61

$$\int_{-\infty}^{\infty} |\Psi|^2 dx = \int_{-\infty}^{\infty} \Psi^* \Psi \, dx$$

$$= \int_{-\infty}^{\infty} A^* e^{-i(kx-\omega t)} A e^{i(kx-\omega t)} dx$$

$$= |A|^2 \int_{-\infty}^{\infty} dx$$

$$= \infty$$

the probability is uniform over all space

this function cannot represent a physical state of a particle

must use gaussian wave packets to represent a particle

6.10

recall $\cos x = \frac{e^{ix} + e^{-ix}}{2}$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\psi = A\left(e^{ix} + e^{-ix}\right)$$

$$= 2A\cos x$$

$\frac{x}{2} + \frac{\sin 2ax}{4a}$; $a = 1$

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 4A^2 \int_{-\pi}^{\pi} \cos^2 x \, dx = 4A^2 \pi = 1$$

$$\Rightarrow \boxed{A = \frac{1}{2\sqrt{\pi}}}$$

$$P = \int_0^{\pi/8} \psi^* \psi \, dx = 4 \cdot \frac{1}{4\pi} \left(\frac{x}{2} + \frac{\sin 2x}{4} \right) \Bigg|_0^{\pi/8}$$

$$= \frac{1}{\pi} \left(\frac{\pi}{16} + \frac{1}{4\sqrt{2}} \right)$$

$$= \boxed{0.119}$$

6.15

$$E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2}$$

$$= n^2 \frac{\pi^2 (\hbar c)^2}{2mc^2 L^2}$$

$$E_1 = \frac{(3.14)^2 (197.3 \text{ eV nm})^2}{2(0.511 \times 10^6 \text{eV})(2000)^2} = \boxed{9.40 \times 10^{-8} \text{ eV}}$$

then $E_2 = 4 \cdot 9.40 \times 10^{-8} = \boxed{3.76 \times 10^{-7} \text{ eV}}$

$$E_3 = 9 \cdot 9.40 \times 10^{-8} = \boxed{8.46 \times 10^{-7} \text{ eV}}$$

now $\frac{3}{2} kT = \frac{3}{2}\left(1.381 \times 10^{-23} \text{ J/K} \times \frac{1 \text{eV}}{1.602 \times 10^{-19} \text{ J}}\right) 13$

$$= 1.68 \times 10^{-3} \text{ eV}$$

$$\frac{1.68 \times 10^{-3}}{9.40 \times 10^{-8}} = n^2 \Rightarrow \boxed{n = 134}$$

6.28

$$\psi = A \sin\frac{\pi x}{L} \sin\frac{\pi y}{L} \sin\frac{\pi z}{L}$$

$$\int \psi^* \psi \, d\vec{r} = A^2 \iiint \sin^2\frac{\pi x}{L} \sin^2\frac{\pi y}{L} \sin^2\frac{\pi z}{L} \, dx\,dy\,dz =$$

look at 1 $\int$

$$\int_0^L \sin^2\frac{\pi x}{L} = \frac{L}{2}$$

$$\Rightarrow \quad A^2 \frac{L^3}{8} = 1$$

$$\Rightarrow \quad A = \left(\frac{2}{L}\right)^{3/2}$$

(6.54)

$$\Psi = \frac{1}{2}\psi_1 + \frac{\sqrt{3}}{2}\psi_2$$

now $\int \Psi^* \Psi \, dx =$

$$\int_0^L \frac{1}{4}\psi_1^*\psi_1 \, dx + \int_0^L \frac{3}{4}\psi_2^*\psi_2 \, dx$$

$$+ \int_0^L \cancel{\frac{\sqrt{3}}{4}\psi_1^*\psi_2 \, dx} \rightarrow 0 + \int_0^L \cancel{\frac{\sqrt{3}}{4}\psi_1\psi_2^* \, dx} \rightarrow 0$$

orthogonality

$$= \frac{1}{4} + \frac{3}{4} = 1 \checkmark$$

possible energies are E_1 and E_2

probabilities are $\frac{1}{4}$ and $\frac{3}{4}$