Phys 242 Exam 1

This is a closed book, closed note exam. Calculators are permitted. An equation sheet is provided on the last page. If you have difficulty with one problem, move on to the next one and come back to the one you are having trouble with later. For full credit please show all work. Good luck!

Problem 1. (Short answer)

a. Briefly explain the twin paradox and its resolution.

b. Consider Newton's second law. Write down the relativistically correct form of this equation in frame K' moving with velocity V with respect to frame K.

c. Is it possible to accelerate a proton to the speed of light? Explain.

c. Apply the binomial expansion to $E = \gamma mc^2$ to find an expression for the non-relativistic energy of a particle. For partial credit, just write down the answer.

e. Write down two Lorentz invariants other than the spacetime interval $x^2 + y^2 + z^2 - (ct)^2$.

Problem 2.

a. Use the Lorentz transformations to show the length contraction relation.

b. Consider two clocks at different positions that read the same time in frame K. Use the Lorentz transformation to find the time difference between the clocks as measured in K'.

c. A meter stick moves with velocity $v = 0.8c$ relative to you. How long does it take the meter stick to pass you?

Problem 3.

Consider the reaction $pp \rightarrow ppp\bar{p}$ where the $\bar{p}$ is an antiproton. Both protons and antiprotons have a mass of 938 MeV/c^2.

a) What energy must the beam protons have in the center-of-mass frame to produce this reaction? The center-of-mass frame is the same as the center-of-momentum frame.

b) What energy must the beam proton have in the laboratory frame to produce this reaction? In the laboratory frame one proton is the beam and the other is the target, which is at rest.

Problem 4.

Phys 242 students are traveling on Rocket A moving towards earth with $v = 0.6c$. A Phys 242 professor is traveling on Rocket B moving towards Rocket A with $v = 0.3c$ away from the earth. Both velocities are with respect to the earth.

a. What is the velocity of Rocket B as measured by Rocket A?

b. If the time interval on Rocket B shows 50 minutes have passed, how much time has passed as measured by the students on Rocket A?

c. If the time interval on Rocket B shows 50 minutes have passed, how much time as passed as measured by someone on earth?

Equations and Constants

Lorentz transformation
$$x' = \gamma(x - Vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - Vx/c^2)$$

Addition of velocity
$$u'_x = \frac{u_x - V}{1 - Vu_x/c^2}$$

Doppler effect
$$f = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} f_0$$

Energy and momentum relations
$$p = \gamma m u$$
$$E = \gamma m c^2 = T + mc^2$$
$$T = \gamma m c^2 - mc^2$$

Four vectors
spacetime (ct, x, y, z)
energy-momentum $(E/c, p_x, p_y, p_z)$

speed of light
$$c = 3 \times 10^8 m/s$$