}{2mL^2}$

$$\alpha^2 + \beta^2 = \left(k^2 + k_{III}^2\right) L^2$$

$$= \left(\frac{2mE}{\hbar^2} + \frac{(V_0 - E)\, m 2}{\hbar^2}\right) L^2$$

$$= \frac{2 m V_0}{\hbar^2} L^2$$

$$= \frac{\cancel{2mL^2}}{\hbar^2} \frac{4 \cancel{\hbar^2}}{\cancel{2mL^2}} = 4$$

$$\alpha^2 + \beta^2 = 4$$

from 6.49

$$K_{III} \tan KL = -K$$

$$K_{III} L \tan KL = -KL$$

$$\left.\begin{aligned} \beta \tan\alpha &= -\alpha \\ \alpha^2 + \beta^2 &= 4 \end{aligned}\right\} \text{ solve graphically}$$

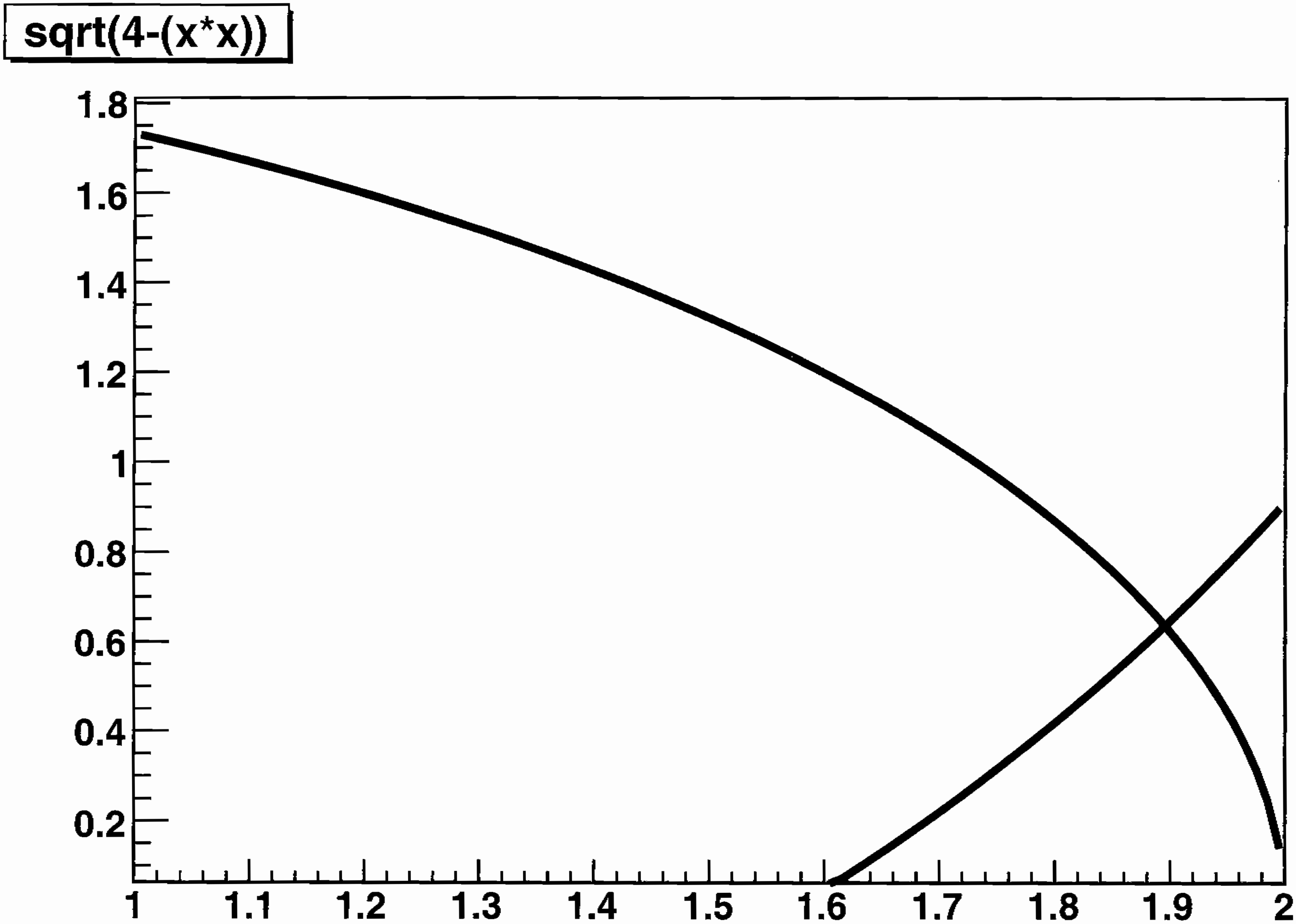
$$\beta = -\alpha \cot\alpha$$

$$\beta = (4 - \alpha^2)^{1/2}$$

$\beta \approx 0.67$ at $\alpha \approx 1.9$

$$\Rightarrow \alpha^2 = \frac{2mEL^2}{\hbar^2}$$

$$\Rightarrow \boxed{E = \frac{\hbar^2}{2mL^2}\alpha^2 = 3.61 \frac{\hbar^2}{2mL^2}}$$


sqrt(4-(x*x))
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

#6

the wave function must vanish at

$x = 0$

we can only have <u>odd</u> harmonic

oscillator wavefunctions $\psi_1(x), \psi_3(x), \ldots$

see p. 223

this means

$$E = (n + 1/2)\hbar\omega \quad \text{with} \quad n = 1, 3, \ldots$$