Phys 242 #3

2.84

$$^2H + {}^2H \rightarrow n + {}^3He$$

$m(^2H) = 2.014102\ u$

$m(n) = 1.008665\ u$

$m(^3He) = 3.016029\ u$

$\Delta m = 2 \cdot 2.014102 - 1.008665 - 3.016029$

$= 3.51 \times 10^{-3}\ u \times 931.494\ \frac{MeV}{c^2 u}$

$= 3.27\ MeV/c^2$

$\boxed{\Delta E = 3.27\ MeV}$

rest energy initial $= 2 \cdot 2.014102\ u \times 931.494\ \frac{MeV}{u}$

$= 3752.25\ MeV$

$3.27 / 3752.25 = \boxed{8.7 \times 10^{-4}\ (\sim 0.09\%)}$

2.93

$\beta = 0.9c \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = 2.29$

$p_p = \gamma m_p \beta$

$= 2.29 \quad 938.27 \quad 0.9c$

$= 1937.28 \text{ MeV}/c$

$p_e = \gamma m_e \beta = 1937.28 \text{ MeV}/c$

$= \frac{1}{\sqrt{1-\beta^2}} \; 0.511 \text{ MeV} \; \beta = 1937.28 \text{ MeV}$

$\frac{\beta}{\sqrt{1-\beta^2}} = 3791.15$

$\frac{\beta^2}{1-\beta^2} = (3791.15)^2$

$$\beta^2 + (3791.15)^2 \beta^2 = (3791.15)^2$$

$$\beta^2 = \frac{(3791.15)^2}{1 + (3791.15)^2}$$

$$\boxed{\beta = 1 - 3.47 \times 10^{-8}}$$

$$\gamma = 3790$$

$$T = \gamma mc^2 - mc^2$$

$$= 2.29\ 938.27 - 938.27 = 1210.37 \text{ MeV}$$

$$T_c = 1210.37 = \gamma(0.511) - 0.511$$

$$\gamma = 2369.62$$

$$\Rightarrow \boxed{\beta = 1 - 8.90 \times 10^{-8}}$$

$$\frac{1}{\sqrt{1-\beta^2}} = \gamma$$

$$1 - \beta^2 = \frac{1}{\gamma^2}$$

$$\beta^2 = 1 - \frac{1}{\gamma^2}$$

2.101

a) $498 - 135 - 135 = 228$ MeV

b) momentum $\Sigma = 0$

$T = 228/2 = 114$ MeV

$E = T + mc^2 = 114 + 135 = \boxed{249 \text{ MeV}}$

$E^2 = p^2c^2 + m^2c^4$

$p^2c^2 = E^2 - m^2c^4$

$= (249)^2 - (135)^2 = 43776$

$\Rightarrow \boxed{p = 209.23 \text{ MeV}/c}$

$\beta = pc/E = \frac{209}{249} = \boxed{0.84}$

#4

$$20\,MT = 20 \times 10^6 \times 4.18 \times 10^9\ J$$

$$= 8.36 \times 10^{16}\ J = mc^2$$

$$\Rightarrow \quad m = \frac{8.36 \times 10^{17}}{(3 \times 10^8)^2} = \boxed{0.93\ kg}$$

#5

$$p p \rightarrow p p \bar{B} B$$

$$s^2 = (\Sigma E)^2 - c^2 \cancel{(\Sigma \vec{p})^2}^{\,0} \qquad \text{for end products at rest}$$

$$= \left(m_p c^2 + m_p c^2 + m_B c^2 + m_{\bar{B}} c^2 \right)^2$$

$$= \left(938 + 938 + 5000 + 5000 \right)^2 = (11876)^2$$

in center-of-momentum frame

$$s^2 = (P_1 + P_2)^2 = (E_1 + E_2)^2 - c^2 \cancel{(\vec{p}_1 + \vec{p}_2)^2}^{\,0}$$

$\vec{p}_1 \rightarrow \quad \leftarrow \vec{p}_2$

$$= (2E_1)^2$$

$$= 4E_1^2$$

or $$4E_1^2 = (11876)^2$$

$$E_1 = 5938$$

$$T_1 = E_1 - m_p c^2 = \boxed{5000 \text{ MeV}} \quad \text{(as expected)}$$

in lab frame

$$s^2 = (p_1 + p_2)^2$$

$$= p_1^2 + p_2^2 + 2 p_1 \cdot p_2$$

$$= m_1^2 c^4 + m_2^2 c^4 + 2\left(E_1 E_2 - \cancel{\vec{p}_1 \cdot \vec{p}_2 c^2}\right)$$

0 since "2" is at rest

$$= m_p^2 c^4 + m_p^2 c^4 + 2 E_1 m_p c^2 = (11876)^2$$

$$2 \cdot 938 \cdot E_1 = (11876)^2 - (938)^2 - (938)^2$$

$$E_1 = 74242.9 \text{ MeV}$$

$$T_1 = E_1 - m_1 c^2 = 74242.9 - 938 = \boxed{73304.9 \text{ MeV}}$$