Phys 242 Exam 3

This is a closed book, closed note exam. Calculators are permitted. An equation sheet is provided on the last page. If you have difficulty with one problem, move on to the next one and come back to the one you are having trouble with later. For full credit please show all work. Good luck!

Problem 1.

a. Consider a particle confined to an infinite square well of width L. Use the Heisenberg uncertainty princple to estimate the ground state energy. For comparison, the actual ground energy is $E = \frac{\pi^2\hbar^2}{2mL^2}$.

b. Consider the double slit experiment where one electron is sent towards the two slits at a time. Is the result of this experiment evidence for the wave nature or particle nature of electrons? Explain.

c. Consider a hydrogen atom in a strong magnetic field. Draw the energy level diagram for the various n=1 and n=3 states. Draw all possible electric dipole transitions between the levels.

d. What is the working principle of an STM? (Keep answer short.)

Problem 2.

a. What is the potential energy term for the harmonic oscillator?

b. Consider the state $\phi(x) = \frac{1}{4}\psi_1(x) + \frac{i}{2}\psi_2(x) + \frac{i\sqrt{11}}{4}\psi_3(x)$ where ψ_1, ψ_2, and ψ_3 are the first three energy eigenstates of the harmonic oscillator. What are possible outcomes of energy measurements and what are their probabilities?

c. Write down $\phi(x,t)$

d. Draw the solution for n = 2 for the harmonic oscillator.

e. Briefly discuss this solution in terms of Bohr's correspondence principle.

Problem 3. Note the points in part d are worth more than the other parts.

Consider a one dimensional step potential.

$V(x) = 0$ for $x < 0$

$V(x) = V_0$ for $x \geq 0$

Consider the case where $E < V_0$

a. What happens classically where there is a particle incident from the left. (Short answer!)

b. The general solution for $x < 0$ (region 1) is

$\phi(x) = Ae^{ik_1 x} + Be^{-ik_1 x}$

The general solution for $x \geq 0$ (region 2) is

$\phi(x) = Ce^{k_2 x} + De^{-k_2 x}$

Consider the asymptotic solutions to eliminate at least one of these constants.

c. Apply the appropriate boundary conditions at x = 0.

d. Calculate the reflection coefficient $R = |\frac{B}{A}|^2$.

e. How does this compare to the classical result?

Problem 4.

The radial wave function for the 1s state of hydrogen is Ae^{-r/a_0}.

a. Normalize this wave function. (There are useful integrals on the equation sheet.)

b. What is the expectation value of r for the 1s state?

c. Sketch the radial probability distribution for the 1s and 2s states.

d. What are the expectation values for L and L_z for the 3d state?

Equations and Constants

$E = \hbar\omega$ and $p = \hbar k$

$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V\Psi(x,t)$

$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$

$\int_0^\infty re^{-ar}dr = \frac{1}{a^2}$

$\int_0^\infty r^2e^{-ar}dr = \frac{2}{a^3}$

$\int_0^\infty r^3e^{-ar}dr = \frac{6}{a^4}$

$c = 3 \times 10^8 m/s$
$\hbar = h/2\pi = 1.05 \times 10^{-34} Js = 6.58 \times 10^{-16} eVs$
$hc = 1.99 \times 10^{-25} Jm = 1239.8 eVnm$
$\hbar c = 3.16 \times 10^{-26} Jm = 197.33 eVnm$
Bohr radius $a_0 = 0.53 \times 10^{-10} m$
$E_0 = 13.6 eV$