Phys 242 Exam 4

This is a closed book, closed note exam. Calculators are permitted. An equation sheet is provided on the last page. If you have difficulty with one problem, move on to the next one and come back to the one you are having trouble with later. For full credit please show all work. Good luck!

Problem 1. Please keep answers short.

a. What happens when you place a magnetic moment (orbital or spin) in a magnetic field? What happens to the associated angular momentum vector?

b. What is the origin of the exchange force between two identical electrons?

c. When building the periodic table one finds that the energy level of the 4s state is lower than that for the 3p state and the energy level of the 5s state is lower than that for the 4d state. Explain why this happens.

d. As two identical atoms are brought close together a splitting occurs in the energy levels. Why does this splitting occur? Which state is lowest in energy?

e. How does the electronic band structure differ for conductors, semiconductors, and insulators?

Problem 2.

a. Briefly describe the Stern-Gerlach experiment. What result was expected? What result was observed?

b. Consider an electron beam that is 100% polarized in the +z direction. The beam is incident on a Stern-Gerlach analyzer that analyzes in the +x and -x directions. The beam emerging with +x polarization (if any) is incident on a second Stern-Gerlach analyzer that analyzes in the +z and -z directions. What fraction of the original beam emerges from the second analyzer with -z polarization?

c. Consider a spin state given by $\chi = \frac{2i}{\sqrt{5}} \uparrow + \frac{1}{\sqrt{5}} \downarrow$. What is the probability that a measurement of the spin will produce spin up?

d. Consider a system of two spin 1/2 identical protons. Write down all the possible spin states of this system. You can use $\uparrow$ and $\downarrow$ to represent spin up and spin down.

e. What is the value of S^2 for each of the states you wrote down?

Problem 3.

Consider a multielecton atom in which one valence electron is in the 4p subshell and one valence electron is in the 4d subshell. The points in part d. are worth more than the other parts.

a. Consider the following four effects: (1) Zeeman effect, (2) Coulomb attraction between electrons and nucleus, (3) Spin-orbit coupling, and (4) Coulomb repulsion between electrons. Order these four effects from high to low in terms of their importance in determining the energy of the 4p4d state.

b. What is the origin of the spin-orbit interaction?

c. Write down an equation for the energy of the spin-orbit interaction. Don't worry about the constants. There are two possible answers here.

d. Considering the effects (2), (3), and (4) given in part a., list all the possible states in spectroscopic notation of the 4p4d configuration and order them from high to low in energy.

e. Now turn on an external magnetic field. Will this atom show the normal Zeeman effect, anomalous Zeeman effect, none, or both?

f. Draw the Zeeman splittings for the lowest energy state you produced in part d.

Problem 4.

Consider the free electron model.

a. Calculate the number of atoms per volume for aluminum. The density of aluminum is $2.70 \times 10^3 kg/m^3$ and the molar mass is $26.98g$.

b. Use your knowledge of metals to estimate/guess the Fermi energy for aluminum. Round your answer to the nearest power of 10. For example, if you think the Fermi energy is 60 MeV, you would write 100 MeV as your answer.

c. Use the results of parts a. and b. to estimate the number of conduction electrons per atom for aluminum.

d. Draw the distribution of occupied energies n(E) as a function of E for temperature T $>$ 0. Indicate on your graph where the Fermi energy is.

e. Is the contribution from conduction electrons to the molar heat capacity a small, medium, or large contribution? Explain.

For reference $C_V = \frac{1}{N}\frac{\partial U}{\partial T}$

Equations and Constants

$E = \hbar\omega$ and $p = \hbar k$

$\mu_l = -\frac{g_l \mu_B}{\hbar} L$ where $g_l = 1$

$\mu_s = -\frac{g_s \mu_B}{\hbar} S$ where $g_s = 2$

$\Delta E = -\mu_l \cdot B$

$\Delta E = -\mu_s \cdot B$

$n(E) = g(E) F_{FD}$

$F_{FD} = \frac{1}{e^{\beta(E-E_F)}+1}$ where $\beta = 1/kT$

$g(E) = \frac{3N}{2} E_F^{-3/2} E^{1/2}$

$T_F = E_F / k_B$

$E_F = \frac{h^2}{8m} (\frac{3N}{\pi L^3})^{2/3}$

$U = \frac{3}{5} N E_F$ at $T = 0$

$c = 3 \times 10^8 m/s$
$\hbar = h/2\pi = 1.05 \times 10^{-34} Js = 6.58 \times 10^{-16} eVs$
$hc = 1.99 \times 10^{-25} Jm = 1239.8 eVnm$
$\hbar c = 3.16 \times 10^{-26} Jm = 197.33 eVnm$
$m_e = 9.1 \times 10^{-31} kg = 0.511 MeV/c^2$
$k_B = 1.38 \times 10^{-23} J/K$
$N_{Av} = 6.022 \times 10^{23} mol^{-1}$