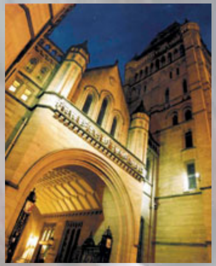
Linear Accelerators: Theory and Practical Applications: WEEK 4

Stanford Linear Accelerator, shown in an aerial digital image. The two roads seen near the accelerator are California Interstate 280 (to the East) and Sand Hill Road (along the Northwest). Image data acquired 2004-02-27 by the United States Geological Survey



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Summary of Week 3

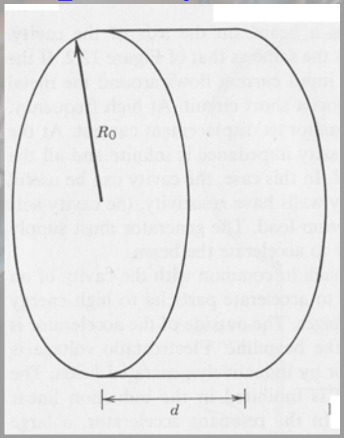
- Basic concepts of SW acceleration were introduced. The energy gain and efficiency were calculated and compared with the TW counterpart.
- Fundamental concept of electron capture and phase stability for ion and relativistic electron linacs were developed.
- Circuit models of infinite periodic structures were explored.
- Dispersion relations for structures consisting of a finite number of cells
- The fundamental issues of the mode stability of π-mode and π /2 mode accelerators were explored
- Circuit models of electron accelerators were developed, used later on in the tutorial and the accuracy was verified.
- For the ILC superconducting cavities the circuit model is accurate to better than 1% in calculations for the mode frequencies!

Overview of Week 4

- Coupled cavity linacs are explored -operated in the pi/2 mode. A detailed circuit model is developed.
- General phase stability criterion are developed for linacs
- The important issue of transient beam loading is discussed in detail
- Approximate 'zero order' design equations are developed for periodic accelerators. This determines ω in terms of a and b or, what is usually the case, b in terms of ω and a.
- Means for determining the characteristic loss factor are described via a wire measurement.
- X-band and L-band wire measurements are discussed: NLC damped detuned structure at 11.424 GHz and crab cavity at 3.9 GHz, respectively.
- General expressions for multi-cell loss factors are developed in terms of the standard single-cell loss factor.

Linac Cell Design

➢ **We will develop an approximate analytical formula relating the iris radius a, and cavity radius b, in a disk loaded slow wave structure to the synchronous particle beam velocity (v_p~c), at a prescribed frequency ω.**



Before considering the iris loaded cavity, we consider a closed cylindrical cavity ('pill-box cavity'). The modes of the cavity are obtained from Maxwell's equations:

$$\nabla \mathrm{x} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla . \mathbf{E} = 0$$

$$\nabla \mathrm{x} \mathbf{B} = -\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

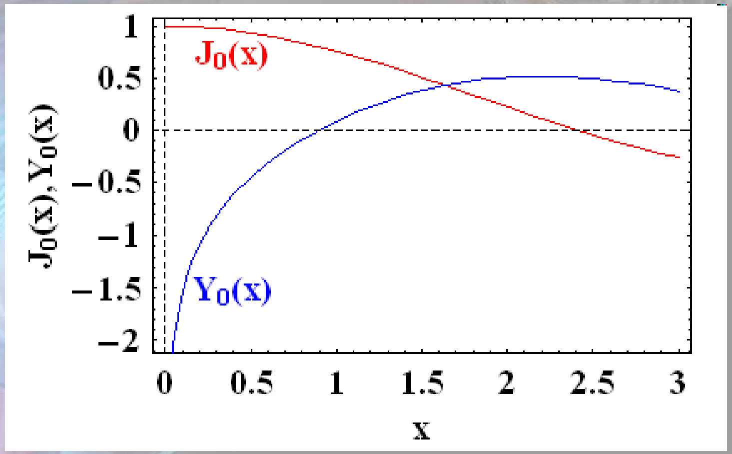
$$\nabla . \mathbf{B} = 0$$

where c is the velocity of light in the cavity medium.

Applying the vector identity: $\nabla \mathrm{x}(\nabla \mathrm{x} \mathbf{V}) = \nabla(\nabla . \mathbf{V}) - \nabla^2 \mathbf{V}$, we obtain:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix} = 0$$ **← we refer to this as the wave equation.**

There are of course many different modes of which correspond to the solution of the wave equation (see Jackson, for example). Once the solution E (H) has been obtained, the remaining field is obtained from the explicit form of Maxwell's equations indicated previously. Here, we focus on:



1. Modes with azimuthal symmetry ($\partial/\partial\theta = 0$)
2. The electric field has no longitudinal variation
3. The only component of the electric field is E_z
4. Fields have a time harmonic variation, $e^{j\omega t}$

The last two assumptions imply field of the form:

$$\mathbf{E} = E_z(r)\exp(j\omega t)\hat{z}$$

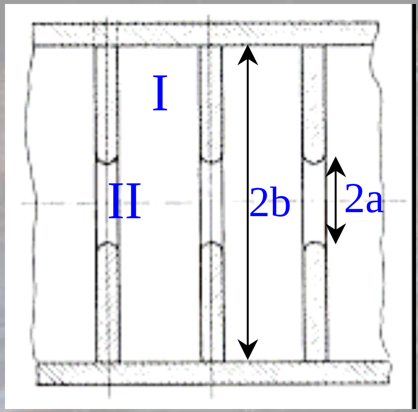
Using the cylindrical form of the Laplacian operator and dropping azimuthal (1) and longitudinal variations (2):

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \frac{\omega^2}{c^2}\right)E_z(r) = 0$$

This is Bessel's equation which has solutions: $J_0(k_n r)$ and $Y_0(k_n r)$

For a true cylindrical cavity we eliminate the Y_0 solution as it gives rise to a non-finite e.m. field on axis.

We will use these solution in the iris loaded cavity (our slow wave structure). We demarkate the cavity into two regions.



Region I (SW region) defined by: a<r < b

Region II (TW region) defined by: 0<r < a

The solution in Region I is given by:

$$E_z^{SW} = \left[E_0^{SW} J_0(\omega r/c) + C Y_0(\omega r/c)\right] e^{j(\omega t - kz)}$$

and on the walls of what we assume to be a perfect conductor, $E_z^{SW} = 0$

$$\Rightarrow E_z^{SW} = E_0^{SW}\left[Y_0(\omega b/c) J_0(\omega r/c) - J_0(\omega b/c) Y_0(\omega r/c)\right] e^{j\omega t}$$

and in Region II:

$E_z^{TW} = E_0^{TW} e^{j(\omega t - k_0 z)}$, where we have assumed the iris spacing is small compared to the wavelength (making the TW field approx. constant across an iris)

The magnetic field is given in terms of E_z in both cases:

$$H_\phi^{SW} = \frac{1}{\mu c^2}\frac{1}{r}\int r\frac{\partial E_z^{SW}}{dt}dr = \frac{jE_0^{SW}}{Z_0}\left[Y_0(\omega b/c) J_1(\omega r/c) - J_0(\omega b/c) Y_1(\omega r/c)\right] e^{j\omega t}$$

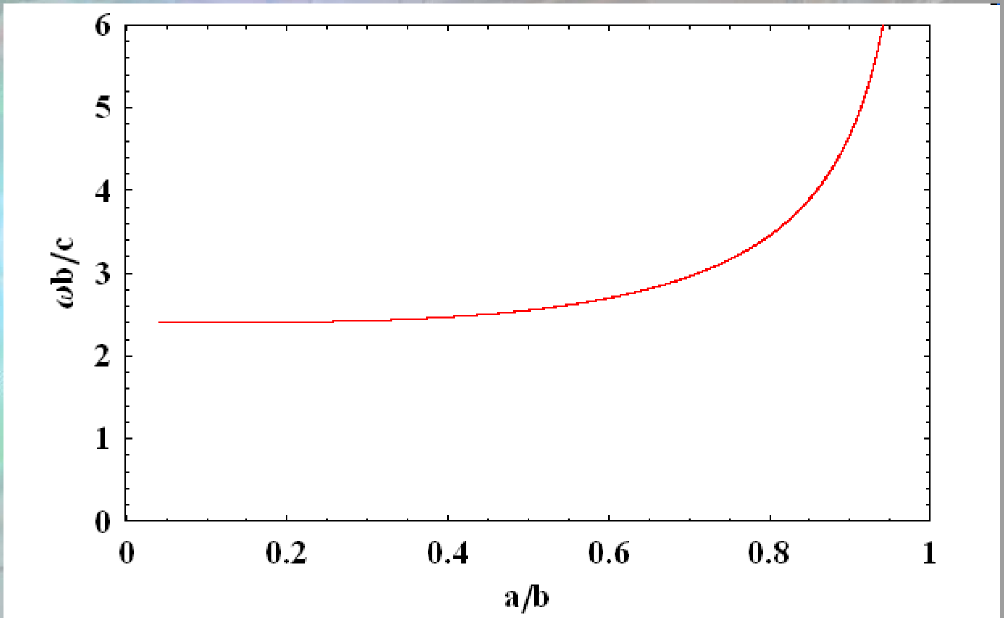
$$H_\phi^{TW} = \frac{j}{Z_0}\frac{\omega r}{2c} E_0^{TW} e^{j(\omega t - k_0 z)}$$

Equating the wave admittance at r=a:

$$\frac{H_\phi^{TW}}{E_z^{TW}} = \frac{H_\phi^{SW}}{E_z^{SW}}$$

$$\Rightarrow \boxed{\frac{\omega a}{c} = 2\frac{Y_0(\omega b/c)J_1(\omega a/c) - J_0(\omega b/c)Y_1(\omega a/c)}{Y_0(\omega b/c)J_0(\omega a/c) - J_0(\omega b/c)Y_0(\omega a/c)}}$$

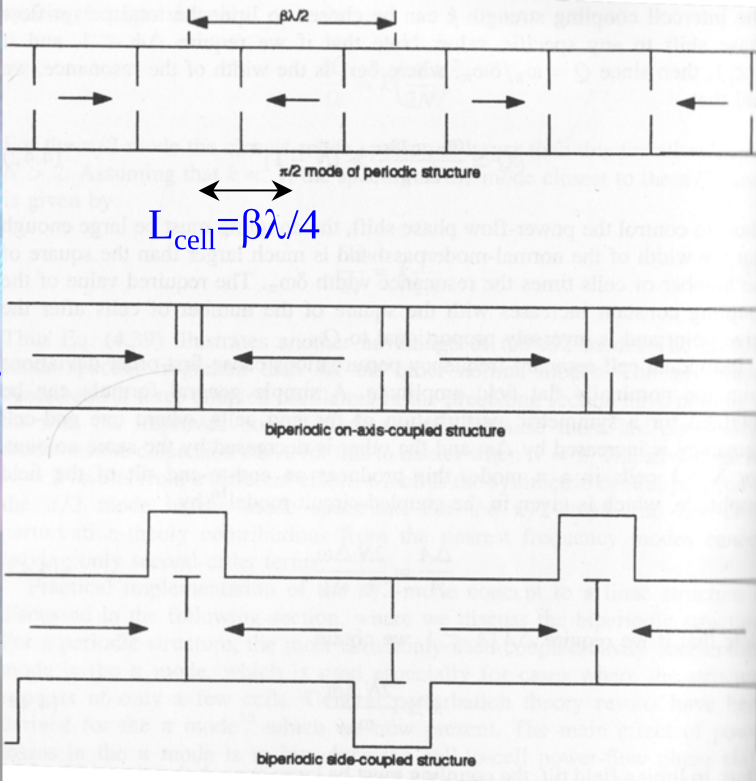
- This is a transcendental equation which determines ω in terms of a and b or, what is usually the case, b in terms of ω and a. Although the assumption of a large number of irises per wavelength is often not satisfied, it still gives a reasonable approximation to design the cells.
- This can be regarded as a zero order design.



- The design is then refined with finite difference or finite element numerical codes –such as Superfish, GdfidL, HFSS, MAFIA, Microwave Studio, Omega2, Omega3, Analyst (a commerical code based on Omega3).

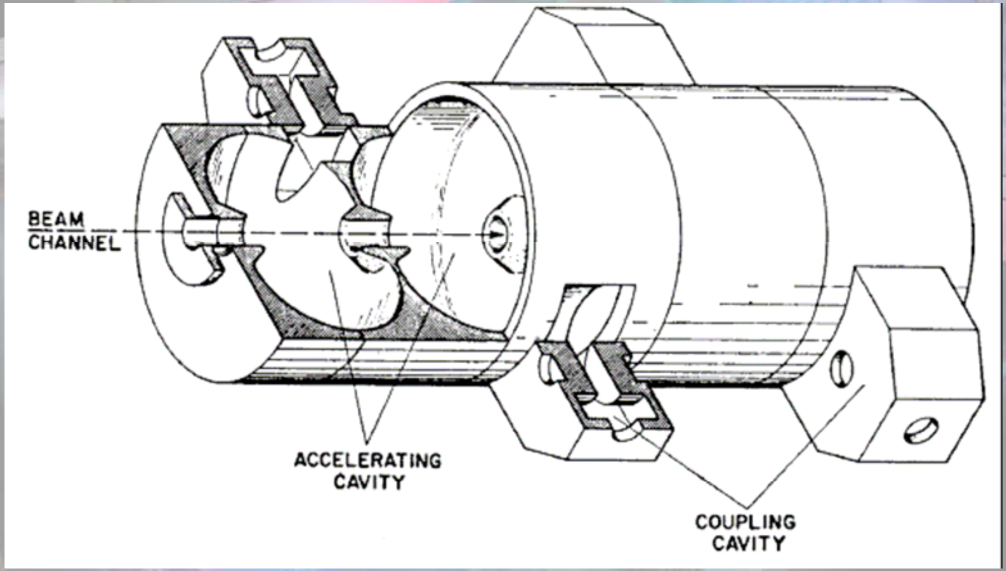
Coupled Cavity Linacs

- As pointed out earlier, the π acceleration mode has a maximum efficiency of transfer of energy to the beam. However, neighbouring modes are readily excited and it is very sensitive to frequency errors, which are inevitable in the fabrication of any accelerating structure.
- The π/2 mode on the other hand is far more stable than the π mode but it is rather inefficient.
- How can one mitigate for the inefficiencies of the π/2 mode accelerator?
- Upon reflection, we can see that the reason for the relatively poor shunt impedance is because there are empty fields within a π/2 set of cavities and this is account for the inherent inefficiency.



- Thus, if one reduces the time the beam spends in the unfilled cavities, the shunt impedance approaches that of the π mode.

- **Thus, the length of the cavities which contain zero energy is reduced and the cavity is retuned to that of the main π/2 mode –so long as the mode frequency of each cell is that of the π/2 mode the overall cavity mode is that of a π/2 mode.**
- **Or we can move the unfilled cavities entirely off axis**
- **In effect we have the shunt impedance of a π mode accelerator with the stability of a π/2 mode cavity.**



Side-Coupled Cavity Linac (LANL Design)

References

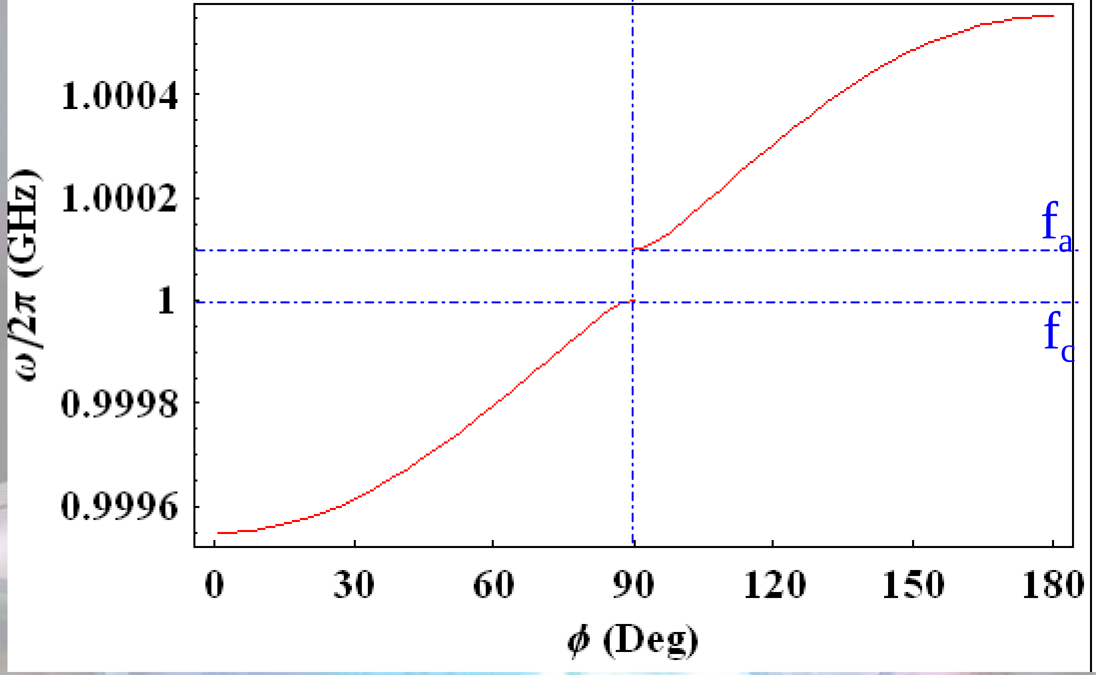
1. S.O. Schriber et al, Proc. 1972 Linear Accelerator Conference, Los Alamos Lab. Report LA-5115, p.140.
2. E.A. Knapp and J.M. Potter, Rev. Sci. Instrum. 39, 979-991 (1968).

The behaviour of these on-axis coupled and side-coupled cavities was analysed using a nearest neighbour and next nearest neighbour circuit model. Here, we restrict the analysis to nearest neighbour. The resulting dispersion equation for 2N coupling cavities with frequency ω_c, which alternate with 2N+1 accelerating cavities with frequency ω_a; the total number being 4N+1:

$$\kappa^2 \cos^2 \frac{\pi q}{2N} = \left(1 - \frac{\omega_a^2}{\Omega_q^2}\right)\left(1 - \frac{\omega_c^2}{\Omega_q^2}\right); \quad q=0, 1, ..., 2N$$

where κ is the coupling constant between the accelerating and coupling cells, referred to as nearest neighbours, and Ω_q is the mode frequency for normal-mode q. The next-nearest-neighbour coupling between adjacent accelerating cells is ignored in this analysis. The quantity $\pi q/2N$ is the phase advance per cavity of a traveling wave. The $\pi/2$ mode corresponds to $q = N$ and thus the LHS is zero. This gives two solutions: $\Omega_q = \omega_a$ and $\Omega_q = \omega_c$.

We now investigate the coupling of modes for a band of phase values.



Dispersion Curve of a Bi-periodic Linear Accelerator

- **The dispersion curves indicate that there are two branches which split at π/2.**
- **The upper branch corresponds to excited accelerating cavities and unexcited coupling cavities**
- **The lower curve corresponds to excited coupling cavities and unexcited accelerating cavities.**
- **These two branches are known as the upper and lower passbands.**
- **The discontinuity at π/2 corresponds to the stop band in which there are no normal mode solutions.**
- **The stop band is removed by tuning the cavities to the same frequency: $\omega_a = \omega_c$ and in practise this is how the device is operated.**

Generalised Phase Stability in Linac

- Here we will see that the frequency of longitudinal motion is generally much less than that of transverse motion; thus, approximately, they are decoupled.
- Suppose there is a synchronous particle that gains energy at a rate

$$\frac{dW_s}{dz} = eE_0 \cos\phi_s, \quad \phi_s = \omega(t - z/v_s)$$

and non-synchronous particles are governed by:

$$\frac{dW_s}{dz} = eE_0 \cos\phi, \quad \phi = \omega(t - z/v_s)$$

The difference between the synchronous and the arbitary particle is:

$$\frac{d\Delta W_s}{dz} = \frac{d}{dz} W - W_s = eE_0 (\cos\phi - \cos\phi_s)$$

Similarly, the phase difference equation is given by:

$$\frac{d}{dz}\phi - \phi_s = \frac{\omega}{v_s}\frac{d}{dt}(z_s - z)\frac{dt}{dz} = \omega\left(\frac{1}{v} - \frac{1}{v_s}\right)$$

Now we express velocity in terms of the relativistic factor:

$$\frac{1}{v}=\frac{1}{\beta c}=\frac{\gamma}{c}(\gamma^2-1)^{-1/2}$$

and the relative energy deviation is proportional to:

$\Delta\gamma=\gamma-\gamma_s$ and we obtain:

$$\frac{\omega}{c}\left[\frac{\gamma}{\sqrt{\gamma^2-1}}-\frac{\gamma_s}{\sqrt{\gamma_s^2-1}}\right]\approx\frac{\omega}{c}\left[\frac{\gamma_s+\Delta\gamma}{\sqrt{\gamma_s^2-1}}\left(1-\frac{\gamma_s\Delta\gamma}{\gamma_s^2-1}\right)-\frac{\gamma_s}{\sqrt{\gamma_s^2-1}}\right]$$

$$\approx-\frac{\omega}{c}\frac{\Delta\gamma}{\left(\gamma_s^2-1\right)^{3/2}}=-\frac{\omega}{c}\frac{\Delta\gamma}{\left(\beta_s\gamma_s\right)^3}$$

Thus, the phase equation becomes:

$$\boxed{\frac{d}{dz}\Delta\phi=\frac{d}{dz}(\phi-\phi_s)=-\frac{k\Delta W}{m_0c^2\beta_s^3\gamma_s^3}}$$

where $k=\omega/c$, $\Delta W=m_0c^2\Delta\gamma$

and this together with the previous energy equation

$$\boxed{\frac{d\Delta W_s}{dz}=\frac{d}{dz}W-W_s=eE_0(\cos\phi-\cos\phi_s)}$$

describe the motion of the particles in phase space $(\Delta W,\phi)$

Taking the derivative of the phase equation w.r.t z and

using the energy equation we obtain the second order differential equation:

$$\frac{d}{dz}\left[\beta_s^3\gamma_s^3\frac{d}{dz}\left(\phi-\phi_s\right)\right]=-\frac{ekE_0}{m_0c^2}\left(\cos\phi-\cos\phi_s\right)$$

There are three features of this equation that will be looked into;

the frequency for small amplitudes, the stability range and, the effect

of variation of variation of coefficients.

For small amplitudes and slowly varying energy we linearise the 2nd

order equation by $\phi=\phi_s+\Delta\phi$:

$$\left(\frac{d^2}{dz^2}+k_\phi^2\right)\Delta\phi,\quad k_\phi=\frac{2\pi}{\lambda_\phi}=\sqrt{-\frac{ekE_0}{m_0c^2\beta_s^3\gamma_s^3}\sin\phi_s}$$

The small amplitude particles perform SHM. In order that the oscillations

be stable, it is required that $\sin\phi_s < 0$, corresponding to acceleration in front of

the wave -otherwise the solutions will be hyperbolic. It is also worth noting that

the frequency of oscillations decreases with increasing energy, γ.

To investigate the phase stability we treat the parameters in the second order differential equation as constant, multiply by $(\phi-\phi_s)'$ and integrate:

$$\boxed{\frac{1}{2}\beta_s^3\gamma_s^3(\phi-\phi_s)'^2 = -\frac{ekE_0}{m_0c^2}\left(\sin\phi-\phi\cos\phi_s+C\right)}$$

Using the previous phase equation

$$\frac{d}{dz}\Delta\phi = \frac{d}{dz}(\phi-\phi_s) = -\frac{k\Delta W}{m_0c^2\beta_s^3\gamma_s^3}$$

we obtain:

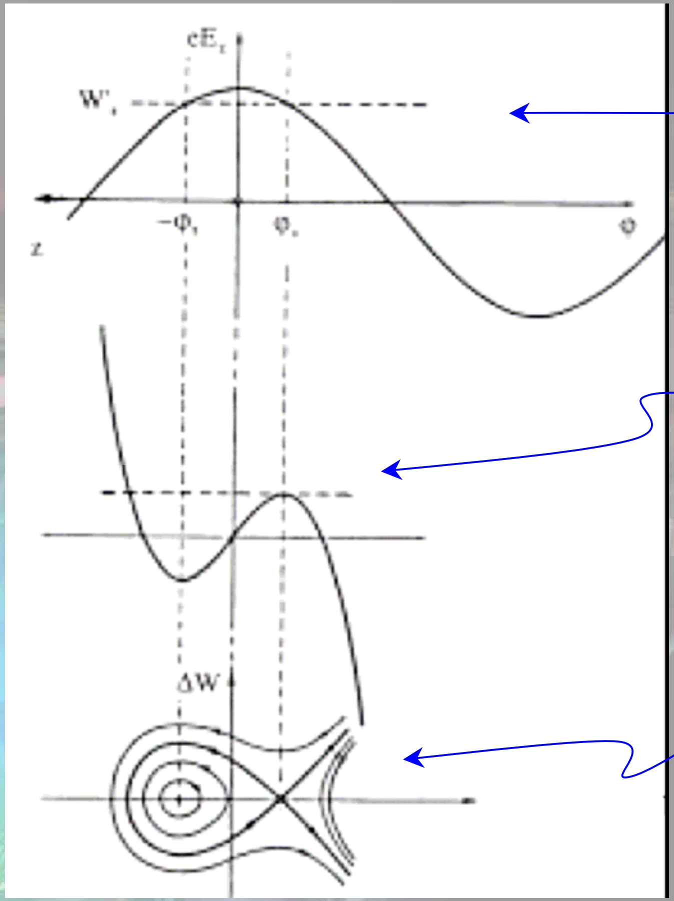
$$\boxed{\frac{k}{2m_0c^2\beta_s^3\gamma_s^3}\left(\Delta W\right)^2 + eE_0\left(\sin\phi-\phi\cos\phi_s+C\right) = 0}$$

Each value of the constant C describes a possible trajectory in phase-space. The point $\phi=\phi_s$ is a stationary but unstable point, and it determines the limiting amplitude of oscillations. The corresponding trajectory is obtained by setting:

$C=\phi_s\cos\phi_s-\sin\phi_s$

and it is referred to as the separatrix as it bounds the region of stable oscillations.

- The area of the stable region, enclosed by the <u>separatrix</u>, is called the bucket(particles in the bucket can be lifted to higher energies).
- Normally, the bucket is not completely filled but only a compact area around $-\phi_s$ which is called a bunch.
- Certainly, the factor $(\gamma_s \beta_s)^3$ varies along the accelerator



- And sometimes one also desires variable E_0 and ϕ_s . Under these circumstances the motion becomes quite complicated. However, for sufficiently slow changes the boundary of a bunch, which originally was determined by one of the oval trajectories will be a new oval trajectory at each moment, calculated from the current values of parameters.
- Such changes are called adiabatic.

➢To illustrate the use of adiabatic changes we refer to Liouville's theorem, stating that in a Hamiltonian system the phase-space area is a constant of motion. We note that the energy and phase equation

$$\frac{d\Delta W_s}{dz} = \frac{d}{dz} W - W_s = eE_0(\cos\phi - \cos\phi_s)$$

$$\frac{d}{dz}\Delta\phi = \frac{d}{dz}(\phi - \phi_s) = -\frac{k\Delta W}{m_0 c^2 \beta_s^3 \gamma_s^3}$$

have precisely the form of canonical equations:

$$\frac{d\Delta W}{dz} = -\frac{\partial H}{\partial \Delta\phi}, \quad \frac{d\Delta\phi}{dz} = -\frac{\partial H}{\partial \Delta W}$$

This valid provided we take the Hamiltonian as eE_0C in

$$\frac{k}{2m_0 c^2 \beta_s^3 \gamma_s^3}(\Delta W)^2 + eE_0\left(\sin\phi - \phi\cos\phi_s + C\right) = 0,$$

$$\Rightarrow H = -\frac{k}{2m_0 c^2 \beta_s^3 \gamma_s^3}(\Delta W)^2 - eE_0\left(\sin\phi - \phi\cos\phi_s\right)$$

Thus, Liouville's theorem applies to the motion in $(\Delta W, \phi)$ phase space and the area of the ellipses remains constant.

Let us consider small ellipses around the stable phase point $\phi = -\phi_s$. Letting $\phi = -\phi_s + \Delta\phi$ and $\Delta W = 0$ we obtain C from the above equation:

$$C = (\sin\phi_s - \phi_s \cos\phi_s) - \frac{1}{2}\Delta\phi^2 \sin\phi$$

The vertical axis of the ellipse is found from $\phi = -\phi_s$ and the above value of C:

$$\Delta W = \left[eE_0 \sin|\phi_s| m_0 c^2 \beta_s^3 \gamma_s^3 / k \right]^{1/2} \Delta\phi$$

Now, we indicate the beginning (end) of the linac with a 1 (2) and apply Liouville's theorem that the total area in phase space remains constant:

$$\Delta\phi_1 \Delta W_1 = \Delta\phi_2 \Delta W_2$$

The oscillations in phase-space are damped:

$$\frac{\Delta\phi_1}{\Delta\phi_2} = \left[\frac{E_1 \sin|\phi_{s1}| \beta_{s1}^3 \gamma_{s1}^3}{E_2 \sin|\phi_{s2}| \beta_{s2}^3 \gamma_{s2}^3} \right]^{1/4}$$

For constant $E\sin\phi_s$ the motion is attentuated according to $\gamma^{-3/4}$

- **The importance of the adiabatic law results from the fact that we need the equations of motion only at the beginning and at the end of the process**
- **How they change in between is irrelevant as long as it is slow.**

- However, there is a limit. Particles close to the separatrix pass close to the unstable point $\phi = \phi_s$ and have very low phase oscillation frequencies.
- Some of these particles will find the parameter change too fast to be adiabatic.

From these equations it is clear that the synchrotron motion and the energy acceptance width are strongly dependent on γ. From the ratio of ϕ at the end and beginning of the linac and from

$$k_\phi = \frac{2\pi}{\lambda_\phi} = \sqrt{-\frac{ekE_0}{m_0 c^2 \beta_s^3 \gamma_s^3} \sin\phi_s}$$ we obtain:

$$\omega_\phi \sim \lambda_\phi^{-1} \sim \gamma^{-3/2},\ \Delta\phi \sim \gamma^{-3/4}$$

i.e. the oscillation frequency decays rapidly with γ and the phase amplitude also diminishes.

The energy acceptance width, that is the bucket width is:

$$\Delta W = \pm 2\left[eE_0 m_0 c^2 \beta_s^3 \gamma_s^3 \left(\sin\phi_s - \phi_s \cos\phi_s \right) / k \right]^{1/2}$$

or

$$\frac{\Delta W}{W} = \pm 2\left[\frac{eE_0 \beta_s^3 \gamma_s}{m_0 c^2 k} \left(\sin\phi_s - \phi_s \cos\phi_s \right) \right]^{1/2} \sim \gamma^{1/2}$$

where we have used

$$\frac{k}{2m_0c^2\beta_s^3\gamma_s^3}(\Delta W)^2 + eE_0(\sin\phi - \phi\cos\phi_s + C) = 0 \text{ and } C = \phi_s\cos\phi_s - \sin\phi_s$$

As an example we take a high energy linac: f=3GHz, $E_0 = 20\text{MV/m}$,

$m_0c^2\gamma = 5\text{GeV}$, $\phi_s = 10^\circ$ which gives $\lambda_\phi = 2\pi/k_\phi = 300\text{km}$, $\Delta W/W = \pm 6.6$

- **The synchrotron oscillation wavelength is very large, several orders of magnitude larger than the betatron wavelength (typically of the order of 100 m).**
- **In high energy linacs longitudinal and transverse motion are highly decoupled. The longitudinal motion can normally be neglected.**

Transient Beam Loading

- Beam loading is defined as the energy reduction of charged particles due to their interaction with an accelerating structure.
- We will treat the multibunch beam loading problem by using the energy conservation law.
- Starting with the solution of the transient beam loading problem for a constant- gradient traveling-wave accelerator structure (SLAC-type structure).
- The solution will be used to analyze the case of a bunch train which is injected into an accelerator section either before or after the section is filled entirely with energy.
- Finally, we will extend the results to discuss the energy compensation problem for multi-bunch operation of future linear colliders.
- To simplify the problem, we will assume that the RF pulse is an ideal step-function without dispersive effects and that the charged particles all travel with the speed of light.
- In the absence of a beam, the steady-state variation of the RF power flow P(z) along an accelerator structure is given by

$$\frac{dP}{dz} = -2\alpha(z)P(z)$$

- where a(z) is the attenuation coefficient of the structure.

➢For the constant-gradient structure, $\alpha(z)$ is a slowly varying function along the structure. The attenuation constant for the entire section of length L is:

$$\tau = \int_0^L \alpha(z)dz$$

➢In the presence of an electron beam and taking into account time, the RF power loss per unit length is given by

$$\frac{dP}{dz} = \left(\frac{dP}{dz}\right)_{wall} + \left(\frac{dP}{dz}\right)_{beam}$$

$$= -2\alpha(z)P(z,t) - i(t)E(z,t)$$

➢where E(z,t) is the amplitude of the electric field at (z, t) on axis, the first term is the power dissipated in the structure walls, and the second term is the power absorbed by the beam.

➢ By seeking the total differential of P(z,t) with respect to z, one obtains

$$\frac{dP(z,t)}{dz} = \frac{\partial P(z,t)}{\partial z} + \frac{\partial P(z,t)}{\partial t}\frac{dt}{dz}$$

➢In order to obtain the expression for dt/dz, let us study a disturbance at (z_1, t_1), which travels with the group velocity v_g and arrives at z at time t:

$$t = t_1 + \int_{z_1}^{z} \frac{dz}{v_g(z)}$$

By differentiating one obtains:

$$\frac{dt}{dz} = \frac{1}{v_g(z)}$$

Thus, we have:

$$\frac{\partial P(z,t)}{\partial z} + \frac{\partial P(z,t)}{\partial t}\frac{1}{v_g(z)} = -2\alpha(z)P(z,t) - i(t)E(z,t)$$

The power is related to the shunt impedance per unit length r, and attenuation α, through: $P = E^2 / 2\alpha r$. Using this together with the assumption that the shunt impedance per unit length does not change along the cavity allows the E-field equation to be obtained:

$$\frac{\partial E(z,t)}{\partial z} + \frac{1}{v_g(z)}\frac{\partial E(z,t)}{\partial t} + \left[\alpha(z) - \frac{1}{2\alpha(z)}\frac{d\alpha}{dz}\right]E(z,t) = -\alpha(z)ri(t)$$

Taking the Laplace transform with respect to time:

$$\frac{\partial E(z,t)}{\partial z} + \left[\frac{s}{v_g(z)} + \alpha(z) - \frac{1}{2\alpha(z)}\frac{d\alpha}{dz}\right]E(z,s) = -\alpha(z)ri(s)$$

For a constant-gradient structure without beam, the attentuation coefficient is given by:

$$\alpha(z)=\frac{(P_{in}-P_{out})/2L}{P(z)}=\frac{(P_{in}-P_{out})/2L}{P_{in}-(P_{in}-P_{out})(z/L)}$$

$$=\frac{(1-e^{-2\tau})/2L}{1-(1-e^{-2\tau})(z/L)}$$

Diffentiating this wrt z:

$$\frac{d\alpha(z)}{dz}=\frac{(P_{in}-P_{out})/2L.(P_{in}-P_{out})/L}{\left[P_{in}-(P_{in}-P_{out})(z/L)\right]^2}=2\alpha^2(z)$$

Subtituting this into the E-field differential equation and integrating:

$$E(z,s)=E(0,s)e^{-st_z}-e^{-st_z}ri(s)\int_0^z e^{st_z}\alpha(z)dz$$

where t_z is the time taken for the energy to propagate from 0 to z:

$$t_z=\int_0^z\frac{dz}{v_g(z)}=\int_0^z\frac{2Q\alpha(z)}{\omega}dz=-\frac{Q}{\omega}\ln\left[1-(1-e^{-2\tau})z/L\right]$$

Replacing $dt_z = \dfrac{1}{v_g(z)} dz = \dfrac{2Q\alpha(z)}{\omega} dz$ in the E-field equation allows the integral to be evaluated:

$$E(z,s) = E(0,s)e^{-st_z} - \frac{\omega r i(s)}{2sQ}\left[1 - e^{-st_z}\right]$$

The energy gain of a synchronous particle passing through the accelerator is given by: $V(t)=\int_0^L E(z,t)dz$. Also, the Laplace transform: $V(s)=\int_0^L E(z,s)dz$

Using the previous expression for $z=\dfrac{L}{1-e^{-2\tau}}\left[1-e^{-(\omega/Q)t_z}\right]$ gives:

$$dz=\frac{\omega L}{\left(1-e^{-2\tau}\right)Q} e^{-(\omega/Q)t_z} dt_z$$

Using this expession in the E-field equation and integrating:

$$V(s) = \frac{E(0,s)\omega L}{\left(1-e^{-2\tau}\right)\left(s+\omega/Q\right)Q}\left[1-e^{-(s+\omega/Q)t_F}\right] - \frac{\omega r i(s) L}{2sQ\left(1-e^{-2\tau}\right)}$$

$$\left[\left[1-e^{-(\omega/Q)t_F}\right] - \frac{\omega}{Q(s+\omega/Q)}\left[1-e^{-(s+\omega/Q)t_F}\right]\right]$$

where $t_F = t_z(L) = 2\tau Q/\omega$ is the filling time of the accelerator section.

Now, let us assume that E(0,t) and i(t) are step functions:

$$\begin{cases} E(0,t)=E_0 U(t) \\ i(t) = i_0 U(t - t_i) \end{cases}$$

where t_i is the time when the beam is injected, i_0 is the average beam current, E_0 is the amplitude of the electric field at z=0 and U(t) is the unit step function.

If there are N equally spaced bunches, each of charge q, and the bunch train has a time span t_b, the average current can be expessed as:

$$i_0 = Nq / t_b$$

Now, we can readily obtain the Laplace tranform of the field and current:

$$\begin{cases} E(0,s)=E_0 / s \\ i(t) = (i_0 / s)e^{-st_i} \end{cases}$$

This enables the E-field and energy gain to be evaluated:

$$E(z,s) = \frac{E_0}{s} e^{-st_z} - \frac{\omega r i_0}{2Q}\left[\frac{e^{-st_i}}{s^2} - \frac{e^{-s(t_i + t_z)}}{s^2}\right]$$

$$V(s)=\frac{E_0\omega L}{\left(1-e^{-2\tau}\right)\left(s+\omega/Q\right)sQ}\left[1-e^{-2\tau}e^{-st_F}\right]-\frac{\omega r i_0 e^{-st_i}L}{2Qs^2}$$

$$\left[1-\frac{\omega}{(1-e^{-2\tau})(s+\omega/Q)Q}\left[1-e^{-2\tau}e^{-st_F}\right]\right]$$

Taking the inverse Laplace transform gives the E-field at any point and the energy gain as a function of time:

$$E(z,t)=E_0U(t-t_z)-\frac{\omega r i_0}{2Q}\left[(t-t_i)U(t-t_i)+(t-t_i-t_z)U(t-t_i)\right]$$

$$V(t)=\frac{E_0L\left[1-e^{-(\omega/Q)t}\right]}{\left(1-e^{-2\tau}\right)}U(t)-\frac{E_0Le^{-2\tau}}{\left(1-e^{-2\tau}\right)}$$

$$\left[1-e^{-(\omega/Q)(t-t_f)}\right]U(t-t_f)$$

$$+\frac{r i_0 L}{2}\left\{\frac{\omega L e^{-2\tau}}{Q(1-e^{-2\tau})}(t-t_i)-\frac{L}{(1-e^{-2\tau})}\right.$$

$$\left.\left[1-e^{-(\omega/Q)(t-t_i)}\right]\right\}U(t-t_i)$$

$$-\frac{r i_0}{2}\left\{\frac{\omega L e^{-2\tau}}{Q(1-e^{-2\tau})}(t-t_i-t_F)-\frac{Le^{-2\tau}}{(1-e^{-2\tau})}\left[1-\frac{\omega}{Q}(t-t_i-t_f)\right]\right\}U(t-t_i-t_F)$$

➢Having obtained these general expressions, let us now apply them to some practical examples.

➢The first example is for the case where the beam is injected exactly after one RF filling time. Most of the traveling-wave linear accelerators work in this mode.

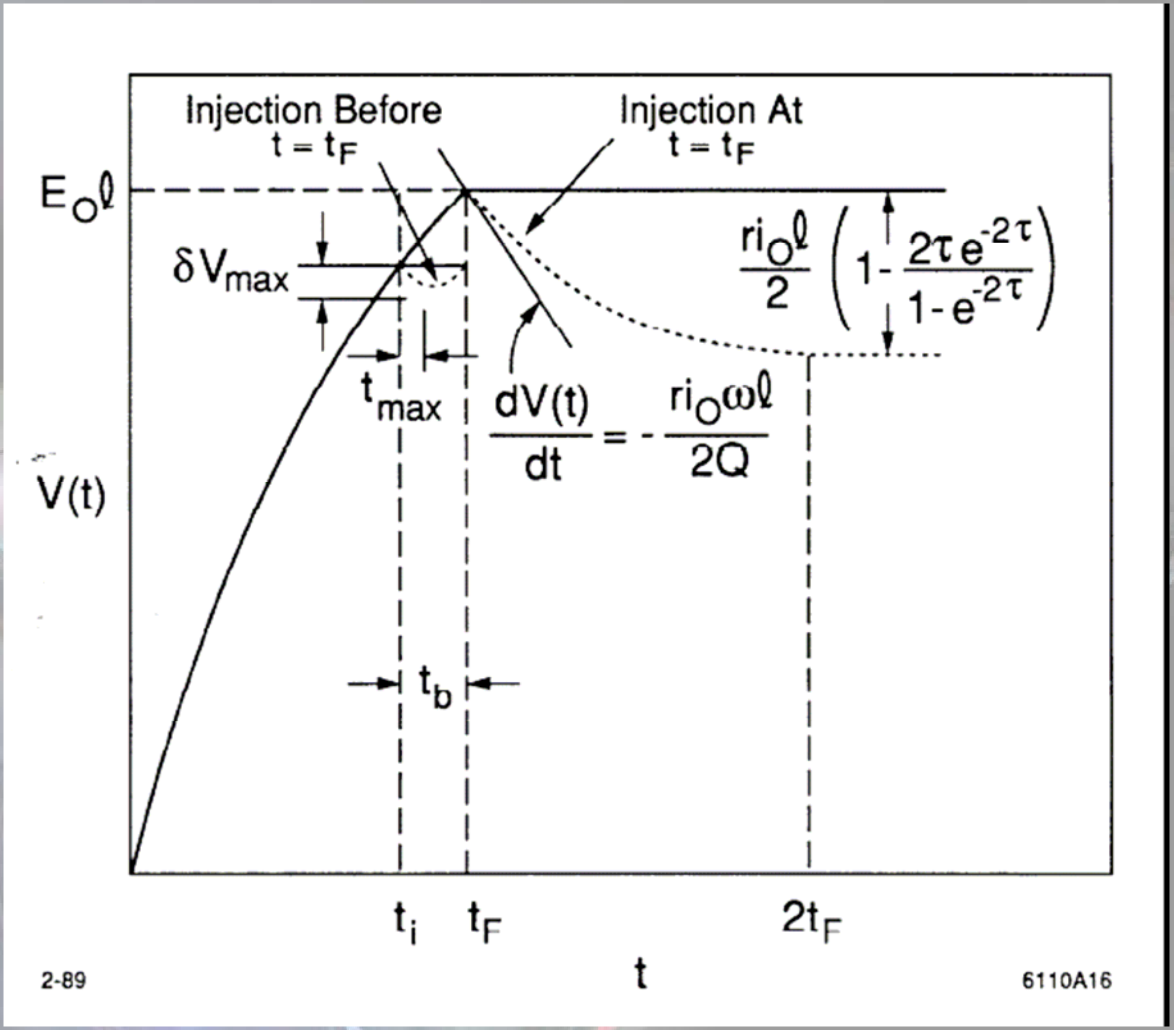
➢The second example is for the case where beam injection takes place before the RF structure is entirely filled up. The linear colliders accelerating multi-bunches will work in this mode.

Beam Injection after one Filling Time

In this case, for convenience we choose the time at which the beam is turned on as zero. The new time then starts after one filling time:

$$V(t)=\begin{cases} E_0L+\dfrac{ri_0}{2}\left[\dfrac{\omega L e^{-2\tau}}{Q(1-e^{-2\tau})}t-\dfrac{L}{(1-e^{-2\tau})}(1-e^{-\omega t/Q})\right] & 0\geq t\leq t_f \\ E_0L-\dfrac{ri_0L}{2}\left[1-\dfrac{2re^{-2\tau}}{(1-e^{-2\tau})}\right] & t\geq t_f \end{cases}$$

where $E_0L=(1-e^{-2\tau})^{1/2}(P_{in}rL)^{1/2}$



Transient Beam Loading for Injection before $t=t_F$ and at $t=t_F$

The transient beam loading for $0 \le t \le t_f$ can be expressed as:

$$\Delta V_b = V(t) - E_0 L = \frac{r i_0 L}{2(1-e^{-2\tau})}\left[1 - \frac{\omega}{Q} e^{-2\tau} t - e^{-\omega t/Q}\right]$$

$$= \frac{r i_0 L}{2(1-e^{-2\tau})}\left[1 - 2\tau e^{-2\tau}\left(\frac{t}{t_F}\right) - e^{-2\tau(t/t_F)}\right]$$

What happens to the derivative of ΔV_b at the beginning of beam injection $(t \le t_F)$?

$$\left.\frac{d\Delta V_b}{dt}\right|_{t=0} = -\frac{i_0 r \omega L}{2Q} = 2k_{loss}\Delta q/\Delta t\big|_{t=0}$$

where the energy loss factor is given by: $k_{loss} = \dfrac{\omega r L}{4Q} = \dfrac{\omega R}{4Q}$

$(r = R/L)$.

Also, what occurs at time t_F after the beam has been turned on?

After t_F :

$$\left.\frac{d\Delta V_b}{dt}\right|_{t=0} = 0$$

This corresponds to the transition from transient to steady state beam loading.

Early Injection for Multibunch Operation

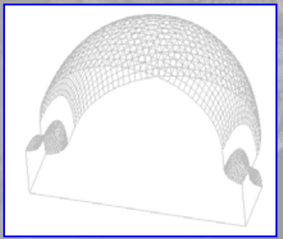
- **In order to increase the luminosity and RF energy transfer efficiency of a linear collider, multi-bunch operation will almost certainly be required.**
- **The beam should then be injected before the accelerator section is completely filled so that to first order, the energy decrease due to beam loading is compensated by the energy increase due to filling.**
- **Homework.** **For a bunch train consisting of bunches spaced by ΔS from their neighbours show that the energy compensation condition and maximum energy sag are given by:**

$$\frac{\Delta S}{L} = \left(\frac{ct_F}{L}\right)\frac{2qk_{loss}/L}{E_0 + Nqk_{loss}/L}$$

$$\delta V_{max} = -\frac{\tau}{2(1-e^{-2\tau})}\left(\frac{1}{ct_F}\right)N^2 k_{loss} q \Delta S$$

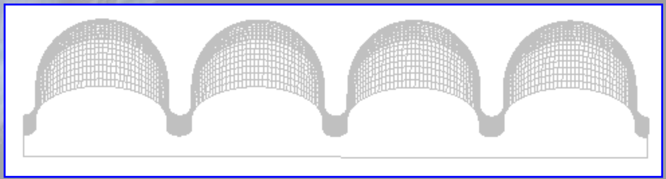
Multi-Cell Loss Factors in Terms of Single-Cell Loss Factors

- Sometimes useful to be able to calculate the loss factor of multiple identical cells in terms of the single-cell value.
- We will calculate the loss factor per unit length.
- This is useful for part I of the computer project.



Single-Cell Cavity

- N.B. here we will be analyzing multiple *identical* cells. This is of course quite different from the multi-cell loss factor calculated for cells with different dimensions (such as for detuned structures, which will be covered later).



Multi-Cell Cavity

- Loss factor is defined as:

$$\Rightarrow k_{loss} = \frac{|V|^2}{4U}$$

- Where V is the voltage (from an integral of Ez along the axis) and U is the energy stored in the mode.
- For a single infinitely repeating cell of period

$$V = \int_0^L E(z)\mathrm{Exp}(ikz)dz$$

where $k = \omega / c$.

*Take care not to confuse the wavenumber k with the loss factor (context should make it clear!).

- For a string of cavities with common period p the resonance condition is: => $E(z+L) = E(z)e^{-i\phi}$ (where ϕ is the phase advance over one cell).
- Thus, the voltage dropped across the complete cavity of N cells is:

$$V = \int_0^{Np} E(z)e^{ikz}dz = \int_0^{p} E(z)e^{ikz}dz + ...\int_{(N-1)p}^{Np} E(z)e^{ikz}dz$$

$$= \int_0^{p} E(z)e^{ikz}dz + ...\int_0^{p} E(z+[N-1]p)e^{ik(z+[N-1]p)}dz$$

$$= V_0[1 + ...e^{-i(N-1)\phi}e^{i(N-1)\omega p/c}]$$

$$= V_0\sum_{n=0}^{N-1} e^{-in(\phi-\omega p/c)}$$

$$= V_0\left[\frac{1-e^{-iN(\phi-\omega p/c)}}{1-e^{-i(\phi-\omega p/c)}}\right] = V_0^{-i(N-1)\delta/2}\frac{\mathrm{Sin}\left(\frac{N\delta}{2}\right)}{\mathrm{Sin}\left(\frac{\delta}{2}\right)}$$

$$\Rightarrow |V|^2 = |V_0|^2\left[\frac{1-\mathrm{Cos}N\delta}{1-\mathrm{Cos}\delta}\right].$$

- Here the phase slip angle has been defined as: $\delta = \phi - \omega p/c = \phi - \phi_{acc}\,\omega/\,\omega_{acc}$.

- **In terms of the modulation index M we have:**

  $$|V|^2 = |V_0|^2 M^2$$

  where:

  $$M = \frac{\mathrm{Sin}\left(\frac{N\delta}{2}\right)}{\mathrm{Sin}\left(\frac{\delta}{2}\right)}$$

- **Taking the limit of the phase slippage going to zero then:**

  $$\lim_{\delta \to 0}\{M\} = N$$

- **i.e. the voltages add: $V = NV_0$, as one might expect!**

- **Now we can calculate the LOSS-FACTOR PER UNIT LENGTH**

  $$\Rightarrow k'_{loss} = \frac{1}{Np}\frac{|V|^2}{4U} = k'_0 \frac{M^2}{N^2}$$

  $$\text{where } k'_0 == \frac{1}{p}\frac{|V_0|^2}{4U_0}$$

- **Here the relation $U = NU_0$ has been used (i.e. the stored energy, U_0, is independent of the phase factor ϕ).**

Summary

$$k'_{loss} = k'_0 \frac{M^2}{N^2}; \quad M = \frac{\mathrm{Sin}(N\delta/2)}{\mathrm{Sin}(\delta/2)}; \quad \delta = \phi - \omega p/c = \phi - \phi_{acc}\frac{\omega}{\omega_{acc}}$$

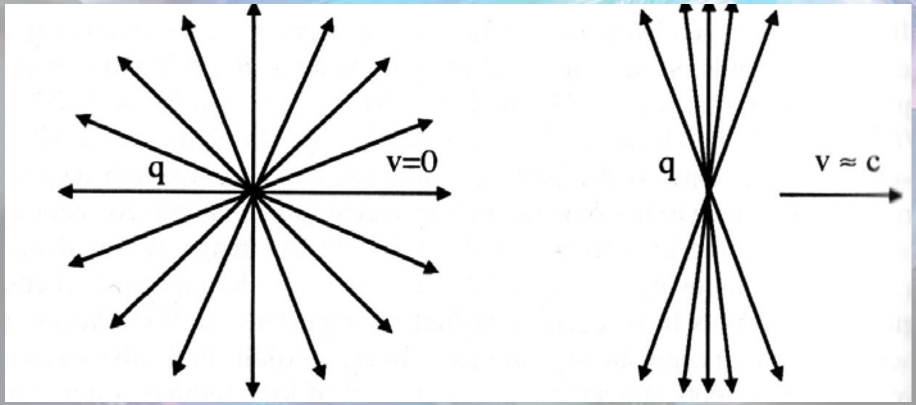
$$k'_0 == \frac{1}{p}\frac{|V_0|^2}{4U_0}; \quad V_0 = \int_0^p E(z)e^{i\omega z/c}dz$$

- For example, consider 10 cells with $\phi = \phi_{acc}$ and $\omega = \omega_{acc}$ => M=N
- => $k'_{loss} = k'_0$ (single-cell and multi-cell loss factor *per unit length* are identical).
- However, $k_{loss} = 10\ k_0$ (the ten-cell loss factor is an order of magnitude larger than the single-cell loss factor)

Coaxial Wire Measurement of Loss Factor and Wake-Field

Motivation and Validation

- The field of a point charge at rest is isotropic. The field of a charge moving relativistically is concentrated in a characteristic 'pancake'. The angle subtended by the field shrinks to zero as particle becomes ultra-relativisitic – i.e. the velocity of the particle approaches the velocity of light.



- In the limit of v->c the field is entirely perpendicular to the motion. In this case, The the radial field is given by:

$$E_r(r,z,\omega) = Z_0 H_\phi(r,z,\omega) = \frac{Z_0 q}{2\pi r}\exp(-j\frac{\omega}{c}z)$$

- The amplitude of the field decays inversely with the radial distance away from the charge. This is also the case for a charge moving in a perfectly conducting pipe (with no obstructions present). In both cases the e.m. field travels with the charge.

- Now consider a perfectly conducting cylindrical waveguide with a circular cross-section of radius b in which a wire of radius a has been inserted on-axis. The resulting coaxial structure allows the propagation of all frequencies. The TEM mode of this structure is given by:

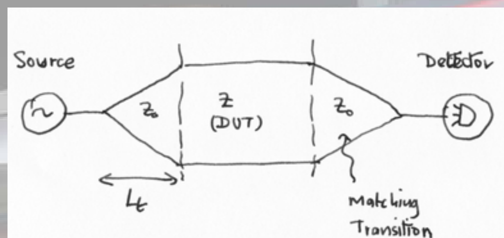
$$E_r(r,z,\omega) = Z_0 H_\phi(r,z,\omega) = Z_0 \frac{A}{r}\exp(-j\frac{\omega}{c}z)$$

where A is a constant depending on the power launched down the structure.

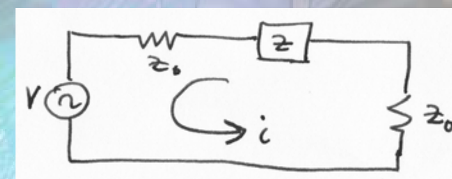
- Thus, we can make an analogue of electron beam traveling down a structure by investigating the progress of the TEM field excited in a waveguide in which a centre conductor has been placed symmetrically within it.

Coaxial Measurement Wire Measurement of Loss Factor from a Lumped Equivalent Circuit Analysis

- The equivalent circuit of the wire inserted into the Device Under Test (DUT) is illustrated below. The impedance Z represents that of the DUT and impedances either end are assumed to matched to the source.



Schematic illustrating DUT and S21 measurement



Equivalent circuit of DUT of impedance Z and matched input and output

- We assume the generator and detector are matched to the source. The power available to the output Z_0 is:

$$P_0 = V^2/(8Z_0)$$

- The transmission coefficient is given by:

$$T = \frac{Z_0}{2P_0}\left|i\right|^2, \quad \text{where } i = \frac{V}{2Z_0 + Z}, Z = R + jX$$

$$T=\frac{4Z_0^2}{(2Z_0+R)^2+X^2}, \quad \text{and at resonance: } T_0=\frac{4Z_0^2}{(2Z_0+R)^2}.$$

We normalise the power with respect to the maximum power that can be transmitted to the output Z_0. Thus, the power dissipated in the DUT is:

$$\frac{P_{diss}}{P_0}=1-T=\frac{R^2+4Z_0R+X^2}{(R+2Z_0)^2+X^2}$$

For a resonance in a parallel R-L-C we have:

$$Z\sim\frac{R_0}{1+j\dfrac{2Q(\omega-\omega_0)}{\omega_0}}$$

Thus, let $\xi=-\tan\psi$ and we have $R=R_0/(1+\xi^2)$ and $X=-R_0\xi/(1+\xi^2)$.

Also, $R^2+X^2=R_0^2/(1+\xi^2)$

Now, the loss factor is given by:

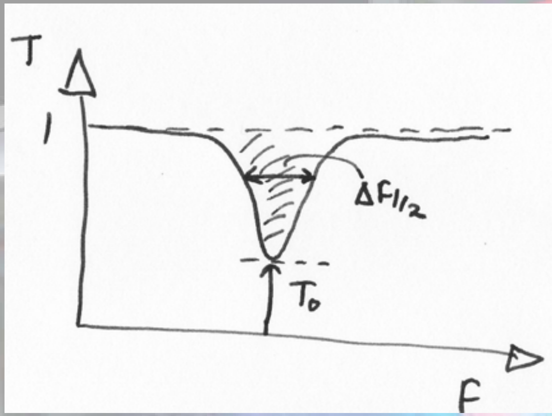
$$\boxed{k_{loss}=\frac{1}{\pi}\int_{mode}R\,d\omega=\frac{\omega_0R_0}{2Q\pi}\int_{-\infty}^{\infty}\frac{1}{1+\xi^2}d\xi=\frac{\omega_0R_0}{2Q}=\frac{\omega_0R_{acc}}{4Q}}$$

Here, $R_{acc}=2R_0$.

N.b. this $R_{acc}=|V|^2/P$ and $R_0=|V|^2/(2P)$

In the experiment the frequency is scanned and a particular dip in T ($=|S_{21}|^2$) is mapped out and this is illustrated below.

The objective of the experiment is to obtain the resonance frequency and the loss factor of each mode of interest. To obtain the mode frequency is quite straightforward as it corresponds to the minimum in T.



Power transmitted from source to detector. The dip corresponds to a resonant mode.

However, to compute the loss factor the area under the 1-T curve needs to be equated to the area under the power dissipated:

$$A=\frac{1}{\pi}\int_{-\infty}^{\infty}(1\text{-}T)\,d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{P_{diss}}{P_0}d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{R^2+X^2+4Z_0R}{R^2+X^2+4Z_0R+4Z_0^2}d\omega$$

$$=\frac{1}{2\pi}\frac{\omega_0}{2Q}\int_{-\infty}^{\infty}\frac{(1+\xi^2)^{-1}(R_0^2+4Z_0R_0)}{(R_0^2+4Z_0R_0)(1+\xi^2)^{-1}+4Z_0^2}d\xi$$

multiplying the numerator and denominator by $(1+\xi^2)$:

$$A=\frac{\omega_0}{4\pi Q}\int_{-\infty}^{\infty}\frac{(R_0^2+4Z_0R_0)}{(R_0+2Z_0)^2+4Z_0^2\xi^2}d\xi$$

$$=\frac{\omega_0}{4\pi Q}\frac{\pi R_0}{2Z_0}\frac{R_0+4Z_0}{R_0+2Z_0}=\frac{\omega_0}{8Z_0Q}\left(\frac{R_0+4Z_0}{R_0+2Z_0}\right)$$

However, $T_0=\frac{4Z_0^2}{(2Z_0+R_0)^2}\Rightarrow 1+\sqrt{T_0}=\frac{R_0+4Z_0}{R_0+2Z_0}$

and we obtain:

$$k_{loss}=\frac{\omega R_0}{2Q}=\frac{4Z_0}{1+\sqrt{T_0}}A$$

If the dip is Lorentzian, i.e. $1\text{-}T=\frac{1\text{-}T_0}{1+(2\frac{f\text{-}f_0}{\Delta f_{1/2}})^2}$ and $A=\frac{\pi}{2}\Delta f_{1/2}(1-T_0)$

$$\Rightarrow \boxed{k_{loss}=2\pi Z_0\Delta f_{1/2}(1-\sqrt{T_0})}$$

Thus, in the experiment all we are required to do is measure the minimum transmission (T_0), the half power points of the mode and the loss factor is readily calculated.

- The longitudinal wake of a particular mode is calculated according to: $W_z(s)=2k_{loss,0}\cos(\omega_o s/c)$. The complete wake-field is obtained by summing over all such modes.

- For the transverse modes, the dipole is the most critical. The dipole wake is computed according to:

$$W_t(s) = \frac{2k_{loss,d}\, r / a_0}{(\omega a_0 / c)} \sin(\omega_d s / c)$$

where r is the offset of the driving bunch, a_0 is the position at which the loss factor is calculated, $\omega_d/(2\pi)$ is dipole mode frequency and $k_{loss,d}$ is the dipole mode loss factor. Again, we sum over all modes to obtain the complete field.

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Wire Measurement of Impedance: Formally Exact Method

- The basis of the method is to take a transmission line and embed the coupling impedance with it. The goal of the analysis being to uncover the embedded impedance in terms of the measured experimental parameters –S matrix parameters.

- We begin with the standard unperturbed transmission line. Here we consider a series impedance Z (=R_0+j ωL_0) and a parallel admittance Y (=jωC_0).

The characteristic impedance of the line is given by:

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R_0 + j\omega L_0}{j\omega C_0}}$$

and the propagation constant:

$$\beta = \sqrt{(R_0 + j\omega L_0) j\omega C_0} \sim \omega\sqrt{L_0 C_0} - \frac{jR_0}{2\sqrt{L_0 C_0}}$$

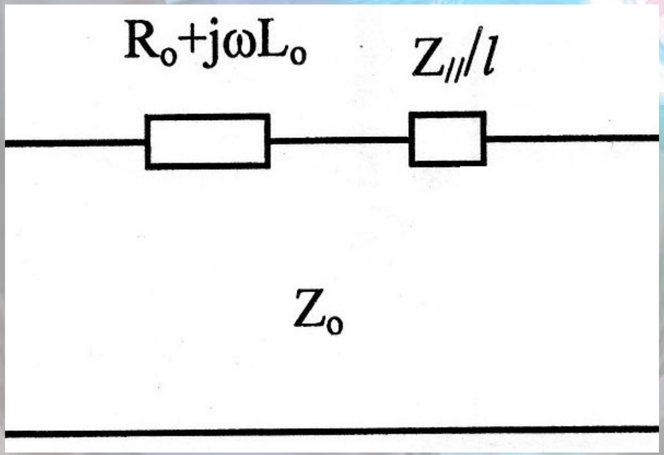
andthis means the transmission scattering matrix component is given by:

$S_{21} = \exp(-j\theta) = \exp(-j\beta l)$, where l is the length of the transmission line

Wire Measurement of Impedance: Formally Exact Method

- **Now we consider the DUT (Device Under Test) embedded within the transmission line as a coupling impedance. We consider the same transmission line with the same physical length, parallel admittance per unit length but with a series impedance per unit length which is a factor of ζ^2 different.**

- **This modified series impedance per unit length can be thought of the sum of the series impedance of the unperturbed line $j\beta Z_0$, in series with an additional impedance per unit length $\boxed{Z_{//}/l = j\,\beta(\zeta^2 - 1)Z_0}$ -the *coupling impedance*. This is illustrated below.**



Transmission line with an embedded coupling impedance $Z_{//}$.

- **In order to calculate this coupling impedance in terms of the measured scattering parameters we need to transform the Z-matrix for the transmission line to the S-matrix**

The impedance matrix for a transmission line is defined as:

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

and for the transmission line DUT:

$$Z = \frac{-j\xi Z_0}{\sin\xi Z_0} \begin{pmatrix} \cos\xi\theta & 1 \\ 1 & \cos\xi\theta \end{pmatrix}$$

The S-matrix is related to the Z-matrix according to:

$S=(Z-Z_0U)(Z+Z_0U)^{-1}$, where U=Identity matrix and Z_0 is the reference impedance (originally matched out with the reference device).

Thus, the S-matrix becomes:

$$S = \frac{\begin{bmatrix} Z_0^2(\xi^2-1)\sin\xi\theta & -2j\xi Z_0^2 \\ -2j\xi Z_0^2 & Z_0^2(\xi^2-1)\sin\xi\theta \end{bmatrix}}{Z_0^2(\xi^2+1)\sin\xi\theta - 2jZ_0^2\xi\cos\xi\theta}$$

Thus the transmission matrix is given by:

$$S_{21,DUT} = \left[\cos\xi\theta + \frac{j}{2}(\xi+\xi^{-1})\sin\xi\theta\right]^{-1}$$

and as the reference tranmission is given by:

$S_{21,REF} = \exp(-j\theta)$ then :

$$\boxed{\frac{S_{21,DUT}}{S_{21,REF}} = \frac{e^{j\theta}}{\cos\xi\theta + \frac{j}{2}(\xi + \xi^{-1})\sin\xi\theta}}$$

No approximations have been made up to this stage.

- Clearly if the reference line has the same impedance as the DUT then ξ=1 and S_{21} = exp(-jθ).

- Taking the natural log of the S_{21} ratio, with some rearrangement, leads to:

$$\log\left(\frac{S_{21,DUT}}{S_{21,REF}}\right) = -j(\xi - \theta) + \log\left[\frac{4\xi}{(\xi+1)^2 - e^{-2j\xi\theta}(\xi-1)^2}\right]$$

- This formula can be solved numerically but in the process of performing experiments it is useful to have analytical expressions. Thus, we explore several approximate formula for the impedance.

1. log $-$ formula

Substitute for ξ in terms of Z_z / Z_0 : $\xi = \sqrt{1 - jZ_z/(Z_0\theta)}$ and expand in powers of Z_z / Z_0 :

$$\frac{Z_z}{Z_0} \simeq -2\log\left(\frac{S_{21,DUT}}{S_{21,REF}}\right)$$

This is valid for a small coupling impedance compared to the unperturbed (reference) characteristic impedance $\left|Z_z / Z_0\right| < 1$

2. Improved log formula

Retain ξ and make an expansion in ξ-1:

$$\log\left(\frac{S_{21,DUT}}{S_{21,REF}}\right) = -j\theta(\xi - 1) + \theta(\xi - 1)^2$$

$$\Rightarrow \frac{Z_z}{Z_0} = -2\log\left(\frac{S_{21,DUT}}{S_{21,REF}}\right)\left[1 + \frac{j}{2\theta}\log\left(\frac{S_{21,DUT}}{S_{21,REF}}\right)\right]$$

Both the log and the improved log formula are first order power series expansions. The improved formula includes a θ expansion as $\xi\text{-}1=\sqrt{1\text{-}jZ_z/(Z_0\theta)} - 1$.

It is equivalent to neglecting the mismatch at the beginning and end of the transition (prove this!), i.e. $S_{21,DUT} \sim e^{-j\xi\theta}$

3. lumped element formula

Taking the original exact formula (not its logarithm) and replace ξ with

$$\xi=\sqrt{1-Z_z/(Z_0\theta)} \text{ in } \frac{S_{21,DUT}}{S_{21,REF}}=\frac{e^{j\theta}}{\cos\xi\theta+\frac{j}{2}(\xi+\xi^{-1})\sin\xi\theta} \text{ gives:}$$

$$\frac{S_{21,DUT}}{S_{21,REF}}=\frac{1}{1+\frac{1}{2}\frac{Z_z}{Z_0}}$$

$$\Rightarrow Z_z=2Z_0\left(\frac{S_{21,REF}}{S_{21,DUT}}-1\right)$$

This corresponds to the transmission amplitude of the transmission coefficent (i.e. $\sqrt{T}$)derived earlier in the lumped element analysis.

Domain of applicability

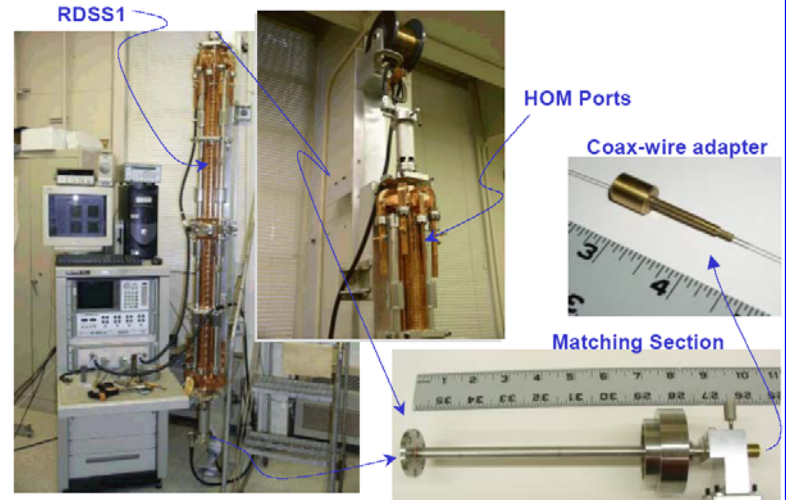
- For longer cavities (structures long compared to wavelength) the improved formula is the most appropriate one to use. The improved formula is often significantly more accurate than the log formula.

- Use of either log formula for short structure can easily give inaccurate results. Use the lumped in this case as it is the most appropriate one for this system.

Wire Measurements On SLAC X-Band Structures

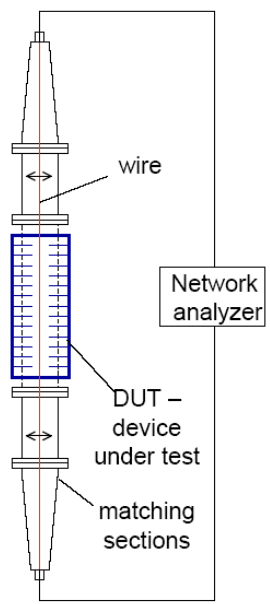
- **Wire Measurement of HOM and Alignment in L-Band Cavities**
- **Measurements made on X-band structures.**
- **Bench measurement provides mode frequencies and kick factors**
- **Measurement in progress on L-band 9-cell cavities (1.3 GHz and 3.9 GHz)**

Measurement Setup



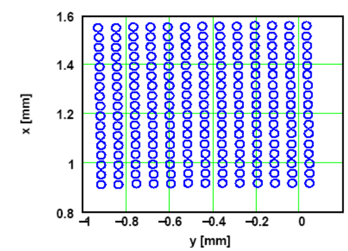
Measurement Setup

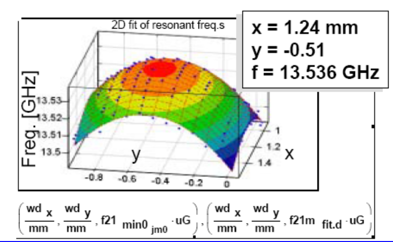
- Principle
  - based on similarity of field in presence of beam and of thin wire
- For dipole modes: 2 options
  - twin-ax
  - single off-axis wire
- Frequency domain method
  - setup for 11-18 GHz (first dipole band for NLC traveling-wave structures)
  - matching sections: S11 < -30dB for most freq.
  - possibility to move wire transversely
- Wire
  - 300μm diameter



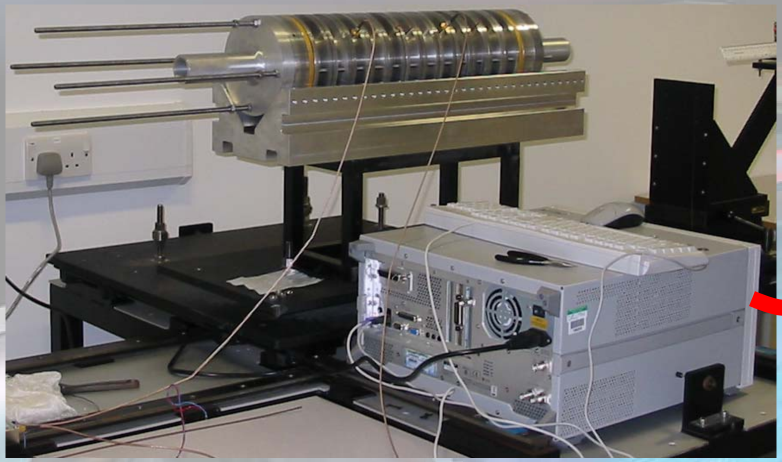
2D scan - single mode

- Scan 2D plane of wire
  - (need correction)
- For each mode
  - follow frequency with wire position
  - find wire position for maximum (or minimum frequency)
  - from here one will find a reference for normalization of wire position (also freq. of unperturbed mode)
  - minimum position will be different for each mode ⇒ information about cell alignment

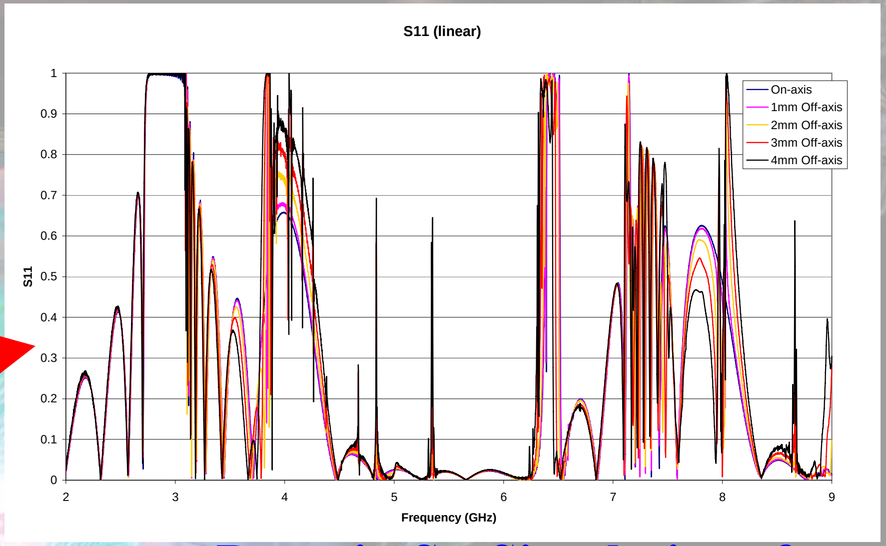




Frequency Domain Wire Measurements of Crab Cavities



Crab Cavity Measurement Set-Up



Frequency Domain S_{11} Simulations for Several Wire Offsets

- Resonances located at dipole modes
- Area under $S_{21} \sim Z_l$ (beam impedance)
- Fourier transform enables wake-field and kick factors to be calculated
- Bench-top measurement allows rapid determination of cavity modes (sync. freqs and kick factors).
- Also allows cavity alignment to be determined.
- Use method to determine modes in main linac cavities (1.3 GHz)