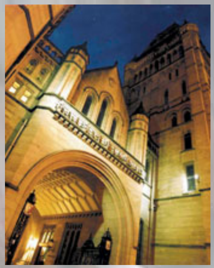
Linear Accelerators: Theory and Practical Applications: WEEK 2

Stanford Linear Accelerator, shown in an aerial digital image. The two roads seen near the accelerator are California Interstate 280 (to the East) and Sand Hill Road (along the Northwest). Image data acquired 2004-02-27 by the United States Geological Survey



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Summary of Week 1

- Linear accelerators embrace:

1. Potential drop accelerators – Cockcroft-Walton, Tandem Van Der Graaf
2. RF linear accelerators – DTL (ion acceleration), RFQ (low energy ion accelerator which has largely replaced those of 1.) and RF electron linear accelerators.

- Explored phase stability of RFQ and means of accelerating and focussing without the need of external magnets. This is an unusual device as both functions are combined in one structure using RF e.m. fields alone.
- Operation of Alvarez DTL indicated that gain from gap to gap grows grows overall the length of the accelerator and, the length of the drift tubes increases in order to remain in synchronism with the accelerated beam. Also, the diameter of the drift tubes reduces as the length increases, in order to maintain the correct accelerating frequency.

Overview of Week 2

- **RF Linear accelerators for electron acceleration. Electrons have a rest mass of ~0.5 MeV and hence they are rapidly relativistic.**
- **The principles of infinite periodic structures will be explored.**
- **Together with dispersion relations for structures consisting of a finite number of cells**
- **The basic concepts of Standing Wave (SW) and Traveling Wave (TW) acceleration are introduced.**
- **Fundamental accelerator physics parameters (shunt impedance, loss factor, etc) are introduced.**
- **Important frequency scaling relations used in designs for shunt impedance, Q and group velocity are developed.**
- **Approximate design formula for shunt impedance of pill-box and periodic structure is discussed**

Introduction

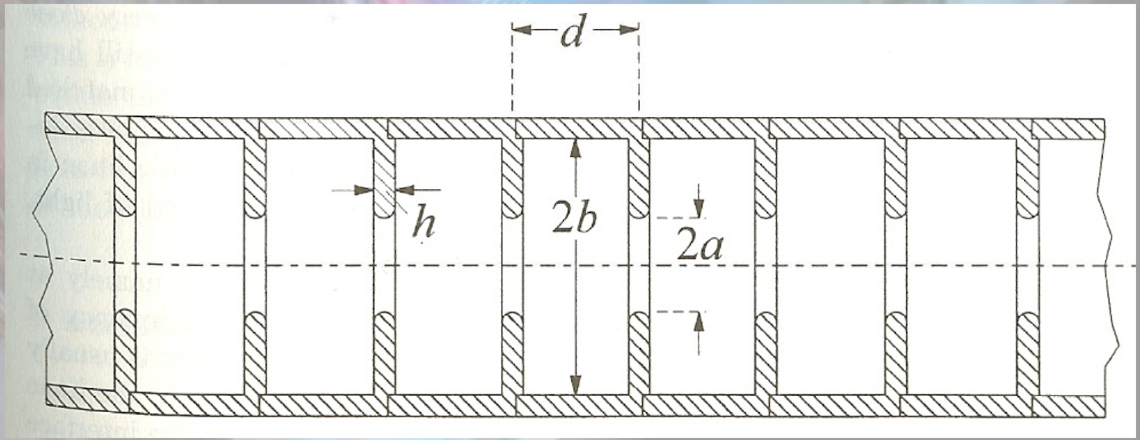
➢RF linear accelerators find application in medical accelerators -X-band accelerator have recently made major in-roads in this area as they are stable and compact. Obviously, a major concern for medical accelerators is field stability –potential parasitic mode excitation and the dangerous miss-steering of the beam can irradiate the patient in an unwanted manner. For this reason coupled cavity linacs operating in the $\pi/2$ mode are preferred.

➢RF linear accelerators find a direct application in electron-positron colliders. For example, the ILC will require ~16,000 superconducting 9-cell cavities, each of which is approximately 1 meter long. This is in order to obtain 500 GeV centre of mass at the collision point (with an intended upgrade path to 1 TeV). The baseline design gradient is 35 MV/m and the frequency of operation is 1.3 GHz.

➢CERN is also developing a room temperature Cu accelerator at 12 GHz with at an accelerating gradient of 100 MV/m. The centre of mass energy reach is 3 TeV

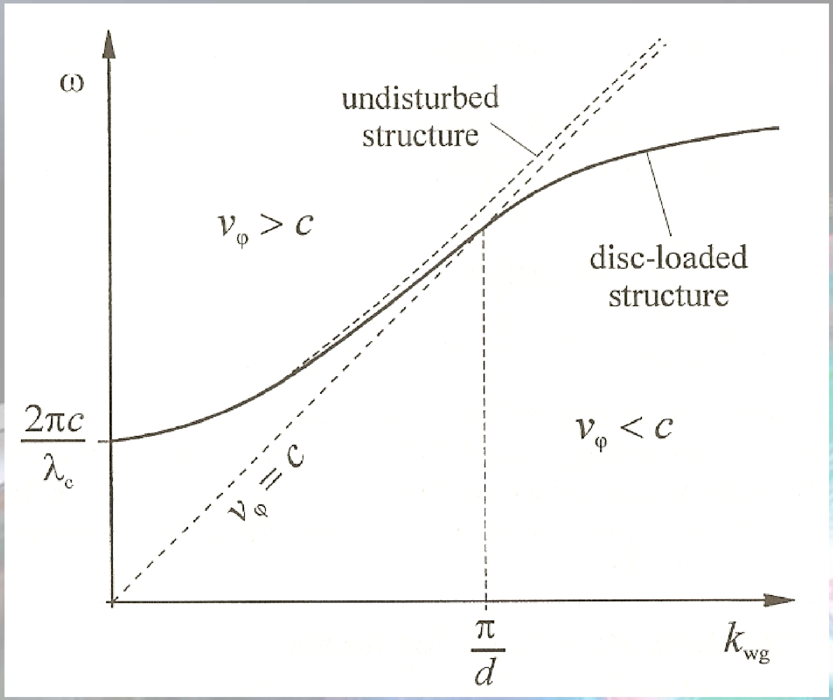
RF Linear Accelerators

- In general, the aim is to transfer energy from the RF wave to electron beam consisting of bunches of charged particles
- If we inject the RF into a waveguide then on average, a electron beam traversing the waveguide gains no energy from the e.m. field.
- Why? Because the phase velocity of the RF wave is larger than the velocity of light and so it runs ahead of the electron beam –the electron beam sees both the accelerating and decelerating part of the RF field and this averages to zero.
- In order to exchange energy, the phase velocity of the RF has to be matched with that of the electron beam. This is achieved by slowing the RF field down in an aptly named "slow wave structure" which consists of the original waveguide loaded down, periodically, with irises. This is also known, for obvious reasons, as a "disk loaded structure" and is illustrated below



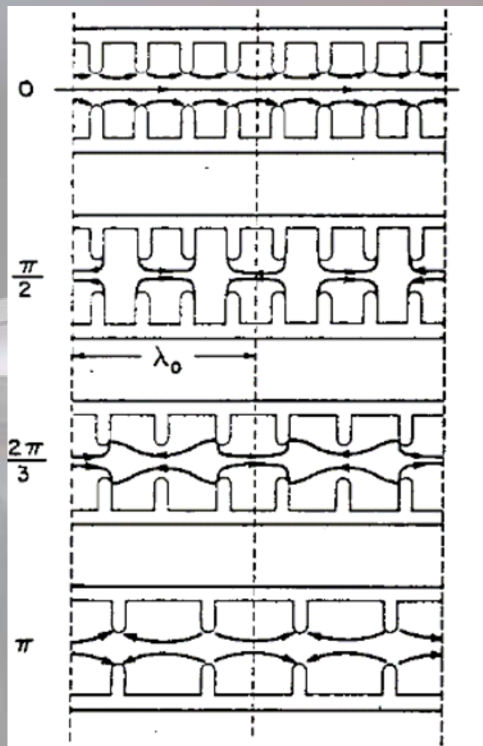
Disk loaded Accelerating Structure Suitable for Electron Acceleration. The characteristic parameters are also indicated -a and b refer to the iris radius and cavity radius, respectively (very common nomenclature)

RF Linear Accelerators



- **The influence of the irises on slowing down the wave is readily seen by referring to the dispersion diagram.**
- **The original smooth waveguide is also illustrated.**
- **The irises form a periodic structure within the cavity, reflecting the wave as it passes through and causing interference.**
- **This process is similar to the interference of light in a diffraction grating**
- **Loss-free propagation in the grating occurs at λ_z = pd, with p=1, 2, 3..**
- **Thus it is clear that $2\pi/p = 2\pi d/\lambda_z$ with p=1, 2 , 3…**
- **Applying this to the disk-loaded linac, we find only certain wavelengths propagate characterised by mode number p.**

RF Linear Accelerators



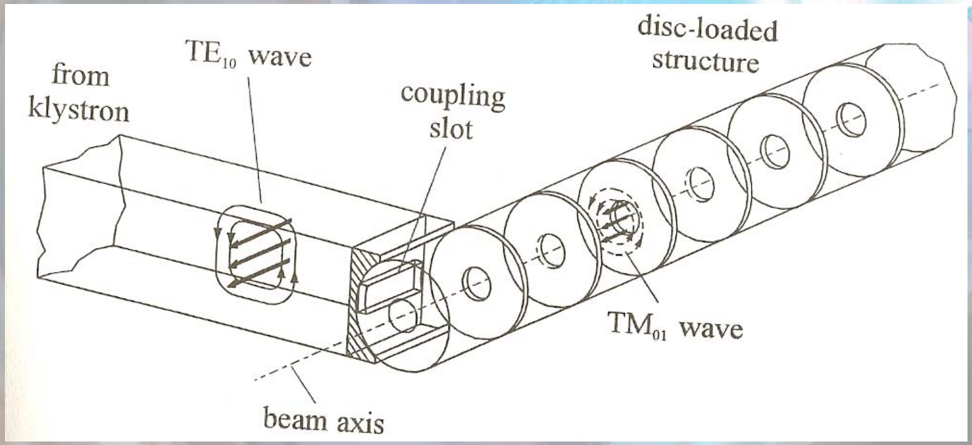
Modal configuration of 4 most used modes

- **One can imagine using any value of p at all.**
- **In practise there are a limited number of modal configurations used and these are illustrated adjacent.**

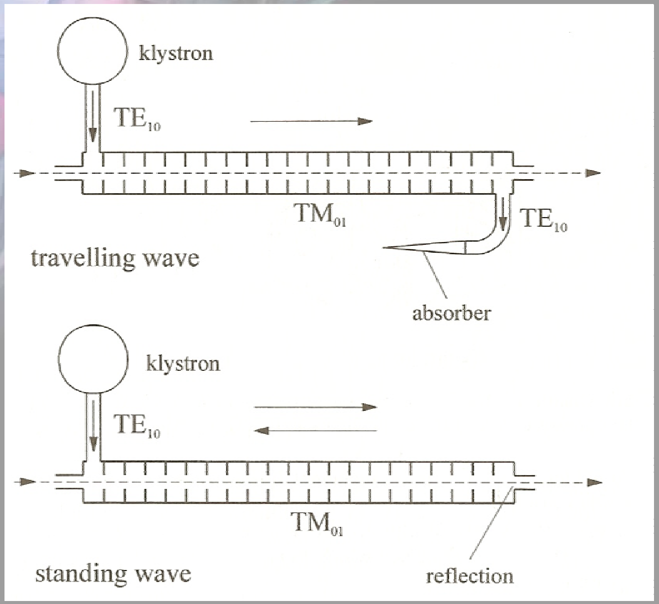
$$k_z d = \begin{cases} \pi & (\text{'}\pi\text{' mode -i.e. } \lambda_z = 2d) \\ 2\pi/3 & (\text{'}2\pi/3\text{' mode -i.e. } \lambda_z = 3d) \\ \pi/2 & (\text{'}\pi/2\text{' mode -i.e. } \lambda_z = 4d) \end{cases}$$

- **The π mode requires a considerable amount of time for the transient oscillations to die away and, it is very sensitive to frequency errors as the neighbouring modes are spaced very close to it. The ILC uses this mode for the superconducting 9-cell Niobium SW cavities (of which there will be 16,000 or more).**

- The $\pi/2$ has a relatively low shunt impedance per unit length but it is relatively stable to the excitation of neighbouring modes.
- This mode is often used in medical accelerators where stability of operation is paramount.
- The $2\pi/3$ mode has a relatively high shunt impedance, reasonable mode separation and shorter settling time than the π mode.
- This is the mode chosen for the operation of the SLAC linac at 2.856 GHz.
- Means of coupling in RF energy is illustrated below.
- In practise the input coupler is carefully designed to minimise high field regions that can lead to electrical breakdown.



Coupling RF Energy into Disk-Loaded Accelerator



Comparison between SW and TW Disk-Loaded Linacs

- The RF field in the disk-loaded linac consists of space harmonics.
- We study these space harmonics by considering a structure consisting of infinite series of irises –periodic boundary conditions are imposed by the disks.
- We obtain a periodic solution of the form:

$$E(r,\theta,z)=e^{-\gamma z}E_1(r,\theta,z)$$

$$H(r,\theta,z)=e^{-\gamma z}H_1(r,\theta,z)$$

where E_1 and H_1 are periodic functions: $E_1(r,\theta,z+d)=E_1(r,\theta,z)$

As one progresses from cell to cell, the field repeats apart from a phase factor, $e^{-\gamma d}$.

As we make a Floquet expansion of the field -the field repeats- then we can make a Fourier series expansion of the field:

$$E_1(r,\theta,z)=\sum_{n=-\infty}^{\infty}E_{1n}(r,\theta)e^{-2jn\pi z/d}.$$

For a loss-less structure (almost true in a superconducting structure) the propogation constant is entirely imaginary: $\gamma=j\beta_0$ and the field becomes: $E(r,\theta,z)=\sum_{n=-\infty}^{\infty}E_{1n}(r,\theta)e^{-j\beta_n z}$, $\beta_n=\beta_0+2n\pi/d$ and β_0 is the propagation constant of the fundamental space harmonic.

$E(r,\theta,z)=e^{-\gamma z}E_1(r,\theta,z)$

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$E(r,\theta,z)=e^{-\gamma z}E_1(r,\theta,z)$

$H(r,\theta,z)=e^{-\gamma z}H_1(r,\theta,z)$

The field must also satisfy the usual transverse boundary conditions.

For the lowest order, monopole mode in a disk-loaded waveguide the field is:

$$E_z = \sum E_{0n} J_0(k_{c,n} r) e^{-j\beta_n z}$$

$$E_r = j\sum \frac{\beta_n}{k_{c,n}} E_{0n} J_1(k_{c,n} r) e^{-j\beta_n z}$$

$$H_\phi = \frac{j}{Z_0}\sum \frac{k_0}{k_{c,n}} E_{0n} J_1(k_{c,n} r) e^{-j\beta_n z}, \text{where:}$$

$$\beta_n^2 = k_0^2 - k_{c,n}^2, \text{ with } k_0 = \omega / c$$

The propagation constant of the fundamental differs from that of the unloaded guide. In fact, one finds that β_0 decreases as the perturbation due to the irises increases. The phase velocity of the space harmonics is given by:

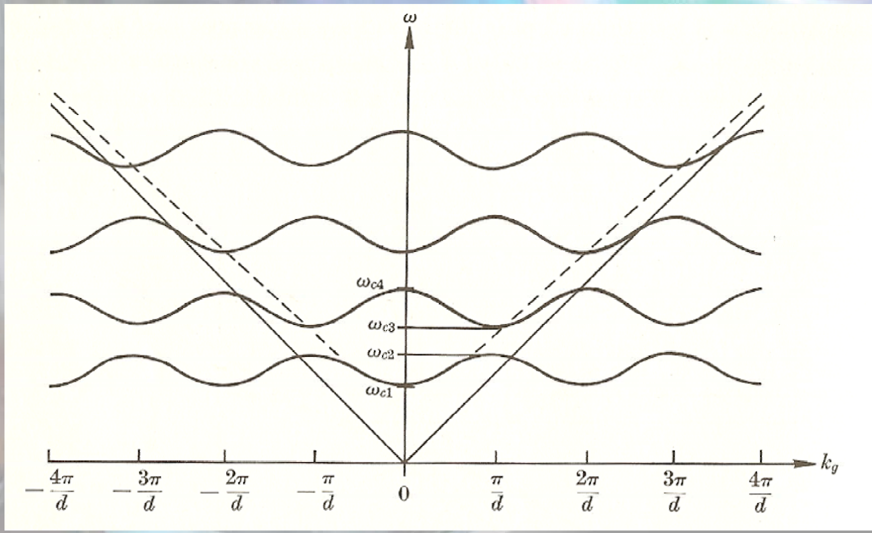
$$v_{pn} = \frac{\omega}{\beta_0 + \dfrac{2\pi n}{d}}$$

and the group velocity is the same for all space harmonics:

$$v_{gn} = \frac{d\omega}{d\beta_n} = \left(\frac{d\beta_n}{d\omega}\right)^{-1} = \left(\frac{d\beta}{d\omega}\right)^{-1} = v_g$$

One can see this by referring to the dispersion diagram.

Thus, for a given frequency, an infinite series of space harmonics are excited. The fundamental space harmonic has the largest Fourier amplitude. Also, for a structure designed to be synchronous with the fundamental space harmonic the integrated effect on the beam is such that all non-synchronous space harmonics average to zero over the length of the cavity



Full dispersion curve (Brillouin diagram) for a loaded waveguide with iris spacing (period) equal to d. For comparison, and indicated with the dotted curve, is the hyperbola corresponding to the uniform waveguide of the same diameter. Multiple pass-bands are illustrated.

- **Thus, one designs the disk-loaded structure to interact with the fudamental space harmonic with a disk to disk period of d=($\zeta\pi$)/(ω/c) with ζ=1 for the 'π' mode and 2/3 for the '2π/3' mode, etc.**
- **Also, at synchronism the light line intersects with the dispersion curves and thus $\beta_0 = \omega/v_b$ (v_b is the velocity of the beam).**
- **The details of the dispersion curves are obtained with finite element or finite difference computer codes such as Superfish, HFSS, MAFIA, Microwave Studio, GdfidL.**
- **However, given a limited number of points on the curves (zero and pi) one can use a circuit model to within remarkably good precision to map out the remaining part of the dispersion curve.**
- **The method outlined for disk-loaded structures applies equally well to more elaborate cavities –such as the superconducting Niobium cells in the ILC which consist of elliptical irises and cavities.**

For a cavity designed to be synchronous with the fundamental space harmonic:

$$\beta_0^2 = \left(\frac{\omega}{c}\right)^2 - \left(\frac{\omega}{c\bar{v}_s}\right)^2 = -\left(\frac{\omega}{\gamma_s \bar{v}_s c}\right)^2$$

$$E_z = E_0 I_0(k_{c,0} r)\cos\phi$$

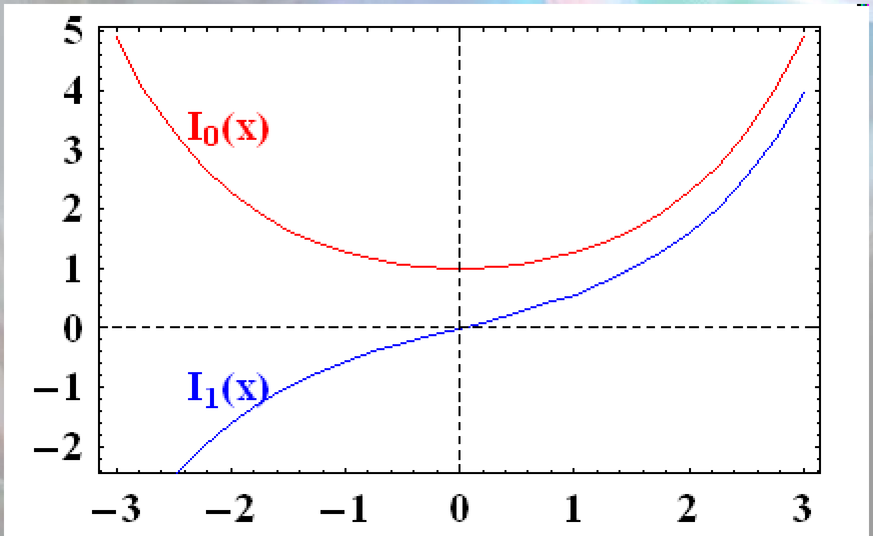
$$E_r = -\gamma_s E_0 I_1(k_{c,0} r)\sin\phi$$

$$B_\phi = -\frac{\gamma_s \bar{v}_s}{c} E_0 I_1(k_{c,0} r)\sin\phi$$

where normalised synchronous velocity is given by $\bar{v}_s = v_s / c$,

$\phi=\omega t$ is the phase of the field relative to the crest of the wave.

We have used modified Bessel functions as $\beta_0^2 < 0$.



The maximum energy gain is related to the coupling of the RF field to the linac cavity and to the shunt impedance of the mode:

$$U=K\sqrt{P_{RF} l r_0}$$

where P_{RF} is the RF power supplied, l the length of the structure, r_0, the shunt impedance per unit length and K a coupling correction factor generally of order K~0.8.

The shunt impedance may be evaluated with a semi-empirical formula to 'reasonable accuracy' (in practise it is an over-estimation):

$$r_0=5.12\text{x}10^8 \frac{\bar{v}_\phi (1-\eta)^2}{p+2.61\bar{v}_\phi (1-\eta)} \left(\frac{\sin D/2}{D/2} \right)^2$$

where $\bar{v}_\phi = v_\phi / c$, $D=2\pi(1-\eta)/p$, p = number of irises per wavelength (mode number), $\eta=h/d$ (h=thickness of iris, d= period = seperation of irises)

➢For example, for the 3km SLAC linac in Stanford, California:

2a = 82.474 mm
2b = 22.606 mm
h = 5.842 mm
d = 35.001 mm

➢And the beam is ultra-relativistic and so the phase velocity is replaced by unity for the 2pi/3 mode (p=3) and we obtain:
$r_0 = 53\ \mathrm{M\Omega/m}$.

➢For 35 MW supplied to the linac, the total accelerating voltage for a structure of length l = 3m is given by: $K(P_{RF} l r_0)^{1/2} = 59.7$ MV

➢These large power are of course supplied in pulse mode and are of duration of a few μs as the heating of the cavity becomes intolerable for longer pulse lengths.

➢The temperature is controlled to within 0.1 degrees –as it is not possible to align cells adequately during operation without this temperature limit.

- **The gradient is predicted to be ~ 19.9 MV/m. In operation 15 –20 MV/m have been obtained.**

- **The NLC achieved 65 MV/m for several room temperature copper structures for 100 ns.**

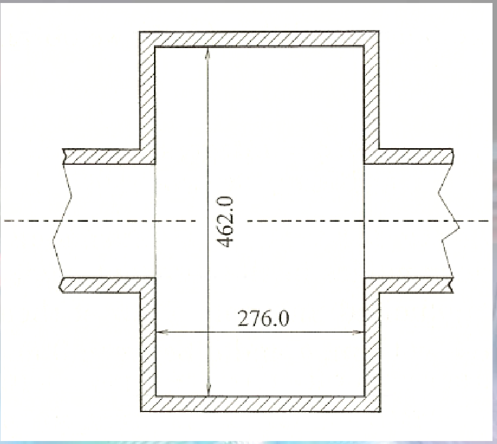
- **Single cell Cu cavities have reached 100 MV/m.**

- **The ILC –using superconducting niobium cavities – has achieved 35 MV/m but the yield is still a significantly low. Reasonable yields have been obtained for superconducting cavities at 15 –20 MV/m and these are in use on the DESY XFEL.**

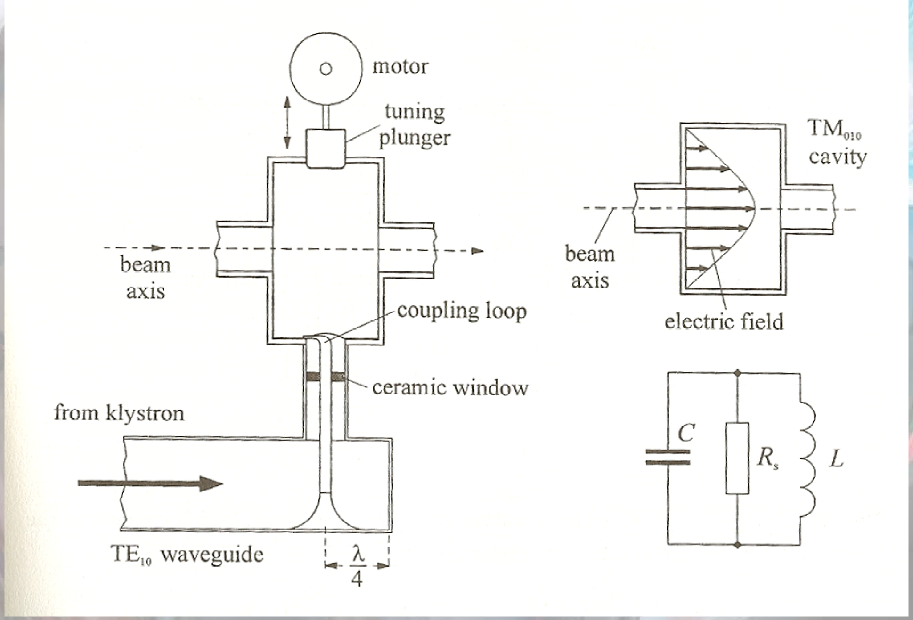
- **Single cell superconducting cavities have reached 50 MV/m. However, to date not one 9-cell ILC cavity has reached this high gradient –there is intense R&D in this area in Cornell University (focusing on a reentrant design), USA and KEK (focussing on a Low Low "Ichiro" design) , Japan, in particular.**

RF Linear Accelerators

- Single cavity structures are also used.
- In damping rings and FFAGS for example.
- Clearly the cost becomes prohibitive when several thousand or more are required.



Single Cell Cavity Resonating at 500 MHz Developed for Storage Ring DORIS



Design of a single cell accelerating cavity using the TM_{010} mode. The frequency is adjusted using the tuning plunger. The resonator is excited using the coupling loop.

- It is advantageous to add as many cells as possible per cavity in order to reduce the number of couplers required.
- There are, of course, practical limitations on the number of cells allowed per cavity:

1. as the number of cells increase the spacing between neighbouring modes decreases
2. The power density requirement increases with the number of cells and hence one may be forced to reduce the number of cells to avoid the surface of the cavity suffering from electrical breakdown. Nonetheless, the ILC had 55 cells with a gradient of 65MV/m in 100ns. The ILC superconducting cavities have adopted a conservative approach as only 9 cells are contained in each cavity –and 9 cavities per module in the present RDR design.

Fundamental RF Linac Parameters

- The limiting quantity in linacs is usually the RF power, either peak power or average power.
- Since the power dissipated per unit length of the structure P'_d is proportional to the square of the RF field, a useful parameter is the <u>shunt impedance per unit length</u> :

$$R' = E_z^2 / P'_d.$$

(Sometimes, R' is defined in terms of r.m.s. rather than peak values, making it a factor two less!)

The shunt impedance is also defined in terms of the stored energy U as:

$$R = \frac{\left| \int_L E_z(z) e^{j\frac{\omega}{c}z} dz \right|^2}{4U}$$

- R' is typically given in M Ω /m. In proton linacs operating at 200 MHz values of 35 MΩ/m are reached, whereas electron linacs at 3 GHz have values around 100 M Ω /m.
- The Q-value is a factor of merit of an RF cavity as a resonator. It is defined as the ratio of the stored energy to the energy dissipated per radian of the RF cycle.

➢The loaded Q_L takes into account also the additional losses due to the coupled external circuits.

➢Since both quantities increase linearly with the number of periods, it is convenient to use both per unit length $Q_0=\omega W'/P'_d$ defines <u>the unloaded Q0</u>.

A parameter which depends on the structure geometry and not losses is the R upon Q: $\frac{R'}{Q} = \frac{E_z^2}{\omega W'}$

It is a measure of how much accelerating field one has for a given energy in a cavity per unit length.

Another important quantity is the group velocity which is the velocity at which signals and energy propagate. To understand this, consider two waves propagating with slightly different frequencies:

$\omega_1 = \omega_m - \Delta\omega, \; \omega_2 = \omega_m + \Delta\omega$

resulting in different phase constants

$$k_1 = k_m - [\left.\frac{\partial k}{\partial \omega}\right|_{\omega_m}]\Delta\omega, \; k_2 = k_m + [\left.\frac{\partial k}{\partial \omega}\right|_{\omega_m}]\Delta\omega$$

Superposition then yields:

$$E_z = E_0 e^{j(\omega t - k_1 z)} + E_0 e^{j(\omega t - k_2 z)}$$

$$= 2E_0 \cos \Delta\omega \left(t - \frac{\partial k}{\partial \omega} z \right) e^{j(\omega_m t - k_m z)}$$

Thus, the high frequency component has a phase velocity

$$v_p = \frac{\omega_m}{k_m}$$

and the beat-signal propagates with a velocity:

$$v_{beat} = \left.\frac{\partial k}{\partial \omega}\right|_{\omega_m} = v_g$$

The group velocity describes the velocity of energy transport:

$$P = W' v_e, \quad v_e = v_g$$

where P is the transported power. For homework, prove $v_e = v_g$!

The group velocity depends intimately on the dimensions of the cavity.

For a disk-loaded periodic structure:

$$v_g / c \sim K(a/b)^4$$

with K a constant depending on the number of irises per wavelength and their thickness.

The group velocity is important for the following three reasons:

1. The filling time -the time to fill a cavity or structure of length l with energy: $T_f = l / v_g$

2. As $P=W'v_g$ and $W' \sim E_{acc}^2$, it is preferable to have a low group velocity in order to maximise the energy density and the accelerating field.

3. R', Q and R'/Q all depend on v_g. As a rule, decreasing v_g increases R' and decreases Q and hence R'/Q is increased.

A wave traveling down the structure is attenuated due to wall losses. The rate of attenuation is obtained from the continuity of power flow:

$$\frac{\partial W'}{\partial t} + \frac{\partial P}{\partial t} + P_d' = 0$$

$$\frac{\partial P}{\partial t} + v_g \frac{\partial P}{\partial t} + \frac{\omega}{Q_0} P = 0$$

and in the steady state, the time derivative is zero:

$$P = P_0 e^{-2\alpha z}, \quad \alpha = \frac{\omega}{2 Q_0 v_g}$$

This can be interpreted in terms of the time required for a field in a cavity to die down to 1/e of its initial value.

In a resonant structure, there will be no variation of energy density with z; thus the stored energy falls according to:

$$\frac{\partial W'}{\partial t}+\frac{\omega}{Q_0}W'=0,\ W'=W_0' e^{-2t/T_0},\ T_0=2Q_s/\omega$$

Thus, the attenuation length l_0, i.e. the length after which the field has decayed to 1/e, and the decay time T_0 are related through

$l_0 = T_0 v_g$.

One of the most important parameters in the design of a linear accelerator is the operating frequency -as other parameter are intimately related to the frequency. However, it is also important to arrive at a design frequency that matches the available power sources -or at least a power source that is potentially on the horizon.

The scaling of parameters with frequency is now discussed. The dissapated power is proportional to the wall current squared times the wall resistance:

$$P_d' = i_w^2 R_w'$$

where $R_w' = 1/(2\pi b_{eff}\sigma\delta)$, $\delta=[2/(\omega\mu\sigma)]^{1/2}$ -δ the skin depth, σ conductivity and b_{eff} the effective cavity radius.

The accelerating field is proportional to the tangential magnetic field at the wall (although with some reshaping of cavities matters are changed a little):

$$E_{acc} \sim H_{tw} \sim i_w / b_{eff}$$

The stored energy and energy transport both scale as:

$$W', P \sim E_{acc}^2 b_{eff}^2$$

Thus, bearing mind $b_{eff} \sim \omega^{-1}$ and putting all components together:

$$\boxed{\begin{array}{l} R' \sim 1/(b_{eff}^2 R_w') \sim \omega^{1/2} \\ Q \sim \omega / R_w' \sim \omega^{-1/2} \\ R'/Q \sim \omega \\ v_g = P/W' \sim \omega^0 \end{array}}$$

For a constant group velocity structure the filling time is also a constant for a given structure length. But to optimise the field, the structure length changes and:

$$T_f \sim \omega^{-3/2}$$

Thus, from the point of view of RF power, the frequency should be as large as possible.

However, we have ignored two further issues:

1. High frequency implies smaller structures and this can lead to RF electrical breakdown
2. The beam excites higher order modes, which constitute a wakefield and these lead to a beam break up instability, at worst, or at the very least a dilution in the beam emittance.

In order to maximise efficiency and prompted by RF breakdown theory, motivated CERN to design a 30 GHz, compact linear collider, known as CLIC. However, intensive experimental work has indicated that the earlier breakdown theory was not adequate and a recent optimised version of CLIC operates at 12 GHz (cf the NLC/JLC design of 11.424 GHz).

The wakefield associated with 2 scales as:

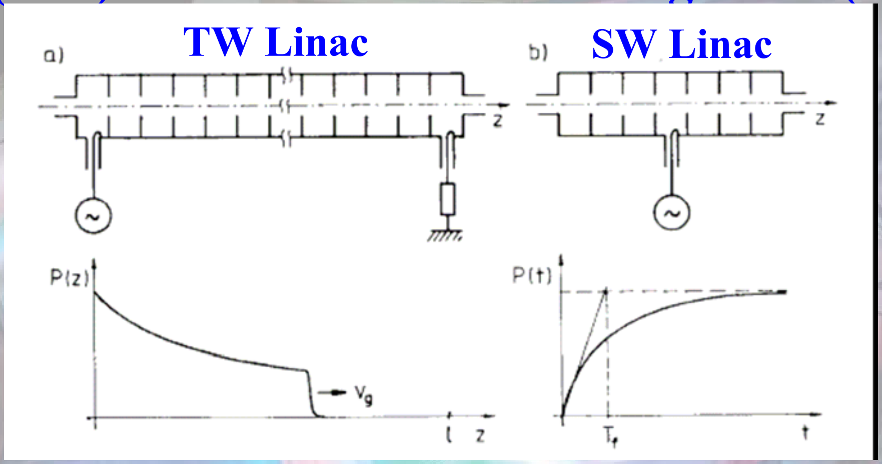
$W_z \sim a^{-2} \sim \omega^2$

$W_t \sim a^{-3} \sim \omega^3$ (and $\partial W_t / \partial z \sim \omega^4$)

This must also be born in mind when designing the linac -the modes must be carefully damped and detuned.

Travelling Wave Linear Accelerators

- Periodic RF structures can be operated in two different ways, as a traveling-wave(TW) accelerator or a standing-wave (SW) accelerator.



- In a TW-structure, the fields build up in space with the wave front traveling with the group velocity. The output of the structure is matched to a load where the left-over energy is dissipated.
- We consider the case where the power transferred to the beam is small compared to the power dissipated in the structure walls –i.e. normal conducting Cu accelerators.
- Two different types of structures are usually distinguished:

1. Constant impedance.
2. Constant gradient.

1. Constant Impedance Linac

In a constant impedance linac the iris diameter remains fixed and the fields decay exponentially down the accelerator with a decay constant α -the fields decay as $W' \sim E^2 \sim \exp(-2\alpha z)$.

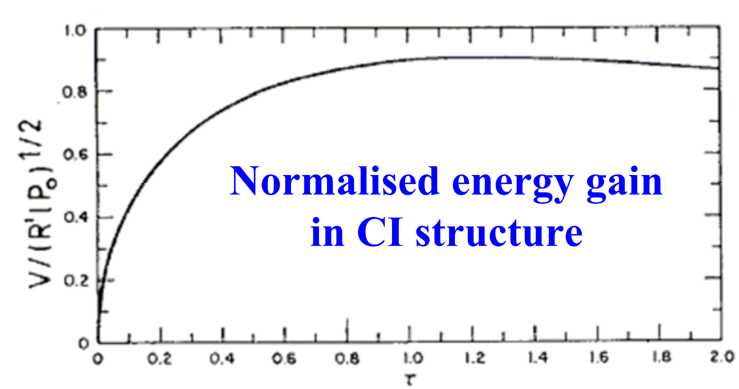
The energy gain for a charge at a constant phase angle ϕ:

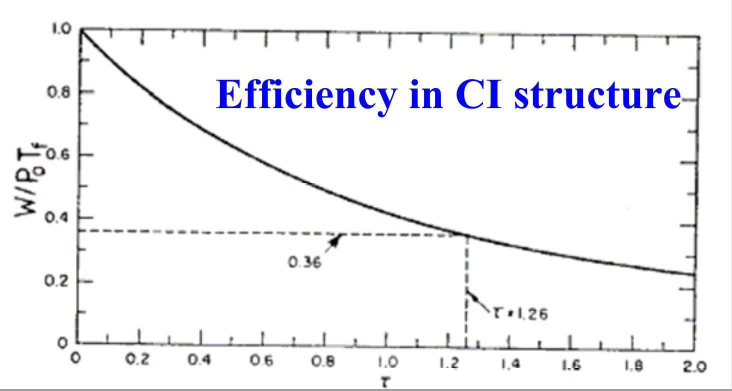
$$V=\int_0^l E(z)dz = E_0\cos\phi\int_0^l e^{-\alpha z}dz = E_0 l\cos\phi\frac{1-e^{-\tau}}{\tau}$$

$$=\frac{1-e^{-\tau}}{\sqrt{\tau}}\sqrt{2R'lP_0}\cos\phi,\ \tau=\alpha l \ \text{ and we have used } E_0^2 = 2R'\alpha P_0.$$

where l is the length of the accelerating structure and P_0, the input power.

The function $V(\tau)$ has a broad maximum at $\tau_{max} = 1.26$. However, for different reasons lower values around 0.8 are preferable with only 3 % decrease of V as compared to V_{max}.





Using the previously derived relations $P=P_0 e^{-2\alpha z}$, $P=W'v_g$ and $T_f = l/v_g$

we derive an expression for the energy stored in the linac after completely filling:

$$W=\int_0^l W'dz=\frac{P_0}{v_g}\int_0^l e^{-2\alpha z}dz=P_0 T_f \frac{1-e^{-2\tau_f}}{2\tau}$$

and $\eta_{st} = W/(P_0\tau_f)$ is referred to as the structure efficiency and is the fraction of energy available for acceleration.

From the curves it is clear that a lower value of τ is prefered.