Inertial Navigation Systems: The Physics behind Personnel Tracking and the *ExacTrak* System

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***Abstract* – Inertial Navigation System (INS) sensors are used in Personnel Tracking. Among these sensors are: accelerometers and gyroscopes each with their capabilities, purposes and physical/theoretical models. This paper focuses on the use of INS sensors for personnel tracking, and specifically, how they work.**

Index Terms – Inertial Navigation System, Microelectromechanical System

INTRODUCTION

Inertial Navigation Systems (INS) are navigation aids that use a computer and motion sensors to continuously track the position, orientation, and velocity of a moving object without the need for external references – or forces. The main sensing mechanics of the INS system within the *ExacTrak* system, developed by P.W.L.S. Innovations, are (1) accelerometer and (2) gyroscope. These two sensors will be the focus of this report.

By tracking both the current angular velocity (gyroscope) and the current linear acceleration (accelerometer) of the system measured relative to the moving system, it is possible to determine the linear acceleration of the system in its inertial reference frame.

THE PHYSICS OF MOTION

The science of mechanics seeks to provide precise and consistent descriptions of the dynamics of systems, that is, a set of physical laws mathematically describes the motions of bodies. For this, we need to define fundamental concepts such as distance and time. Combining these concepts allows us to ultimately define **velocity** and **acceleration**.

It is held to be true that Newtonian Laws of Motion affect all systems, namely that:

(1) A body at rest remains at rest unless acted upon,
(2) A body acted upon by a force moves in such a matter that the time rate of change of momentum equals force (F = ma).
(3) If two bodies exert forces on each other, these forces are equal in magnitude and opposite in direction (equal and opposite reaction).[1]

These laws are the basis of physics – the basis for all motion for that matter, and they come in handy when integrating acceleration to find distance, for instance.

Inertia

Inertia is the tendency of all objects to resist a change in motion. It is directly proportional to an object's mass, so the heavier the object is, the more inertia it has, and would remain in motion forever if it was in a frictionless environment (Newton's Laws).

Utilizing this understanding, we can now move to the concept of reference frame, or *inertial frame*. Newton realized that for the laws of motion to have meaning, the motion of bodies must be measured relative some reference frame. If Newton's laws are valid in this frame, it is referred to as an *inertial frame*. Further, and finally, if Newton's laws are valid in one reference frame, then they are valid in any other frame in uniform motion.

Velocity

Velocity is the time rate of change of position, and is important because of the following relationship:

$$v = \frac{dx}{dt}\,[{}^{m}/_{s}]$$

Acceleration

Acceleration is the time rate of change of velocity, and is important because of the following relationship:

$$a = \frac{dv}{dt}\,[{}^{m}/_{s^2}]$$

THE ACCELEROMETER

Accelerometers measure the linear acceleration of a system in the inertial reference frame, but in directions that can only be measured relative to the moving system, since the accelerometers are fixed to the system and rotate with the system, but are not aware of their own orientation.

Principle of Operation

An accelerometer consists of two surface micromachined capacitive sensing cells (g-cell) and a signal conditioning Application-specific IC (ASIC) contained in a single package. The g-cell is a mechanical structure formed from semiconductor materials, and can be modeled as a set of beams attached to a movable central mass that move between fixed beams.

The movable beams can be deflected from their rest position by subjecting the system to acceleration, as seen in Figure 1.

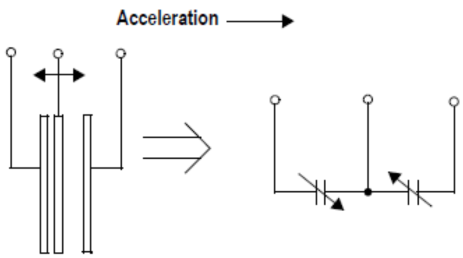

FIGURE 1
A CAPACITIVE ACCELEROMETER

As the beams attached to the central mass move, the distance from them to the fixed beams on one side will increase by the same amount that the distance to the fixed beams on the other side decreases. The change in distance is a measure of acceleration. The g-cell beams form two back-to-back capacitors. As the center beam moves with acceleration, the distance between the beams changes and each capacitor's value will change, ($C = A\varepsilon/D$). Where A is the area of the beam, ε is the dielectric constant, and D is the distance between the beams. The ASIC uses switched capacitor techniques to measure the g-cell capacitors and extract the acceleration data from the difference between the two capacitors. The ASIC also signal conditions and filters (switched capacitor) the signal, providing a high level output voltage that is ratiometric (scales linearly with supply voltage) and proportional to acceleration.

An accelerometer measures the acceleration and gravity it experiences. Acceleration is the rate of change velocity, and velocity is the rate of change of the position, thus:

$$\int a = v \text{ and} \int v = x$$

Further, defining an integral as the area under a curve, where the integration is the sum of small areas whose width is near zero, we see that the sum of the integration represents the magnitude of a physical variable [f(x)]. Taking for example, Figure 2, a sampled accelerometer's signal similar to a sine wave,

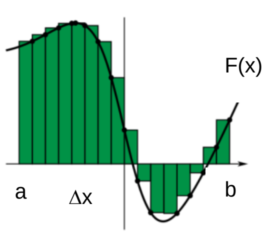

FIGURE 2
EXAMPLE SINE WAVE, F(X)

are the following equations, interpreting the graph as integration:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$\text{Where: } \Delta x = \frac{b-a}{n}.$$

By taking the previous concept, we can now deduce that sampling a signal gets us instant values of its magnitude, so small areas can be created between two samples, specifically, sampling time represents the base of this area, and the sampled value represents the height.

The effects of gravity and acceleration are indistinguishable (following Einstein's equivalence principle), so as a consequence, the output of an accelerometer has an offset due to gravity. This means that an accelerometer at rest on the earth's surface will actually indicate 1 g along the vertical axis. To obtain the acceleration due to motion alone, this offset must be subtracted.[2]

Calibration

Even though acceleration can be positive or negative, samples are always positive, therefore, an offset adjustment must be done - in other words, a reference is needed. This function is defined as the calibration routine. Calibration is performed on the accelerometer when there is a no movement condition, and the output or offset obtained is considered the zero point reference. Values lower than the reference represent negative values (deceleration) while greater values represent positive values (acceleration).

The accelerometer output varies from 0V to V_{dd} and is typically interpreted by an analog to by an A/D. The zero value is near $V_{dd}/2$. The calibration value obtained will be affected by the board's orientation and the static acceleration (earth's gravity) component in each of the axis. If the module is perfectly parallel to the earth's surface, the calibration value should be very close to $V_{dd}/2$. From the sampled signal minus the zero reference we obtain true sampled acceleration. A_1 represents a positive acceleration. A_2 represents a negative acceleration. If we considered this data as sampled data, the signal should be similar to the Figure 3.[2]

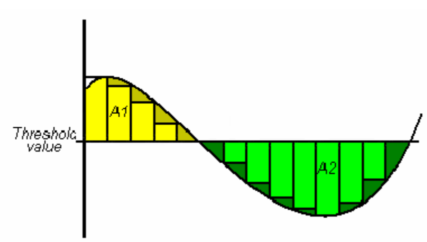

FIGURE 3
SAMPLED DATA, A_1 & A_2

The Gyroscope

Gyroscopes measure the angular velocity of the system in its inertial reference frame. By using the original orientation of the system in the inertial reference frame as the initial condition and integrating the angular velocity, the system's current orientation is known at all times.

Principle of Operation

A gyroscope consists of two sensor elements with vibrating dual-mass bulk silicon configurations that sense the rate of rotation about the X and Y axis:

$$\tau = \frac{dL}{dt} = \frac{d(I\omega)}{dt} = I\alpha,$$

where the vectors τ and L are, respectively, the torque on the gyroscope and its angular momentum, I is its moment of inertia, the vector ω is its angular velocity, and the vector α is its angular acceleration.

It follows from this that a torque τ applied perpendicular to the axis of rotation, and therefore perpendicular to L, results in a rotation about an axis perpendicular to both t and L. This motion is called *precession*. The angular velocity of precession Ω_p is given by:

$$\tau = \Omega_p \times L$$

Making Sense of the Raw Data

Accelerometer Data

This section cover (in detail) how acceleration data can be converted – via some integration – into distance (with some error, which Kalman Filtering will take care of).

Starting with the definition of instantaneous acceleration, $a = dv/dt$, which we rewrite as

$$dv = a\,dt,$$

we take the definite integral of both sides:

$$\int_{v=v_0}^{v} dv = \int_{t=0}^{t} a\,dt$$

giving,

$$v - v_0 = at.$$

Next, with the definition of instantaneous velocity, $v = ds/dt$, which we rewrite as

$$ds = v\,dt,$$

again, we take the definite integral of both sides, and sub in for v_0.

$$\int_{s=s_0}^{s} ds = \int_{t=0}^{t} (v_0 + at)\,dt$$

giving,

$$s - s_0 = v_0 t + \frac{1}{2}at^2.$$

This double integration yields the Mechanical Physics Basic Kinematic Equations:

$$v - v_0 = at$$

$$s = s_0 + v_0 t + \frac{1}{2}at^2.$$

From here, and knowing that our accelerometer reads only changes in acceleration, we look for position (x) in terms of only x and a:

$$s = s_0 \cancelto{0}{+ v_0 t} + \frac{1}{2}at^2$$

yields:

$$s = s_0 + \frac{1}{2}at^2.$$

Finally, since we are working with ever-changing accelerations, we refer to current samples of acceleration with the constant, "K," and modify our Kinematic Equations:

$$\boxed{v(K) = v(K-1) + a(K)t}$$

$$\boxed{s(K) = s(K-1) + \frac{1}{2}a(K)t^2}.$$

Accelerometer Error

An important thing to note about getting position from an accelerometer is that the error in position "integrates," meaning that if the noise or error in the accelerometer follows a normal distribution (overestimates and underestimates equally) then the position estimate should be reasonable. If however, the accelerometer is biased (tends to overestimate more than underestimate, or vice versa) then the error in your position estimate will grow exponentially. On top of this, ANY error is kept in your calculation through the iterative integration, so calculating position the accelerometer can have large errors. There are several error sources that cause an accelerometer output to deviate from its correct value. They are configuration (or misalignment) errors and the accelerometer errors embedded in the device itself. The configuration errors of an accelerometer are the *location* and *orientation* errors of the accelerometer. The error sources of a MEMS accelerometer are: *scale factor error*, *bias*, and *noise*.

How do you fix the error associated with integrating?

One way to eke out better information from accelerometers is to use a complicated form of time dependent probability theory. This is known as Kalman Filtering. Kalman Filtering is commonly used in the navigation systems of airplanes, where knowing the location accurately, and precisely if possible, is important.

Gyroscope Data

This section covers how gyroscopic data can be converted – via some integration – into angular attitude, or orientation (with some error, which Kalman Filtering will take care of).

Starting with the definition of instantaneous velocity, when we take the time rate of change of distance, we find velocity:

$$dx = v_x,$$

with x being the position on the x-axis and v_x being the velocity along the x-axis. The same definition holds for anglular motion. While velocity is the speed at which the position changes, angular velocity, ω, is nothing more than the rate at which the angle is changing, so

$$\partial\omega = \measuredangle rate = gyro\ output,$$

Finally, knowing that the inverse of a derivative is an integral, we alter our equalities into:

$$\int \measuredangle rate = \int gyro\ output = \Delta\omega,$$

In other words, integrating the gyroscope data, gives us our attitude angle, and since data from gyroscopes measure changes in degree of rotation as proportionally conditioned changes in voltage:

$$\Delta\omega \propto \Delta V.$$

So with that knowledge, individual gyroscopes can be characterized simply by collecting ω vs. V data.

KALMAN FILTERING

Kalman Filtering will be defined and discussed in later documents.

REFERENCES

[1] Thornton, Stephen T., Marion, Jerry B., *Classical Dynamics of Particles and Systems*, 2004.

[2] Freescale Semiconductor, *Application Note AN3397*, 2007, <http://www.freescale.com/files/sensors/doc/app_note/AN3397.pdf>.