2.41

$$s = x^2 + y^2 + z^2 - c^2 t^2$$

$$= \gamma^2 (x' + v t')^2 + y'^2 + z'^2$$

$$- c^2 \gamma^2 (t' + v x'/c^2)^2$$

$$= x'^2 \gamma^2 + 2\gamma^2 \cancel{x'} v t' + \gamma^2 v^2 t'^2$$

$$+ y'^2 + z'^2$$

$$- c^2 \gamma^2 t'^2 - c^2 \gamma^2 \cancel{2} \frac{t' v x'}{c^2}$$

$$- c^2 \gamma^2 v^2 x'^2 / c^4$$

$$= x'^2 \gamma^2 (1 - v^2/c^2) + y'^2 + z'^2$$

$$- c^2 \gamma^2 t'^2 (1 - v^2/c^2)$$

$$= x'^2 + y'^2 + z'^2 - c^2 t'^2 = s'^2$$

(2.42)

$$\Delta s^2 = \Delta x^2 - c^2 \Delta t^2$$

$$\Delta s^2 < 0 \Rightarrow \Delta x^2 < c^2 \Delta t^2$$

assume $\exists$ frame K' | $\Delta t' = 0$ (simultaneous)

$$\Delta x^2 - c^2 \Delta t^2 = \Delta x'^2 - c^2 \Delta t'^2 = \Delta x'^2$$

but $\Delta x^2 < c^2 \Delta t^2$

$\Rightarrow \Delta x'^2 < 0$

proof by contradiction so no frame K' exists

2.95

$$E \xleftrightarrow{4.3 \text{ ly}} P$$

Mary S' ages 22 years

Frank S

in S', Mary sees distance $L_0/\gamma = 8.6\,\text{ly}/\gamma$

$$v = d/t = \frac{8.6\,\text{ly}}{\gamma \cdot 22} = \frac{8.6}{22} c \sqrt{1-\beta^2}$$

$$\beta^2 = \left(\frac{8.6}{22}\right)^2 (1-\beta^2)$$

$$\beta^2\left(1 + \left(\frac{8.6}{22}\right)^2\right) = \left(\frac{8.6}{22}\right)^2$$

$$\beta^2 = \frac{(8.6/22)^2}{(1+(8.6/22)^2)} = \frac{0.153}{1.153} = 0.133$$

$$\boxed{\beta = 0.364}$$

2.100

$$f' = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} f_0$$

$$z = \frac{\lambda' - \lambda_0}{\lambda_0} = \frac{\lambda'}{\lambda_0} - 1 = \frac{f_0}{f'} - 1$$

$$\frac{f_0}{f'} = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$$

$$\Rightarrow \quad z = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} - 1$$

$$(z+1)^2 = \frac{1+\beta}{1-\beta}$$

for $z = 1.9$, $\beta = 0.787$
$z = 4.9$, $\beta = 0.944$

$$(z+1)^2 - \beta(z+1)^2 = 1 + \beta$$

$$\beta\left(1 + (z+1)^2\right) = -1 + (z+1)^2$$

$$\beta = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$

Frank in S sees Mary's clock run slow

$$\Delta t = \gamma \Delta t'$$

$$= \frac{1}{\sqrt{1-\beta^2}} \Delta t'$$

$$= \frac{1}{\sqrt{1-(0.364)^2}} \cdot 22$$

$$= 23.6 \text{ yr}$$

or Frank will be $\boxed{53.6}$

#5

a. $$u' = \frac{u - v}{1 - (v/c^2)\,u}$$

$$\Rightarrow \quad v = \frac{u - u'}{1 - uu'/c^2}$$

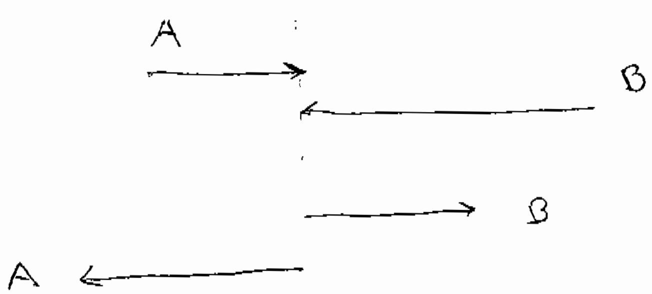
A 500 m, 0.8c S

B 1000 m, 0.6c S'

$$= \frac{-0.8c - 0.6c}{1 - \frac{(-0.8c)(0.6c)}{c^2}}$$

$$= \boxed{-0.946c}$$ is the relative velocity

b.



$$\gamma_A = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.8)^2} = 1.666$$

$$\gamma_\beta = \phantom{1/\sqrt{1-\beta^2}} = 1/\sqrt{1-(0.6)^2} = 1.25$$

observer on earth sees

$$L_A = L_0/\gamma = 500/1.666 = 300$$

$$L_B = L_0/\gamma = 1000/1.25 = 800$$

rocket A $\quad X_A = v_0 t + X_0$

$$= 0.8c\,t - 300$$

rocket B $\quad X_B = v_0 t + X_0$

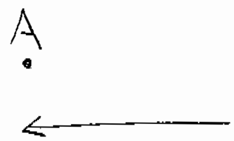
$$= 0.6c\,t + 800$$

tails pass when $X_A = X_B$

$$0.8c\,t - 300 = -0.6c\,t + 800$$

$$1.4c\,t = 1100$$

$$t = \frac{1100}{1.4c} = \boxed{2.62\ \mu s}$$



$$\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.946)^2} = 3.08$$

A sees B's rocket as $1000/3.08 = 324.67$ long

A sees tail of B front of A at $\frac{d}{v} = \frac{324.67}{0.946c} = 1.14\ \mu s$

A sees tail of B at tail of A at

$$1.14 + \frac{500}{0.946c} = 2.9\ \mu s$$

1.76